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A 1D quasilinear equation describing the current drive excitation by helicons in a tokamak plasma

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A quasilinear equation which allows describing evolution of the electron distribution function and generation of non–inductive currents by helicons is obtained. It is shown that in the analyzed case the Fokker–Planck equation can be approximated by a one-dimensional equation in the longitudinal electron velocity space with a diffusion coefficient proportional to the helicon power absorbed by electrons due to Landau damping.

Keywords: tokamak, current drive, helicon, quasilinear diffusion

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At present, a topical issue is developing efficient methods for stationary non–inductive generation of current drive in the tokamak plasma with thermonuclear parameters. The possibility of maintaining the current with the aid of a fast wave with a frequency much lower than the lower hybrid resonance frequency but significantly higher than the ion cyclotron resonance frequency [1] is recognized as most promising. The use of this wave, i.e., helicon, will allow reduction of the influence of linear interaction with ions and of nonlinear (parametric) effects. To describe the current generation by helicons, it is necessary to analyze the quasilinear evolution of the electron distribution function due to resonance interaction with these waves. The last task (the problem of wave–particle interaction) is rather vast and often gets raised in the plasma physics and astrophysics [2]. In particular cases of current generation and interaction between the pumping wave and alpha-particles during the low hybrid heating [3] and current generation by electron cyclotron waves [4], the general partial differential equation in velocity projections, which describes the quasilinear diffusion, can be reduced to a one–dimensional (1D) equation. In the case of the slow longitudinal wave propagation, the diffusion coefficient involved in this equation is proportional to the power lost on the magnetic surface due to the resonance interaction with waves whose longitudinal phase velocity coincides with the longitudinal velocity of particles [5]. This fact significantly simplifies the calculation of the generated current density profile because in this case there is no need in information on the structure of the tokamak plasma high–frequency fields and it is possible to restrict ourselves to using the ray tracing method [6] in analyzing the wave part of the problem. Unfortunately, so far there is no justification for this approach for the case of intermediate–frequency fast wave (helicon) which is necessary for applying efficient program codes [6] developed for describing the current generation in the tokamak plasma by slow intermediate–frequency waves in planning experiments with helicons. In this work we will fill this gap and obtain an appropriate 1D kinetic equation.

Consider a package of electromagnetic waves with intermediate frequencies $\omega_{ci} \ll \omega < \omega_{LH} \ll |\omega_{ce}|$ ($\omega_{LH} = \omega_{pi}/\sqrt{1+\omega_{pe}^2/\omega_{ce}^2}$ is the low–hybrid frequency, $\omega_{pi,pe}$, $\omega_{ci,ce}$ are the ion and electron plasma and cyclotron frequencies) which propagates at an angle to the external magnetic field $\mathbf{B} = Be_z$ in homogeneous plasma:

$$\mathbf{E}_0(\mathbf{r}) = \int_{-\infty}^{\infty} \frac{dk_z}{4\pi}\mathbf{A}(k_z)\exp(ik_x(k_z)x + ik_z z - i\omega t) + c.c., \quad (1)$$

where $\mathbf{A} = \mathbf{e}_G A_0$, A_0 is the amplitude,

$$\mathbf{e}_G = \left(1, -ig/(n_z^2 + n_x^2 - \varepsilon), n_x n_z/(n_x^2 - \eta)\right)$$

are the polarization vector components,

$$\varepsilon = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} < 0, \ \eta = -\frac{\omega_{pe}^2}{\omega^2} < 0$$

are the diagonal elements of the „cold“ plasma dielectric permittivity tensor, while $g = \frac{\omega_{pe}^2}{\omega_{ce}\omega} \gg 1$ is the off–diagonal tensor element,

$$n_x = ck_x/\omega = \sqrt{\left(g^2/(n_z^2 - \varepsilon)\right) - (n_z^2 - \varepsilon)} \approx g/n_z,$$

$$n_z = k_z c/\omega$$

are the refractive index elements. Then consider the kinetic equation for homogeneous magnetized plasma which describes the electron distribution function [2]:

$$\left(\frac{\partial}{\partial t} + v_i\frac{\partial}{\partial x_i} - \frac{|e|}{m_e}(E_i + e_{ijk}v_j B_k)\frac{\partial}{\partial v_i} - \omega_c e_{ijz} v_j \frac{\partial}{\partial v_i}\right) f_e = \mathrm{St}(f_e), \quad (2)$$

where e_{ijz} is the fully antisymmetric unity tensor, $\mathrm{St}(f_e)$ is the Landau collision integral. Let us try to find the equation (2) solution as $f_e = nf_0 + f^{(1)}$, where n is the plasma density, f_0 is the quasi–equilibrium distribution function independent of the particle rotation gyroangle, $f^{(1)} = (2\pi)^{-1} \int_{-\infty}^{\infty} dk_z f^{(1)}(k_z)$ is its linear correction. Substitute this expansion into (2) and obtain equations for the partial linear correction to the distribution function whose frequency and wave number are prescribed by the wave field:

$$\left(-i\alpha + i\lambda\cos\theta + \frac{\partial}{\partial\theta}\right) f^{(1)}(k_z) - \frac{n|e|}{2m_e\omega_c} \times \left(\mathbf{A}(k_z) + \frac{\mathbf{v}\times\big(\mathbf{k}\times\mathbf{A}(k_z)\big)}{\omega}\right)\frac{\partial f_0}{\partial \mathbf{v}} = 0, \quad (3)$$

where $\alpha = (\omega - k_z v_z)/\omega_c$, $\lambda = k_x v_\perp/\omega_c$, $\omega_c = |\omega_{ce}|$, θ is the azimuthal angle in the cylindrical frame of references $\big(v_\perp, \theta, v_z\big)$ in the space of velocities.

Integrating equation (3) and using relation $\exp\big(i\lambda\sin\theta\big) = \Sigma_p J_p(\lambda)\exp(ip\theta)$, find the linear correction to the electron distribution function

$$f^{(1)} = \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} f^{(1)}(k_z) = i\frac{n|e|}{2m_e\omega_c} \times \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \sum_p \frac{\exp(ip\theta - i\lambda\sin\theta)}{\alpha - p}\mathbf{a}_p(k_z)\cdot\mathbf{A}(k_z), \quad (4)$$

where components $\mathbf{a}_p$ are

$$\big(a_{xp}, a_{yp}\big) = \big(J_p^+(\lambda), -iJ_p^-(\lambda)\big) \times \left(\left(1 - \frac{k_z v_z}{\omega}\right)\frac{\partial}{\partial v_\perp} + \frac{k_z v_\perp}{\omega}\frac{\partial}{\partial v_z}\right) f_0,$$

$$a_{zp} = \left(J_p(\lambda)\frac{\partial}{\partial v_z} + \frac{pJ_p(\lambda)}{\lambda}\left(\frac{k_x v_z}{\omega}\frac{\partial}{\partial v_\perp} - \frac{k_x v_\perp}{\omega}\frac{\partial}{\partial v_z}\right)\right) f_0,$$

$$J_n^+(\lambda) = nJ_n(\lambda)/\lambda, \quad J_n^-(\lambda) = J_n'(\lambda).$$

To take into account the Landau resonance interaction with electrons, it is sufficient to retain the zero term $p = 0$ in the infinite series in numbers of electron cyclotron harmonics in (4):

$$f^{(1)} = i\frac{n|e|}{2m_e\omega_c} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \frac{\exp\big(-i\lambda\sin\theta\big)}{\alpha}\mathbf{b}(k_z)\cdot\mathbf{A}(k_z)\frac{\partial}{\partial v_z} f_0, \quad (5)$$

where vector $\mathbf{b}$ has components

$$\mathbf{b} = [0, iJ_1(\lambda)k_z v_\perp/\omega, J_0(\lambda)]_{\omega = k_z v_z},$$

in which only resonance terms at $\omega = k_z v_z$ are taken into account. Substitute (5) into (2):

$$\left(\frac{\partial}{\partial t} + v_i\frac{\partial}{\partial x_i} - \omega_c e_{ijz} v_j \frac{\partial}{\partial v_i}\right) n f_0 - \frac{|e|}{2m_e} \times \int_{-\infty}^{\infty} \frac{dk_z}{2\pi}\left(\mathbf{A}(k_z) + \frac{\mathbf{v}\times(\mathbf{k}\times\mathbf{A}(k_z))}{\omega}\right)\frac{\partial f^{(1)}(k_z)^*}{\partial\mathbf{v}} - \frac{|e|}{2m_e}\int_{-\infty}^{\infty}\frac{dk_z}{2\pi}\left(\mathbf{A}(k_z)^* + \frac{\mathbf{v}\times\big(\mathbf{k}\times\mathbf{A}(k_z)^*\big)}{\omega}\right)\frac{\partial f^{(1)}(k_z)}{\partial\mathbf{v}} = \mathrm{St}(nf_0). \quad (6)$$

Let us average the left and right parts of (6) over the azimuthal angle, use the Sokhotsky formula $(\omega - k_z v_z)^{-1} = P(\omega - k_z v_z)^{-1} - i\pi\delta(\omega - k_z v_z)$ where $\delta(\ldots)$ is the delta–function, average over the random phase $\langle\big(\mathbf{b}\cdot\mathbf{A}(k_z)\big)^*\big(\mathbf{b}\cdot\mathbf{A}(k_z')\big)\rangle = 2\pi|\mathbf{b}\cdot\mathbf{A}(k_z)|^2$ and then obtain the equation for the quasilinear diffusion of the electron distribution function due to their interaction with an electromagnetic wave (helicons) in the presence of collisions.

$$\frac{\partial}{\partial t} f_0 - \frac{\omega_{pe}^2}{16\pi n m_e}\frac{\partial}{\partial v_z}\left(\int_{-\infty}^{\infty}\frac{dk_z}{2\pi}|\mathbf{b}\cdot\mathbf{A}(k_z)|^2 \times \delta(\omega - k_z v_z)\right)\frac{\partial}{\partial v_z} f_0 = \mathrm{St}(f_0). \quad (7)$$

One can see that in the case of electron interactions with an intermediate–frequency wave this is a 1D equation in longitudinal particle velocities, in which the diffusion coefficient depends on the transverse velocity. Nevertheless, let us assume that the distribution function gets factored with retaining the Maxwell's character with respect to transverse velocities, i.e., $f_0 = f_M(v_\perp) f_{0z}(v_z)$, which corresponds to the case of a strong diffusion causing formation of a „plateau“. Let us apply operation $\int_0^{\infty} \ldots v_\perp dv_\perp$ to equation (7):

$$\frac{\partial}{\partial t} f_{0z} - \frac{\partial}{\partial v_z}\int_{-\infty}^{\infty}\frac{dk_z}{2\pi} D_{zz}(k_z)\delta(v_z - v_f)v_f\frac{\partial}{\partial v_z} f_{0z} = \mathrm{St}(f_{0z}), \quad (8)$$

$$D_{zz}(k_z) = \frac{\omega_{pe}^2}{4nm_e v_{te}^2\omega}\int_0^{\infty}|\mathbf{b}\cdot\mathbf{A}(k_z)|^2\exp\left(-\frac{v_\perp^2}{v_{te}^2}\right)v_\perp dv_\perp. \quad (9)$$

To clarify the physical sense of the obtained diffusion coefficient, consider specific loss of a wave beam (1) in plasma due to the Landau resonance, which is $Q = (2\pi)^{-1}\int_{-\infty}^{\infty} dk_z Q(k_z)$, where $Q(k_z) = \langle A_m^*(k_z) j_m\rangle/(8\pi)$ is the partial contribution of the field component k_z, $\mathbf{j} = -|e| \int \mathbf{v} f^{(1)} d\mathbf{v}$ is the electron current density in the wave field. As a result, obtain

$$Q(k_z) = \frac{\omega_{pe}^2 \omega}{4 v_{te}^2 k_z^2} \frac{\partial f_{0z}}{\partial v_z}\bigg|_{\omega/k_z} \int_0^\infty |\mathbf{b} \cdot \mathbf{A}(k_z)|^2 \exp\left(-\frac{v_\perp^2}{v_{te}^2}\right) v_\perp dv_\perp$$

$$= \frac{\varepsilon''_{yy} |A_y|^2 + \varepsilon''_{yz} \mathrm{Im}(A_y^* A_z) + \varepsilon''_{zz} |A_z|^2}{8\pi}. \quad (10)$$

The last term of the right−hand part of (10) describes the Landau damping, the first describes the magnetic pumping, the second is the interference term [7]. Compare (10) with (9) and notice that partial diffusion coefficient (9) in equation (8) is proportional to the partial contribution $\mathbf{A}(k_z)$ to the specific loss:

$$D_{zz}(k_z) = Q(k_z) \Bigg/ \left| n m_e \frac{\omega^2}{k_z^2} \frac{\partial f_{0z}}{\partial v_z} \right|_{w/k_z}. \quad (11)$$

Thus, in the considered case of helicon absorption, the partial diffusion coefficient in the velocity space may be found by analyzing partial component $Q(k_z)$ of the wave beam energy release similarly to the slow−mode case, and does not need calculation of the electric field spatial distributions. This fact allowed us to restrict ourselves to consider only ray traces behavior and energy absorption along them in analyzing the efficiency of non−inductive current generation and its profile.

In this work, electron interaction with intermediate−frequency electromagnetic waves (helicons) according to the Landau resonance mechanism has been considered. As a result, we have derived a quasilinear equation that allows description of the electron distribution function evolution and current drive excitation during the plasma heating with helicons. It has been shown for the first time that, if in the case under consideration the distribution function gets factored with retaining the Maxwell's form with respect to transverse velocities, then the quasilinear equation may be reduced to a one−dimensional one in the space of longitudinal electron velocities. The diffusion coefficient of this 1D equation (11) is proportional to the specific power absorbed during the particle−wave interaction (10).

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Conflict of interests

The authors declare that they have no conflict of interests.

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