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Generation of surface liquid flow in channels by capillary vibrations and waves

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The generation of a directed flow on the water surface in channels with sources and resonators of capillary oscillations is detected and investigated. The surface flow is caused by the movement of the liquid through the gaps between the resonators, as well as between the resonator and the channel walls, under a curved surface that is locally deformed by the sources of capillary vibrations, the transfer of energy of the locally curved surface of the liquid by capillary waves, and the transmission of wave momentum to the particles of the liquid surface in one direction. It is shown that capillary waves together with the energy transfer an excess surface, the flux density of which is equal to the flux of the surface deformation. Moving devices with a capillary-wave accelerator of the surface liquid flow are demonstrated.

Keywords: capillary oscillations and waves, surface liquid flow, channel, flux of surface deformation.

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Introduction

Capillary waves on the liquid surface are excited due to inertial oscillations of the liquid together with the vessel, as well as oscillations of various bodies on the liquid surface. A simple method for excitation of capillary waves is the use of rods and plates vibrations whereof are excited by a piezoelectric converter and transmitted to the liquid. Vibrating rods and plates in contact with the free liquid surface excite capillary waves with a distributed amplitude, which, in their turn, generate jet and eddy currents on the liquid surface. Simultaneous generation of several eddy currents on a liquid surface is observed in case of interaction between the liquid and a horizontally placed vibrating plate with free edges, since nodes of flexural oscillations in such a plate are located along its edges and such nodes give rise to sources of capillary oscillations and waves on the free liquid surface [1]. The pattern of eddy currents on a liquid surface, emanating from each node of plate edge oscillations can be identical, despite the fact that oscillations in adjacent nodes on the plate edges occur in the opposite phase. Excitation of two-dimensional capillary waves on a liquid surface by vibrating elastic bodies, in particular, in the form of Faraday ripples, and the formation of surface flows are threshold, which is typical for parametric excitation of non-linear capillary waves of a finite amplitude [2]. Capillary oscillations and waves at sufficiently large amplitudes of source ventilation can manifest nonlinearity related to a local deformation of the free liquid surface near the source; manifestation of effective vibrational surface tension and Marangoni vibrational convection are possible [3].

A local deformation of the free liquid surface is also possible in case of a short-term interaction with individual particles in the form of liquid drops of a different nature. The studies of high-rate processes of submersion of a free-falling water drop into stationary water by high-resolution photography and video recording [4] have provides the images of capillary waves and microcurrents both on the surface of drop, submerging into a liquid, and on the surface of the receiving liquid. The studies of the drop splash [5] also demonstrated the generation of capillary waves on the surface of submerging liquid drops and origination of currents where the drop substance assembled into thin line structures called ligaments. Ligaments can also form in a two-layer flow of viscous liquids flowing in relation to each other. Viscous waves and ligaments in the form of liquid interlayers, carried by them, are found on the phase interface of liquids moving in a microchannel with a different volume flux [6]. Another method for local excitation of capillary waves and currents on the free liquid surface is exposure of a liquid surface to laser pulses. The authors of [7] carried out a number of experiments for laser excitation of Marangoni convection and capillary waves on the ferromagnetic liquid surface under the action of 1 W focused laser radiation. It was shown that the thermal action of continuous radiation causes convection with protrusion formation on the surface of even a deep liquid, while the action of modulated radiation causes generation of capillary waves. The use of a laser makes it possible to obtain Marangoni thermocapillary convection in separate liquid drops as well, as it was done in [8] with irradiation of a drop, eddy currents wherein were visualized by flows of carbon nanotubes. By studying the dynamics of a viscous liquid layer in the form of silicon oil on the surface of a vibrating plate with free edges, excited by apiezoelectric converter, we have obtained the phenomena of liquid layer motion along the vibrating plate surface and its surface areas with the formation of individual drops [9]. Strongly marked generation of capillary waves and vibrational convection with the formation of eddy currents that evolve into a return trickle manifest themselves in oil drops on areas of the plate edge with flexural oscillation nodes. On the whole, capillary waves excited both by inertial vibrations of the liquid and by localized sources on the liquid surface transfer the liquid particles thus generating eddy and jet currents. Study of mechanisms that trigger the generation of surface flows in a liquid by capillary waves is of interest for the creation of devices and technologies to obtain controlled directed flows.

The aim of the present paper was the study of flows on the water surface generated by capillary oscillations and waves, excited by locally distribution sources at their in-phase oscillations and directed flows in channels with sources and resonators of capillary oscillations.

1. Experimental setup

The device for excitation of in-phase capillary oscillations contains a FML-27T-3.9A1-100 piezoelectric converter with a flat round housing as an oscillation source and a connected elastic waveguide in the form of two identical metal rods 0.6 mm in diameter and 40 mm in length with free ends spaced at 12 mm from each other. Rectangular plates made of steel 0.1 mm thick and of polyethyleneterephthalate (PET) 0.3 mm thick were also used as an elastic waveguide. In the case of slab waveguides, oscillators were arranged on their free butt end by removing a plate part, or short wire rods were glued to the free butt end of the plate. The device is fastened on a tripod by means of screw clamps via a holder soldered to the edge of the piezoelectric converter housing on the area diametrically opposite to the waveguide. The free rod ends are brought to the surface of water filled in a transparent glass Petri dish 100 mm in diameter placed on a table with a lifting mechanism. The water surface is lit by a semiconductor LED pulsed light source, light pulse frequency is adjusted by an electronic key controlled by a square-wave generator. When alternating voltage is supplied from the low-frequency generator signal amplifier to the piezoelectric converter electrodes, flexural oscillations are excited in its housing, are transmitted to the rods and impart oscillatory motion to the rod ends. The wetted surfaces of the rod ends during oscillations on the water surface cause in-phase excitation of capillary oscillations that propagate along the water surface in the form of capillary waves. A wave interference pattern is observed under stroboscopic lighting of the liquid surface, surface currents generated by waves are visualized by graphite or phosphor particles. Capillary waves and currents on the water surface were photographed by a digital photo camera.

2. Results and discussion

2.1. Surface flow from two sources of capillary oscillations

Capillary oscillations on the free water surface, excited by two sources vibrating in phase, propagate in the form of cylindrical capillary waves propagating from the areas of rods-contact with the water surface (Fig. 1). Waves on the water surface interfere and, at certain rod vibration frequencies, capillary oscillations in the form of a standing wave are excited on the water surface area between the rod ends; the wave crest lines are transverse to the segment between the rod ends. A standing capillary wave forms in case of wave excitation by a frequency at which an integer number of wavelength halves fits on a segment of the water surface line between the sources. At the same time, two pairs of eddy currents are generated on the water surface; they are visualized by tracer particles which move along closed extended trajectories (Fig. $1, a$). The overall current in each pair of currents, distanced on both sides from each oscillation source, is directed away from the sources along the line that intersects the sources. The backward motion of particles in eddy currents is towards the liquid surface area between two wave sources perpendicularly to the line segment between them. As rod oscillation frequency increases, capillary wave length decreases, standing waves form again on the water surface area between the rod ends at certain rod oscillation frequencies. Thereat, velocities of tracer particles in eddy currents increase, and the formation of secondary small-scale vortices is observed (Fig. $1, b$). On the whole, the pattern of eddy plane currents on the water surface, generated by two capillary wave sources vibrating in phase (Fig. $2, a$), is identical to the current pattern on the water surface near the edges of a vibrating plate in it, on the surface of a liquid layer and a certain drop on a vibrating plane and the soap film of water on the plate holes.

2.2. Deformation of the free liquid surface by capillary oscillations and deformation flow in a cylindrical capillary wave

Generation of eddy currents on a liquid surface by two spaced-apart sources of capillary waves can be explained as follows.

The free rod ends, due to the wetting phenomenon, perform capillary rise of the liquid, as a result of which the area of liquid contact with the rod surface increases, while the free water surface deforms. The wetting angle periodically changes during rod vibrations, thus causing a local change in the deformed free liquid surface near the phase interface. If we distinguish an element of liquid surface ΔS_0 in a coordinate system fixed in relation to the vibrating surface, its energy under capillary oscillations will be

$$\Delta W_s = \sigma(\Delta S_0 + \Delta S_\nu),$$

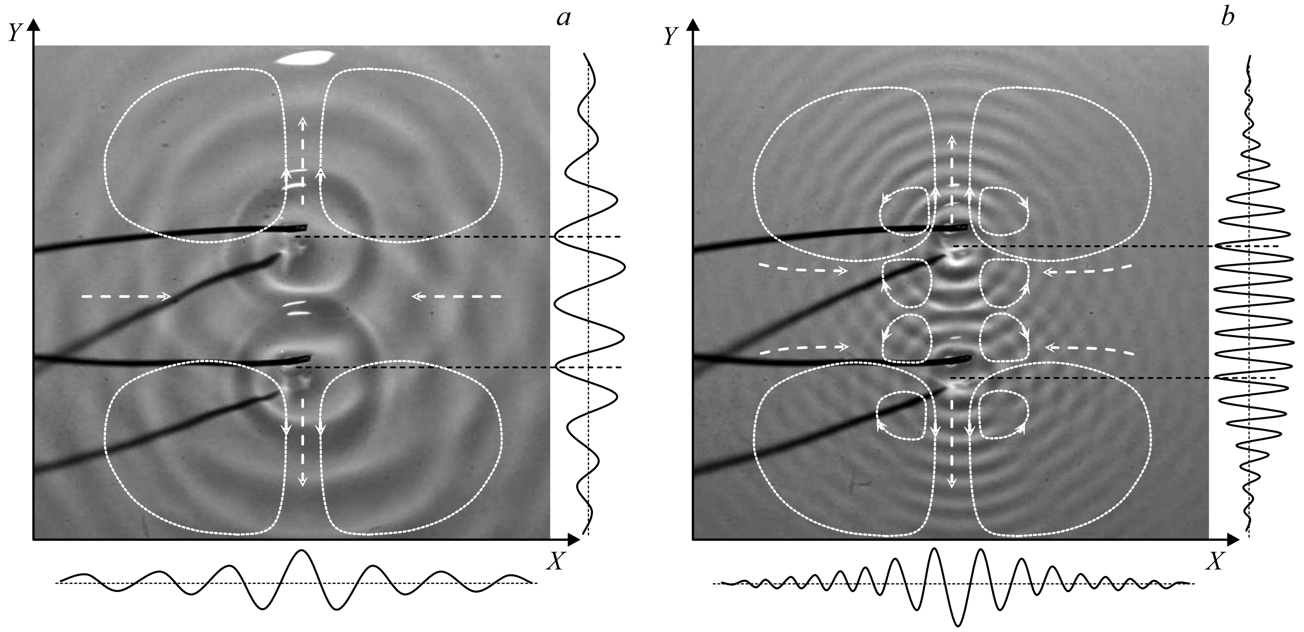

Figure 1. Interference pattern of capillary waves, layout of particle paths on the water surface and profiles of waves excited by two sources vibrating in phase with the frequency of 46 (*a*) and 200 Hz (*b*).

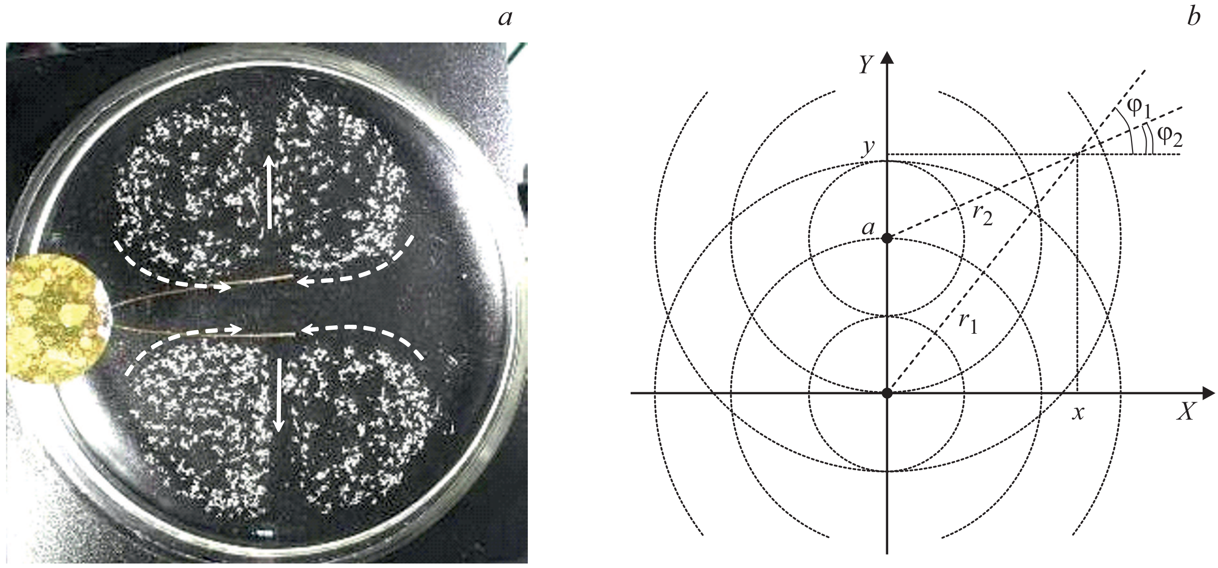

Figure 2. Pattern of phosphor particle distribution in eddy currents generated on the water surface by waves from two sources vibrating in phase with the frequency of 46 Hz (the arrows show the current directions) (*a*) and the wave interference layout (*b*).

where ΔS_v is area increment of a liquid surface element in the oscillatory process. This energy can be described via relative deformation (hereinafter simply deformation) of a liquid surface element $\delta = \Delta S_v/\Delta S_0$ using the equation

$$\Delta W_s = \sigma(1+\delta)\Delta S_0.$$

Thus, the energy of an element of the free liquid surface under capillary oscillations changes due to the surface deformations by oscillations. Density of excess energy of a liquid surface element is equal to the product of surface tension and deformation $\sigma_v = \sigma\delta$. Energy of an element of the free liquid surface and its area under capillary oscillations near the source change at a doubled oscillation frequency under the pulsating law, therefore it can be considered than the vibrating surface acquires excess averaged energy in the oscillation period as compared to the non-moving surface area. On the average, density of excess surface energy is equal to $\Delta\overline{w}_s = \overline{\sigma}_v = \overline{\delta}\sigma$. Such excess density of surface energy has the physical sense of effective vibrational surface tension, which in the experiments leads to the Marangoni vibrational convection. Vibrational convection on the water surface in the form of currents evolving into jets is observed, for instance, during parametric excitation of capillary oscillations by a vibrating plate [1]. Vibrational convection is usually explained by a gradient of effective surface tension. In the considered case, effective surface tension is directly proportional to the liquid surface deformation, and vibrational convection can be explained by a gradient of surface deformation.

In order to establish the dependence of liquid surface deformation on density of energy, transmitted by a vibrating source to this surface, let us consider the excitation of capillary oscillations and waves on the free liquid surface by one source — a rod the end whereof is brought to the liquid surface. The rod motion during vibrations is transmitted to the liquid and its particles directly near the rod surface acquire kinetic energy of density w_k proportional to the squared oscillation velocity v of the rod surface: $w_k = (1/2)\rho_l v^2$, where ρ_l is liquid density. In the conditions of radial symmetry of the phase interface of the liquid and the rod, rod vibrations excite capillary oscillations on the free liquid surface which propagate along the liquid surface in the form of cylindrical capillary waves. The area of deformed liquid surface with the maximum oscillation amplitude is at certain distance r_0 away from the center of contact with the vibrating rod surface, thereat, the liquid surface particles shift both in the transverse and in the longitudinal direction to the said surface. In this case, the equations for transverse and longitudinal offsets η and ξ in a cylindrical capillary wave can be written as

$$\eta = \eta_0\sqrt{r_0/r}\cos(\omega t - kr)$$

and

$$\xi = \xi_0\sqrt{r_0/r}\sin(\omega t - kr),$$

where η_0 and ξ_0 are amplitudes of liquid particle shifts in oscillations near the phase interface with the surface of the vibrating rod, ω is wave frequency, $k = \omega/c_c = 2\pi/\lambda_c$ is wave number, wherein c_c and λ_c — are respectively capillary wave phase velocity and length, $r \geq r_0$ is distance. Liquid surface particles under oscillations in a cylindrical wave move along elliptical trajectories described by the equation

$$\eta^2/\eta_0^2 + \xi^2/\xi_0^2 = r_0/r.$$

Density of kinetic energy of liquid surface particles is determined via their velocities in oscillatory motion $w_k = (1/2)\rho_l[(d\eta/dt)^2 + (d\xi/dt)^2]$. Substitution of derivatives into this equation with account of trigonometric relations $\sin^2 x = (1/2)(1 - \cos 2x)$ and $\cos^2 x = (1/2)(1 + \cos 2x)$ gives the following expression for kinetic energy density:

$$w_k = (1/4)\rho_l\omega^2(\eta_0^2 + \xi_0^2)(r_0/r) + (1/4)\rho_l\omega^2(\xi_0^2 - \eta_0^2)(r_0/r)\cos 2(\omega t - kx).$$

According to this equation, kinetic energy density of liquid surface particles in a capillary wave changes with a doubled wave frequency, while its average magnitude for the oscillation period is equal to $\bar{w}_k = (1/4)\rho_l v_0^2(r_0/r)$, where $v_0 = \omega\sqrt{\eta_0^2 + \xi_0^2}$ is velocity of shift of liquid surface particles near the source at distance $r = r_0$.

Oscillations and waves decay in actual liquids due to viscosity. Paper [10] gives an analysis of dispersion and decay of finite-amplitude capillary waves where it is stated that capillary wave frequency in a viscous liquid is determined by expression $\omega = \omega_0 + i\gamma = \omega_0\sqrt{1-\xi^2}$. Decay modes depend on viscous frequency coefficient $\xi = \gamma/\omega_0$. In case of wave excitation on the free surface of a homogeneous liquid, the wave decay coefficient is determined as $\gamma = 2\nu k^2$, which is suitable to this paper for waves excited on the water surface.

The dispersion equation for capillary waves on the free surface of a viscous liquid, depth H of which is half the wavelength and more, $H \geq \lambda_c/2$, in compliance with the frequency equation can be written as

$$\omega^2 = \omega_0^2 - 4\nu^2 k^4,$$

where $\omega_0^2 = (\sigma/\rho_l)k_0^3 = \kappa_0^2 c_{c0}^2$.

The dispersion equation provides the following expression for capillary wave velocity

$$c_c = (\omega_0/k)\sqrt{1 - 4\nu^2 k^4/\omega_0^2}.$$

This expression can be written via frequency f and length λ_c of excited waves which can be determined from an experiment:

$$c_c = f\lambda_c\sqrt{1 - 16\pi^2\nu^2/f^2\lambda_c^4}.$$

According to this expression, viscous losses of capillary wave velocity must increase with an increase in frequency and decrease in wavelength. The calculations made based on the physical characteristics of water, for which density ρ_l is $10^3\,\mathrm{kg/m^3}$, surface tension σ is $72.8 \cdot 10^{-3}\,\mathrm{N/m}$ and kinematic viscosity ν is $1.006 \cdot 10^{-6}\,\mathrm{m^2/s}$, allow for making a conclusion that viscous losses of wave velocities at low wave frequencies are insignificant. Thus, based on the pattern of waves excited on the water surface by two synchronously vibrating sources, capillary wave length λ_c at the source oscillation frequencies of 46 and 200 Hz is 6 and 2 mm respectively. The wave velocities at these frequencies, determined from an experiment using formula $c_c = f\lambda_c$, are equal to $c_c = 276.0 \cdot 10^{-3}$ and $400.0 \cdot 10^{-3}$ m/s. Addend $16\pi^2\nu^2/f^2\lambda_c^4$ in the subradical multiplier for the velocity of waves at the frequency of 46 Hz is $58 \cdot 10^{-6}$, at the frequency of 200 Hz — $25 \cdot 10^{-5}$, which is slightly below unity. However, experimentally determined values of capillary wave velocity are slightly smaller than their values calculated using formula $c_c(f) = \sqrt[3]{2\pi f\sigma/\rho_l}$, where water viscosity is omitted. The calculations give the following values of the velocity of waves on the water surface: $c_c(46) = 276.1 \cdot 10^{-3}$

and $c_c(200) = 450.6 \cdot 10^{-3}$ m/s. Wavelength for high frequency decreases, while wave number increases, therefore, viscous losses of wave velocity increase.

Excitations of monochromatic waves, where the phase and group velocities are equal to each other, are observed on the water surface in the experiments with surface flow generation by capillary oscillations. It should be noted that the capillary wave velocity for a zero-viscosity liquid is equal to $c_c = \sqrt{(\sigma/\rho_l)k}$, group velocity is $d\omega/dk = (3/2)c_c$. On the whole, liquid viscosity reduces the phase and group velocities of capillary waves on the free liquid surface.

Wave energy in a viscous liquid decreased under the exponential law $\mathbf{E} \propto \exp(-2\gamma t)$ [11]. Since capillary oscillations, excited by sources, are forced and propagate along the liquid surface in the form of progressing waves, wave energy at a certain distance from the source will decrease by $\exp(-4\nu k^2\Delta x/c_c)$ times. The exponent degree can be reduced to expression $(-16\pi^2\nu\Delta x/f\lambda_c^3)$. From here we can estimate a distance from the source at which wave energy will decrease by e times. Again, this distance, based on the experiments for the wave frequency of 46 Hz, is approximately 62 mm or $\Delta x \approx 10\lambda_c$, for the frequency of 200 Hz — $\Delta x \approx 10$ mm or $\Delta x \approx 5\lambda_c$. Thus, a low water viscosity allows for observing a wave field on almost the entire water surface in the Petri dish 100 mm in diameter. For comparison, progressing capillary waves on the surface of PMS-100 silicate oil, kinematic viscosity of which is $\nu = 100 \cdot 10^{-6}$ m^2/s, are not excited by the rod sources. Only parametric excitation of standing waves is possible on the oil layer surface [9].

Energy of wave viscous damping with distance from the source for wave energy density can be taken into account by introducing multiplier $\exp[-\alpha(r - r_0)] = \exp[-4\nu k^2(r - r_0)/c_c]$. Therefore, the expression for kinetic energy density of liquid surface particles in a cylindrical wave on the average will be as follows

$$\bar{w}_k = (1/4)\rho_l v_0^2(r_0/r)\exp[-4\nu k^2(r - r_0)/c_c].$$

Densities of potential and kinetic energy in waves are equal to each other, and density $\bar{w}_p$ of potential energy on the average must be also equal to the kinetic energy density on the average: $\bar{w}_p = \bar{w}_k$. Thereat, energy density $\bar{w}$ in the wave on the average is equal to the doubled kinetic energy density on the average

$$\bar{w} = 2\bar{w}_k = (1/2)\rho_l v_0^2(r_0/r)\exp[-4\nu k^2(r - r_0)/c_c].$$

The dispersion equation for capillary waves provides an expression for energy density in the wave $\rho_l(\omega_0^2/k^2 - 4\nu^2k^2) = \sigma k$, where the left and right members have the measurement units of energy bulk density [J/m^3] and pressure [N/m^2] respectively. This expression indicates that energy density in a capillary wave is (in magnitude) equal to pressure under the free liquid surface with a curvature equal to the wave number. Let us write an equation for energy density in a capillary wave near the source as follows

$$\rho_l(\omega_0^2/k^2 - 4\nu^2k^2) + \bar{w} = \sigma(1 + \bar{\delta})k.$$

From here, deformation of the liquid surface under capillary oscillations excited by the source is equal to the ratio of the density of energy, transmitted to the liquid by the source, to the energy of density of oscillations in the capillary wave away from the source, where the non-linear contribution of the source to energy wave can be disregarded:

$$\bar{\delta} = 2\bar{w}_k/\rho_l(\omega_0^2/k^2 - 4\nu^2k^2) = (1/2)[v_0^2/(\omega_0^2/k^2 - 4\nu^2k^2)] \times (r_0/r)\exp[-4\nu k^2(r - r_0)/c_c].$$

The maximum deformation magnitude equal to

$$\bar{\delta} = (1/2)[v_0^2/(\omega_0^2/k^2 - 4\nu^2k^2)],$$

falls on the surface area in the immediate vicinity of the capillary oscillation source, where $r = r_0$. An expression for deformation can be also written as a relation

$$\bar{\delta} = 2\bar{w}_k/\sigma k = (2/\sigma k)\bar{w}_{k0}(r_0/r)\exp[-4\nu k^2(r - r_0)/c_c],$$

where $\bar{w}_{k0} = (1/4)\rho_l v_0^2$. Thus, deformation of the free liquid surface by capillary oscillations and waves excited by the source is directly proportional to the density of kinetic energy imparted by oscillations to liquid surface particles.

In a cylindrical capillary wave, deformation of the liquid surface decreases as distance from the source increases, and can be represented as

$$\bar{\delta} = \bar{\delta}_0(r_0/r)\exp[-4\nu k^2(r - r_0)/c_c].$$

Then density of excess surface energy of a liquid in a cylindrical wave is equal to

$$\Delta\bar{w}_s = \sigma\bar{\delta}_0(r_0/r)\exp[-4\nu k^2(r - r_0)/c_c].$$

Density of the flow of energy carried by waves is equal to the value of the Umov vector (this vector for electromagnetic waves is usually called the Umov–Poynting vector): $\mathbf{S}_U = wc_w\mathbf{n}$, where w is energy density, $c_w = d\omega/dk$ is group velocity of waves, $\mathbf{n}$ is unit vector.

Wave intensity I means the average value of the Umov vector $I = \bar{w}c_w$. Density of a pulse on the average in a wave of phase velocity c is equal to

$$\bar{p} = (\bar{w}/c^2)c_w = I/c^2.$$

In our paper, flows on the water surface are generated by monochromatic waves having equal phase and group velocities. Therefore, the expression for the density of surface energy flux on the average in a cylindrical capillary wave or intensity of a cylindrical wave can be written via liquid surface deformation as follows

$$I_s = \sigma\bar{\delta}c_c.$$

Surface energy flux dW_s/dt along the liquid surface at certain density r from the source is equal to the product of surface energy flux density and length L of a circuit in the form of a circumference of radius r: $dW_s/dt = I_s L$. Since

$$I_s = \sigma\overline{\delta_0}(r_0/r)\exp[-4\nu k^2(r-r_0)/c_c]c_c$$

and $L = 2\pi r$, surface energy flux is equal to

$$dW_s/dt = 2\pi r_0\sigma\overline{\delta_0}\exp[-4\nu k^2(r-r_0)/c_c]c_c.$$

Surface energy carried over by a wave within source oscillation period T can be found from the integral

$$dW_s = \int\limits_0^T 2\pi r_0\sigma\overline{\delta_0}\exp[-4\nu k^2(r-r_0)/c_c]c_c dt.$$

It is equal to

$$\Delta W_s = 2\pi r_0\sigma\overline{\delta_0}\exp[-4\nu k^2(r-r_0)/c_c]c_c T$$

and is proportional to excess surface $\Delta S = 2\pi r_0\overline{\delta_0}\lambda_c$, created by the source within the oscillation period. Energy carried over by a wave within the oscillation period is equal to the product of surface tension and excessive surface $\Delta W = \sigma\Delta S$, since $\lambda_c = c_c T$. In this case, the wave also carries over the excess surface S_δ, flux of which is equal to

$$dS_\delta/dt = (1/\sigma)dW_s/dt = 2\pi r_0\overline{\delta_0}\exp[-4\nu k^2(r-r_0)/c_c]c_c$$

and has measurement unit [m^2/s]. Density of excess surface flux is equal to the surface deformation flux with measurement unit [m/s]:

$$s_\delta = \overline{\delta}c_c.$$

Density of surface energy flux on the average, upon excitation of finite-amplitude capillary oscillations on the free liquid surface by a source, is equal to the product of the surface tension coefficient and the flux of deformation generated by waves on the liquid surface: $S_{US} = \sigma s_\delta = \sigma\overline{\delta}c_c$. Flux intensity is equal to the product of surface tension and deformation flux: $\mathbf{I}_s = \sigma\mathbf{s}_\delta$.

The equation for wave pulse density can be written as

$$\overline{p_s} = I_s/c_c^2 = (\sigma/c_c^2)\overline{\delta}c_c = (\rho_l/k)\overline{\delta}c_c.$$

The ratio of liquid density to wave number in this expression has the physical sense of surface liquid density $\rho_s = \rho_l/k$ with measurement unit [kg/m^2]. Then surface pulse density can be written via a product of surface density and deformation flux $\overline{p_s} = \rho_s\overline{\delta}c_c = \rho_s s_\delta$. Surface pulse density has the measurement unit of surface flow [(kg/s)/m = (kg/m^2)/(m/s)], while surface pulse flow has the measurement unit of liquid flow [kg/s].

In order to write an equation for motion of the free liquid surface due to transmission of a wave pulse, the relation for pulse density and energy density can be written as $\overline{\mathbf{p}}_s = (\Delta\overline{\mathbf{w}}_s/c_c^2)c_c$. The time derivative of pulse density is equal to $d\overline{p_s}/dt = (1/c_c)d\Delta\overline{w}_s/dt$. In its turn the time derivative of energy density is equal to the product of the coordinate derivative of the energy density and the velocity of energy transfer by waves $d\Delta\overline{w}_s/dt = (\partial\overline{w}_s/\partial r)dr/dt$. Since energy transfer rate is equal to wave velocity $dr/dt = c_c$, the time derivative of wave pulse density is equal to $d\overline{p_s}/dt = \partial\Delta\overline{w}_s/\partial r$. The coordinate derivative of energy density can be expressed as a product of surface tension and the gradient of surface deformation by waves: $\partial\Delta\overline{w}_s/\partial r = \sigma\,\mathrm{grad}\,\overline{\delta}$. Accordingly, the time derivative of density of a cylindrical wave pulse on the liquid surface is proportional to the surface deformation gradient: $d\overline{p_s}/dt = \sigma\,\mathrm{grad}\,\overline{\delta}$.

Assuming that a wave pulse is transmitted to liquid surface particles, the following equation of motion of liquid surface particles can be written:

$$\rho_s du_s/dt = -\sigma\,\mathrm{grad}\,\overline{\delta}.$$

From here, acceleration of a liquid unit surface due to transmission of a cylindrical wave pulse is equal to

$$du_s/dt = c_c^2\,\mathrm{grad}\,\overline{\delta}.$$

Thereat, the liquid unit surface in coordinate interval $\Delta r = r_2 - r_1$ from the source acquires velocity Δu_s equal to

$$\Delta u_s = (1/\rho_s)\Delta p_s = [\delta(r_1) - \delta(r_2)]c_c.$$

If coordinates are counted along the liquid surface from the wave source, where $r_1 = r_0$, this velocity will be $\Delta u_s = \delta_0(1 - r_0/r)c_c$.

By definition, deformation of a liquid surface is proportional to the density of surface particles-kinetic energy, therefore $\mathrm{grad}\,\overline{\delta}(2/\sigma k)\partial\overline{w}_k/\partial r$. In its turn, the surface pressure gradient that accelerates the liquid surface particles is conditioned by the coordinate derivative of kinetic energy density, from which $\mathrm{grad}(p_s) = (\sigma k/2)\,\mathrm{grad}\,\overline{\delta}$.

Now let us consider the current in the liquid near-surface layer in the presence of a cylindrical capillary wave, i.e. in the skin layer bulk. Density of liquid kinetic energy on the average in the skin layer bulk is determined via surface deformation by waves using expression $\overline{w}_k = (1/2)\sigma k\overline{\delta}$. Thereat, pressure difference is equal to difference of kinetic energy density of liquid particles $\Delta p_{V_s} = -\Delta\overline{w}_k = -(1/2)\sigma k\Delta\delta$. From here, acceleration in the liquid layer caused by the pressure gradient is equal to $du_{V_s}/dt = -(1/\rho_l)\mathrm{grad}(p_{V_s}) = c_c^2\,\mathrm{grad}\,\overline{\delta}$. This expression indicates that liquid particles in the skin layer due to the surface deformation gradient, caused by a capillary wave source, acquire an acceleration proportional to the squared phase velocity of waves. Thereat, the liquid moves in the skin layer towards the wave source. Acceleration of liquid surface particles due to transmission of the pulse wave is proportional to squared group velocity of waves. The previous experimentally found movements of the liquid layer on the surface of vibrating rods and plates towards oscillation node areas, where kinetic energy density, on the average, of liquid particles is the maximum, can be also due to the fact that liquid surface deformation on these areas in also the maximum.

If capillary waves are excited on a free water surface by two sources vibrating in phase, the total density of wave energy at a point of the surface depends on density from each source, while density of the total wave pulse depends on direction as wall.

Let us consider the free liquid surface in the XOY Cartesian coordinate system, where sources are located on the Y axis (along the $x = 0$ line) at distance a from each other, while the X axis (the $y = 0$ line) intersects one of the sources (Fig. 2, b). Distances from the sources up to a surface point with coordinates (x, y) are equal to $r_1 = \sqrt{x^2 + y^2}$ and $r_2 = \sqrt{x^2 + (y - a)y^2}$, angles between radius-vectors $\mathbf{r}_1, \mathbf{r}_2$ and the X axis are φ_1 and φ_2.

Since $r_1 \geq r_0$ and $r_2 \geq r_0$, the equation for density $\Delta\bar{w}_s$ of excess liquid surface energy under wave excitation by two sources will be written as

$$\Delta\bar{w}_s = \sigma(\bar{\delta}_1 + \bar{\delta}_0) = \sigma\bar{\delta}_0(r_0/r_1 + r_0/r_2),$$

there, wave pulse density in the liquid surface regions $y \leq 0$ and $y \geq a$ is equal to

$$\bar{p}_s = \rho_s\bar{\delta}_0 r_0[1/r_1 + (1/r_2)\cos(\varphi_1 - \varphi_2)]c_c.$$

Wave pulses in the region of surface $0 \leq y \leq a$ are opposite to each other, and pulse density is equal to

$$\bar{p}_s = \rho_s\bar{\delta}_0 r_0[1/r_1 - (1/r_2)\cos(\varphi_1 + \varphi_2)]c_c.$$

Analysis of the equations for wave pulse density indicates that its magnitude is the maximum on the liquid surface areas $y \leq 0$ and $y \geq a$ along the line that intersects the wave sources. Accordingly, the observed surface currents are conditioned by wave pulse transmission to liquid surface particles.

Deformation of the liquid surface by waves from two sources is equal to

$$\bar{\delta} = \bar{\delta}_0(r_0/r_1 + r_0/r_2)$$

and also is the maximum along the line intersecting the sources. Therefore, the gradient of this deformation conditions the movement of the liquid skin layer in the direction perpendicularly to the said line. These liquid accelerations, directed towards the sources, are the maximum along the $y = 0$ and $y = a$ lines.

Thus, the pattern of the experimentally observed surface currents under excitation of capillary waves on the water surface by distributed sources indicates the generation of surface currents by capillary waves due to wave pulse transmission to wave surface particles. Since the wave pulse density is proportional to the surface deformation flux, waves transfer an excess surface periodically generated by capillary oscillation sources.

Capillary oscillations in the form of a standing wave are excited on the water surface area between the sources under interference of waves having an equal frequency and, accordingly, equal length λ_c, emanating from two sources spaced at a, which is equal to or divisible by half the wavelength $a = n(\lambda_c/2)$. Capillary oscillations outside this area of the water surface propagate in the form of progressing waves that transmit the pulse to the water particles on its free surface. Thereat, the total density of the pulse of interfering capillary waves on the water surfaces is the maximum along the line that connects the sources, on both sides from the sources, where the surface flow with the maximum density is observed. There is no transfer of the energy and pulse in capillary oscillations between the rod ends. However, capillary oscillation amplitude, density of kinetic energy of liquid particles and, accordingly, free surface deformation on this water surface area are greater as compared to other areas. Since kinetic energy density defines hydrodynamic pressure in a liquid, a pressure gradient is created in it and accelerates the particles of the near-surface liquid layer into the region between the vibrating rod ends. This gradient is also determined by the gradient of surface deformation on the average. As a result, plane jet currents originate on the water surface. Thus, capillary waves excited by two sources vibrating in phase carry over the excess surface, generated by the sources as a result of a local surface deformation, and produce differently directed surface flows.

It has also been noted in the experiments that surface currents tend to localize along the line that intersects the capillary oscillation sources. This is due to two reasons. Firstly, currents along the water surface can be considered as flow tubes where hydrodynamic pressure is proportional to the squared averaged velocity imparted to liquid surface particles as a result of capillary wave pulse transmission. Accordingly, a pressure gradient is created in the near-surface liquid layer; it is equal to the gradient of the hydrodynamic pressure due to which liquid surface particles are accelerated towards currents with a higher averaged velocity and, respectively, with a higher density of the capillary wave pulse. Since kinetic energy density in a capillary wave is directly proportional to deformation of the free liquid surface, the surface flow narrowing can be explained by the presence of a liquid surface deformation gradient. Liquid particle accelerations in the surface flow can be found from expression

$$\mathbf{du/dt} = c_c^2\left[(\partial\bar{\delta}/\partial x)\mathbf{i} + (\partial\bar{\delta}/\partial y)\mathbf{j}\right].$$

Secondly, liquid particles in a surface flow with a lower velocity must be accelerated towards a higher-velocity flow also due to the liquid viscosity.

2.3. Surface flows on the water surface in channels under excitation of capillary oscillations by sources vibrating in phase

In order to obtain a directed surface flow generated by capillary waves on the water surface in a Petri dish, a part

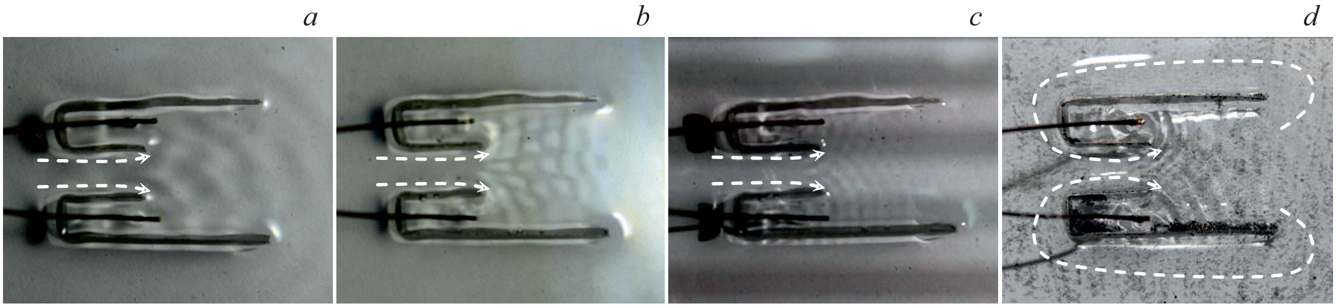

Figure 3. Capillary waves and flow on the water surface in a channel from two sources vibrating in phase at the frequency of 42 (*a*), 130 (*b*), 255 (*c*) and 144 Hz (*d*).

of the water surface was restricted by a channel where the free water surface for capillary waves is a waveguide. The channel walls are composed of two plates 5 mm wide and 0.1 mm thick, bent from one side in the U shape. The plates are glued to the base or connected by jumper in mirror symmetry to the U-shaped parts opposite to each other, between which a slit is left. The slit width is equal to the width of the plates U-shaped parts and is 5 mm. The device was installed on the bottom of a Petri dish where water was poured so that its free surface does not wet the upper plate edges. Thereat, a channel was created in the Petri dish; it was fully open from one end and had two U-shaped cavities, separated by a narrow inlet channel, on the other end. Capillary oscillations and waves were excited in the channel by means of rod oscillators as follows: the ends of the piezoelectric oscillator rods were brought to the water surface opposite to the centers of the U-shaped cavities in the channel. The part of the free water surface, restricted by the U-shaped cavity, for certain capillary oscillation frequencies is a quarter-wave resonator due to wave reflection from the closed cavity side. The U-shaped parts of the channel can be respectively called resonators.

The experiments have established that, at certain vibration frequencies of the rods on the water surface in a flat channel there is a pattern of distributed two-dimensional capillary waves emanating from the open sides of the resonators U-shaped cavities and propagating along the channel in one direction. Two-dimensional progressing waves with a transverse component in the form of a standing wave originate as a result of wave interference. Seeding of the water surface in the channel with tracer particles show that a surface flow of the liquid originates in the channel; it is directed inside the channel through the inter-resonator slit where waves are absent. Flow motion with the maximum velocity is observed when the rod vibration frequency satisfies the condition of excitation of capillary waves whose half-length is equal to or divisible by the width of the cavity of the U-shaped resonators. Moreover, the velocity of liquid particle motion in the flow is affected by the position of the rod ends in the cavity of the U-shaped resonators. To generate an effective flow in the channel, distance from the rod end to the rear wall in each U-shaped resonator shall be divisible by a quarter of the capillary wave length. Thus, directed motion of the liquid surface flow takes place in two-dimensional capillary waves excited in the channel by capillary oscillation sources (vibrating in phase) opposite to the U-shaped cavities of the capillary wave quarter-wave resonators. Tracer particles that move in the channel together with the flow make it possible to reveal flow acceleration near the edges of the U-shaped cavities of the resonators (Fig. 3, *a–c*). The particle paths also show that the flow in the channel generates surface currents enclosed via the water surface outside the device (Fig. 3, *d*). Particle paths in the region of wave disturbance in the broad part of the channel are chaotic and two-dimensional, thereat, their motion, on the average, along the wave propagation direction remains the same.

The surface flows on the water surface were also obtained using a 16×28 mm waveguide structure containing 4 quarter-wave resonators and 3 inter-resonator slits. The resonator cavities and slits have the dimensions of 4×5 mm. In-phase excitation of capillary oscillations was performed by means of rod oscillators brought to the free water surface in the center of the cavity of quarter-wave resonators spaced at 8 mm. The oscillators are glued to the waveguide edge in the form of PET plate sized $36 \times 10 \times 0.3$ mm, while vibrations of the plate itself were excited by means of the FML-27T-3.9A1-100 piezoelectric converter. Fig. 4 shows the pattern of two-dimensional capillary waves excited at different frequencies in a relatively narrow channel of 28 mm, containing 4 quarter-wave resonators and 4 oscillators. Surface water flows are generated in the device near the resonance frequencies of capillary oscillation; water particles in these flows move through the open channels — slits between the resonators towards the water surface area disturbed by two-dimensional waves. Secondary eddy currents are also observed in the region of the water surface area with wave disturbance.

The moving flow of liquid layer particles in the channel carries a pulse. Accordingly, the conservation laws make it possible to obtain motion of a non-fastened device with a channel on the liquid surface in the backward direction towards the flow in the channel.

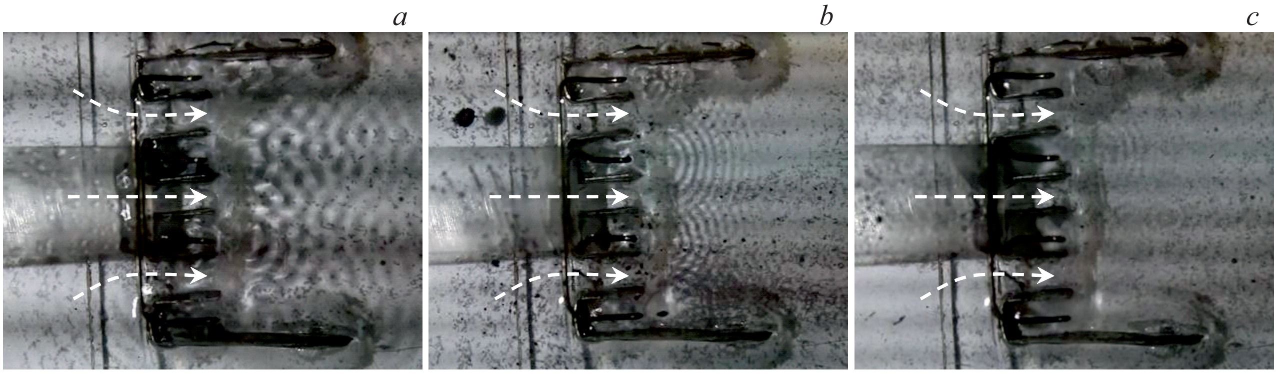

Figure 4. Capillary waves and flow on the water surface in a channel from four capillary oscillation sources vibrating at the frequency of 252 (*a*), 670 (*b*) and 3.9 kHz (*c*).

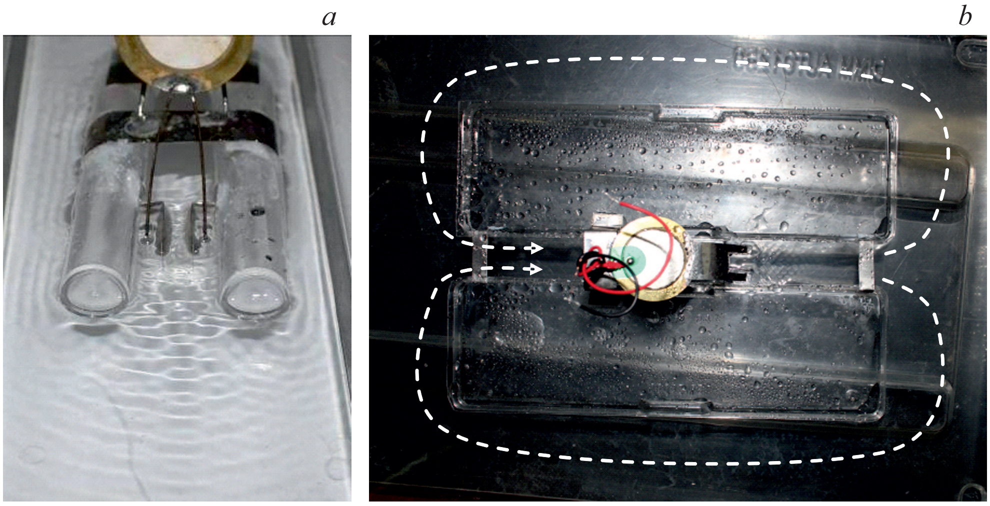

Figure 5. Devices with a capillary-wave mover under wave excitation by the frequency of 137 (*a*) and 212 Hz (*b*).

The device with a capillary-wave mover (Fig. 5, *a*) contains a housing made of hollow polymer cylinders 12 mm in diameter and 75 mm in length. The cylinders are installed in parallel and are glued together by metal clamps with a gap 15 mm wide between them. Bent L-shaped plates 8 mm wide are symmetrically glued to the cylinders in the gap, the long plate ends and the cylinder sides facing them form the U-shaped cavities sized 12×5 mm. A slit 5 mm wide is left between the plates. The FML-27T-3.9A1-100 piezoelectric converter with two oscillators made of metal rods 0.6 mm in diameter and 30 mm in length is installed on the device housing. The free rod ends are brought in the U-cavity to their open sides. The piezoelectric converter electrodes are connected by flexible thin wires to the secondary winding of the output transformer of the low-frequency generator signal amplifier. When the device is placed on a water surface, its housing is wetted by water, while the gap forms a through channel between the cylinders. The bent plates in the gap limit a water surface part and form the cavities of capillary wave quarter-wave resonators. Thereat, the free rod ends touch the water surface in the resonator cavity and ensure the capillary rise of water. When alternating voltage with the frequency of 137 Hz is supplied to the piezoelectric converter electrodes, the rods vibrate at one of the resonance frequencies of capillary oscillations and excite two-dimensional progressive waves in the channel which generate a directed surface flow of water. At the 20 V voltage on the piezoelectric converter electrodes, the device of total weight $8 \cdot 10^{-3}$ kg moves at the velocity of 20 mm/s. The oscillation amplitude of the rod ends in contact with water does not exceed 0.1 mm.

A catamaran-like device with a capillary-wave mover (Fig. 5, *b*) was made for studying the formation of a directed flow in the channel by capillary wave sources vibrating in phase. Its housing is assembled of two polymer $145 \times 45 \times 8$ mm trays rectangular in shape, connected in parallel with a gap 15 mm wide between them. Bent L-shaped plates are glued on the opposite sides of the gap to the trays, the long plate sides and the tray sides facing them form the U-shaped cavities sized 10×5 mm. A slit 5 mm wide is left between the plates. The FML-27T-3.9A1-100 piezoelectric converter with a slab waveguide 12 mm wide

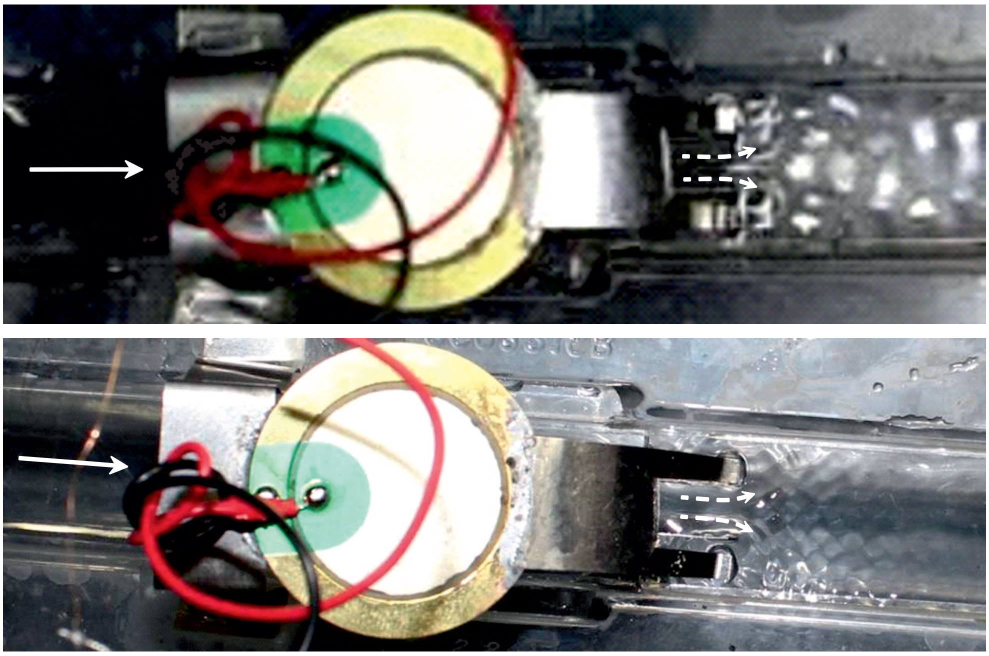

Figure 6. Pattern of two-dimensional capillary waves in the channel of the moving device at the frequencies of 76 (top) and 214 Hz (bottom).

and 0.1 mm thick is installed on the device housing, two short oscillators 2 mm wide are brought away from the free waveguide. The device weight is $16 \cdot 10^{-3}$ kg. When the device is placed on a water surface, its housing is wetted by water and a channel forms between the trays; the free water surface in the said channel for capillary waves is a waveguide limited by the channel width.

When the device is connected to an alternating current source at certain excitation frequencies of the oscillator, the device excites a progressing capillary wave of a distributed amplitude in the channel, so that a directed flow is generated in the channel. The device moves in the opposite direction to the flow in the channel. Seeding of the water surface with tracer particles has shown that the flow from the channel is divided into two closed flows which envelope the device housing and enter the channel from the opposite side. Device movements were also observed at other wave excitation frequencies (616, 704 Hz), but these waves are virtually invisible due to their small amplitude. When electric voltage with the frequency of 3.5 kHz is supplied to the piezoelectric converter electrodes, the device acquires a rather high velocity of 60 mm/s, but this is due to excitation of resonance oscillations of the converter itself and the formation of water jets flowing out from under the oscillator edges.

Fig. 6 shows the pattern of two-dimensional waves with the frequency of 76 and 214 Hz in the channel of the device with two in-phase capillary oscillation sources. The solid arrow shows the direction of surface flow motion on the area of the free water surface, unperturbed by waves, in the channel. The broken arrows show the parts of tracer particle parts in the flow near the edges of the U-shaped cavities of the quarter-wave resonators of capillary oscillations. Seeding of the water surface flow in the device channel with tracer particles and observation of the path of these particles has shown than particles are accelerated in the flow near the edges of the cavities of the quarter-wave U-shaped resonators of capillary waves. At the specified wave excitation frequencies, the average particle velocities on the area of the unperturbed free water surface were estimated at approximately 15 and 20 mm/s, while the device velocities were 2 and 3 mm/s.

Thus, motion is imparted to the device due to excitation of capillary oscillations by two sources, vibrating in phase, and due to formation of a directed surface flow of water in the channel. The oscillators provide in-phase excitation of capillary oscillations on the free water surface in the channel opposite to the open cavities of the U-shaped resonator, with a periodic local deformation of the water surface. Capillary oscillations on the water surface subsequently propagate in the form of waves progressing along the channel away from the open sides of the U-shaped resonator and generating a deformation flux and, respectively, an excess surface flux. The slit between the quarter-wave resonators in the channel provides continuity of the free liquid surface through which the liquid surface is pulled in during its deformation by the oscillators. At certain voltage frequencies on the piezoelectric converter electrodes, the oscillators excite capillary waves of a length divisible by the doubled length of the cavity of the quarter-wave resonators and, accordingly, to the doubled width of the slit in the inter-resonator channel. Thereat, interference and

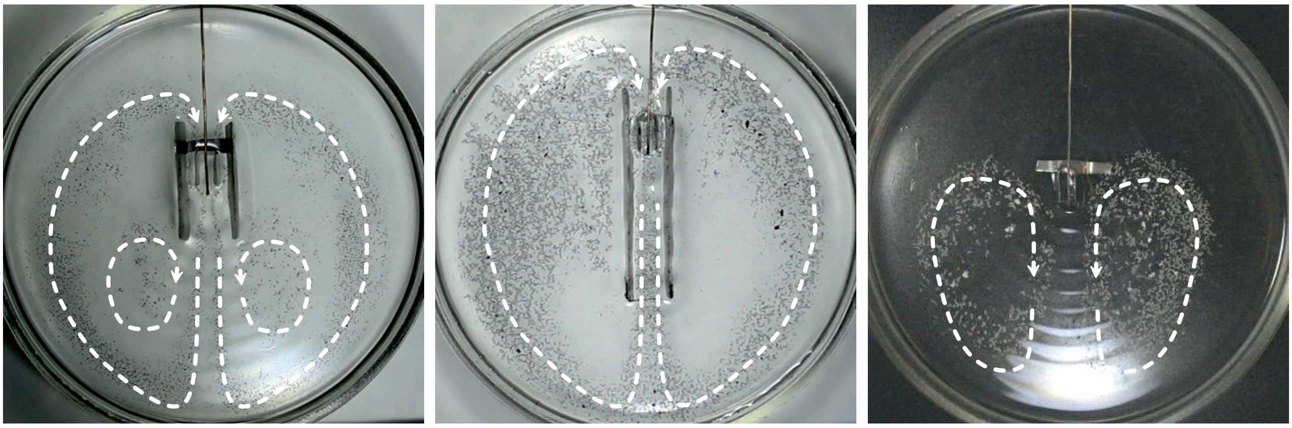

Figure 7. Pattern of waves and currents generated on the water surface by a source and a resonator, located in the short (*a*) and long (*b*) channels and by one source and a resonator (*c*).

reflection of the waves from the channel walls results in a two-dimensional distributed progressing wave that has a transverse component in the form a standing wave in its channel. As a result, the surface flow in the channel, generated by waves, is enhanced due to energy transfer, deformation and excess surface.

2.4. Flows in the channels with one capillary wave source and one resonator

The further studies have demonstrated that a surface water flow can be also generated in channels with one capillary oscillation source and one quarter-wave resonator installed in the channel center. The channel has slits of a width, equal to the resonator cavity width, between the channel walls and the resonator walls. On the whole, the free water surface for capillary waves in such a channel is a waveguide segment that contains a quarter-wave resonator in the center. Fig. 7, *a* shows the pattern of waves and currents on the water surface in a Petri dish under excitation of capillary oscillations with the frequency of 56 Hz on the water surface area limited by a channel sized 28×12 mm, wherein a rod source of capillary oscillations is opposite to the open cavity of the U-shaped resonator sized 8×4 mm in the channel center. It can be seen that capillary oscillations in the channel propagate only in one direction in the form of a standing wave, thus creating a common surface flow of water. Tracer part paths indicate that the flow consists of two plane currents, particles wherein are accelerated on the water surface areas in the immediate vicinity of the edges of the capillary oscillation resonator. The currents in these flows are enclosed outside the channel structure. Moreover, secondary eddy currents on two sides of the flow emanating from the channel, are also observed on the water surface.

The wave interference pattern in the device with a long channel of 50×10 mm and a resonator with a 3.5×8 mm cavity is mainly noticeable on the water surface in the channel itself, while currents pass through the entire channel and close outside the channel (Fig. 7, *b*). The device with one source and resonator with a 4×8 mm cavity, not limited by a channel (Fig. 7, *c*), excites semicylindrical waves and eddy currents similar to those observed on one of the water surface halves in the Petri dish under wave excitation by two sources vibrating in phase (Fig. 2, *a*).

Capillary waves excited by a rod and a resonator in the channel are semicylindrical. As a result of wave reflection from the channel walls, they interfere on the water surface in the channel, the crest lines in them are distorted by an accelerating flow (Fig. 8). Since the maximum velocity is in the flow center, the wave crest lines in the flow center sharpen.

A pattern of waves flows similar to the one observed on the limited water surface in the Petri dish also originates on large virtually unlimited surfaces. Thus, Fig. 9, *a* shows waves and currents generated by a capillary wave source in a channel with a resonator on the free water surface area of 80×210 mm. The free water surface in a short channel sized 12×22 mm with an installed 4×8 mm resonator and a source of capillary oscillations (Fig. 9, *b*) is a waveguide segment for capillary progressing waves of a certain frequency.

Experiments with excitation of capillary oscillations on the water surface in a Petri dish, resulting in the origination of surface flows, were also carried out using a source and a quarter-wave resonator installed in a circular channel 15 mm wide, limited by cylindrical rings. The rings 10 mm wide were cut out from polymer pipes and placed in a Petri dish with water. The free water surface, limited by the rings, is an enclosed waveguide for capillary waves. Under excitation of capillary oscillations by a rod oscillator opposite to the U-shaped cavity of a quarter-wave resonator sized 5×5 mm, oscillations in the channel propagate in the form of waves and create a continuous closed flow (Fig. 10, *a*). Flow velocity in the channel at the oscillator oscillation frequency of 56 Hz was 18 mm/s. Formation of a surface flow which

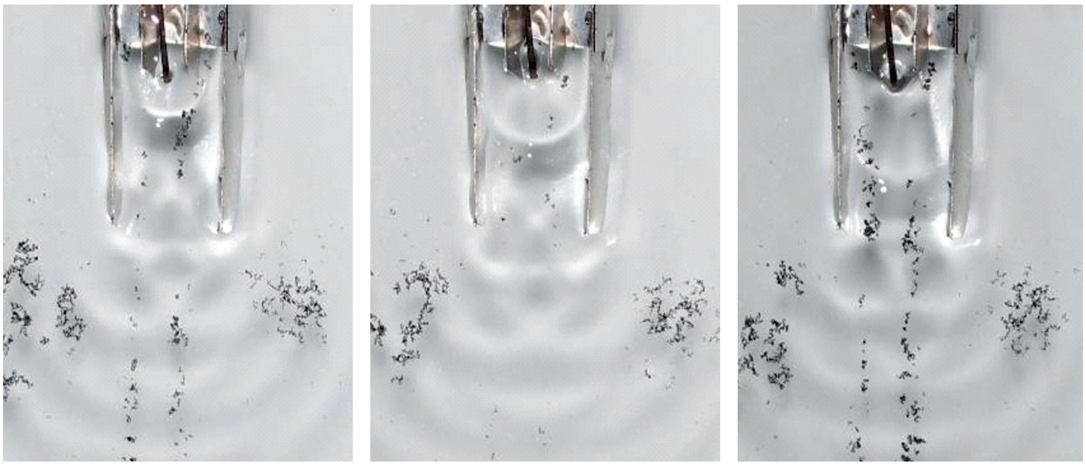

Figure 8. Pattern of waves and surface flows on the water surface, excited by a rod vibrating with the frequency of 56 Hz in a channel with one resonator.

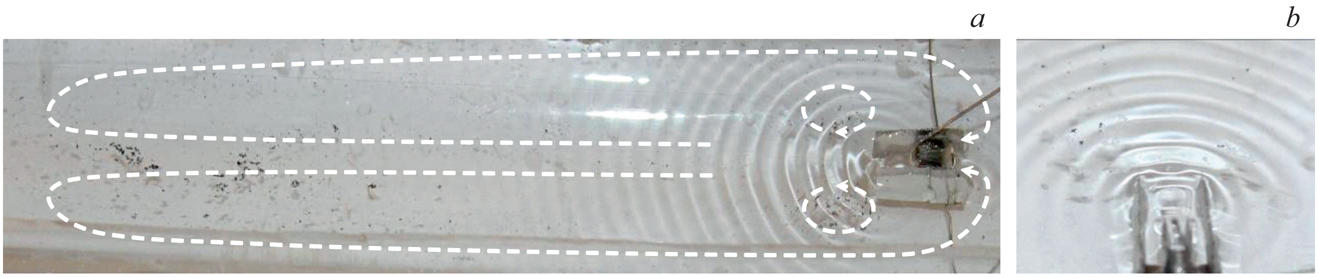

Figure 9. Waves and currents on a large water surface generated by one source and resonator in an open channel.

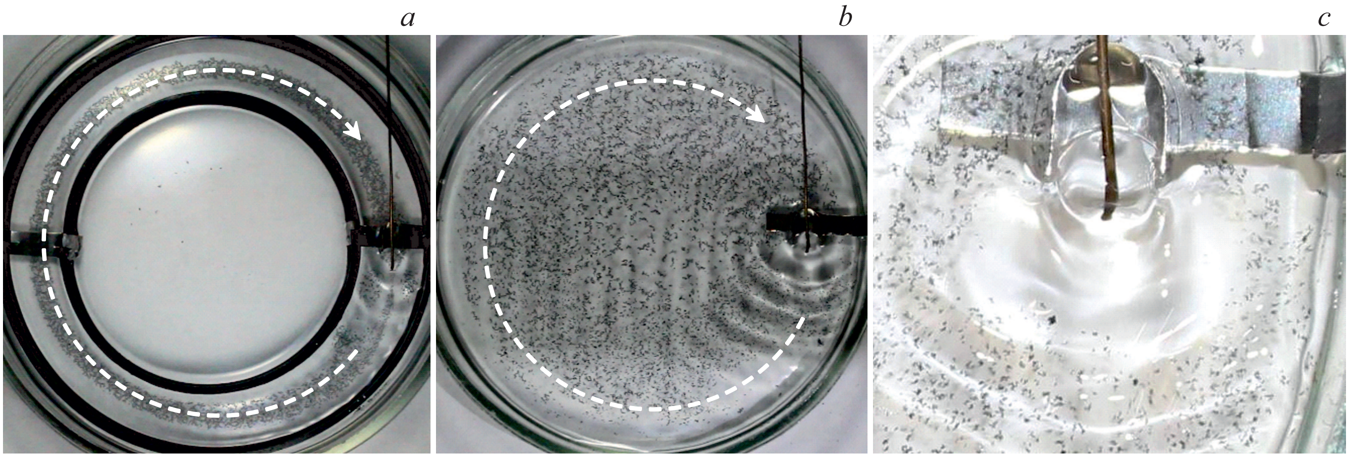

Figure 10. Waves and currents in a circular channel (a), on a round water surface (b) and near a quarter-wave resonator (c), generated by a source vibrating at 56 Hz.

involves the whole water surface in a Petri dish also turned out to be possible, when the quarter-wave resonator and the capillary oscillation source are located near the inner dish wall (Fig. 10, b). To do so, a slit was left between the dish wall and the resonator; slit width was approximately equal to the resonator cavity width. The average velocity of the surface flow near the Petri dish wall at the source vibration frequency of 56 Hz was 16 mm/s. The enlarged image of the water surface near the quarter-wave resonator cavity (Fig. 10, c) reveals symmetric nodes of capillary oscillations that also emanate from the edges of the U-shaped resonator, which indicates oscillations of the resonator edges. This

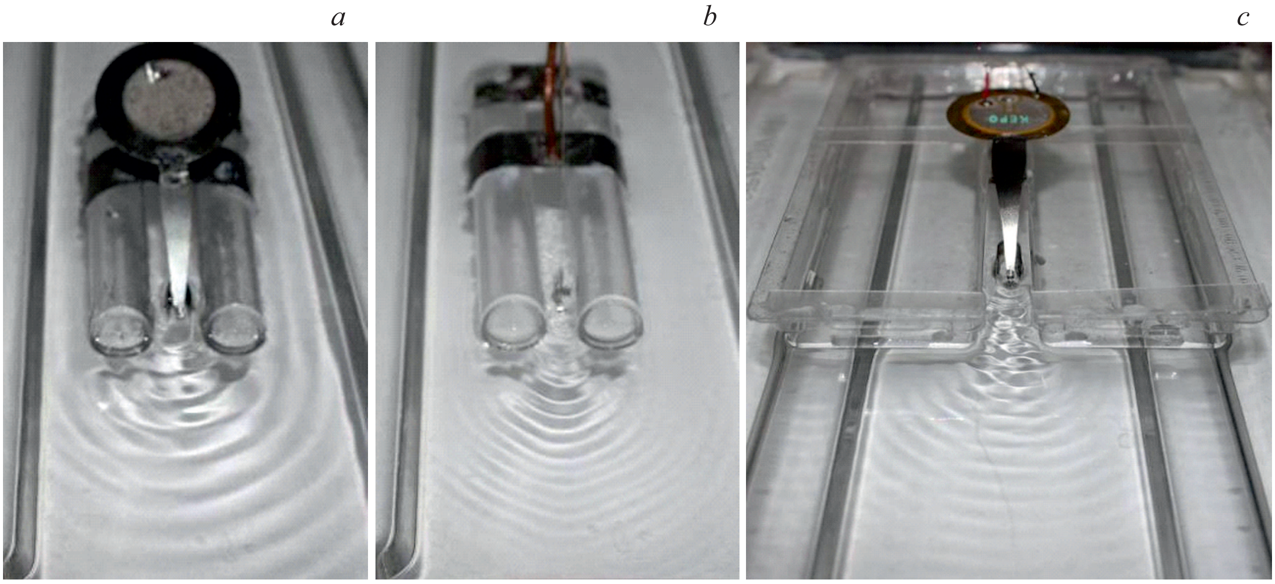

Figure 11. Pattern of two-dimensional capillary waves generated by moving devices with one source and one resonator of capillary oscillations at the excitation frequencies of 46 (a), 280 (b) and 58 Hz (c).

is because the source has a relatively large oscillation amplitude for parametric excitation of capillary oscillations that lead to a local deformation of the free liquid surface.

Devices with a capillary-wave flow accelerator (Fig. 11) were made to obtain motion from a surface flow generated by one source and one resonator in a channel. The device weighing $8 \cdot 10^{-3}$ kg with a housing made of hollow cylinders, channel width between which is 10 mm, with the 7NB-31R2DM-1 piezoelectric converter and the plate vibrator whose free end is opposite to the cavity of the quarter-wave resonator of capillary waves in the channel, is shown in Fig. 11, a. The device, placed on the water surface, receives motion from the flow in the channel generated by capillary waves excited by the vibrating plate edge. When 20 V and 46 Hz alternating voltage is supplied to the piezoelectric converter electrodes, the device moves at the rate of 8 mm/s. The channel in the second device of the same weight (Fig. 11, b) has the width of 5 mm, there is no quarter-wave resonator, the FML-20T-6.0A1-100 piezoelectric converter and the steel plate whose free butt end touches the water surface, are located in the vertical plane. When 280 Hz alternating voltage is supplied to the piezoelectric converter electrodes, the plate excites resonance capillary oscillations due to wave reflection from the channel walls. The result is the formation of a progressing distributed wave that generates a unidirectional water flow in the channel. When the voltage on the piezoelectric converter electrodes is 25 V, device velocity is 22 mm/s.

Fig. 11, c shows the device on the platform made of $145 \times 45 \times 8$ mm rectangular trays with a gap 15 mm wide between them. The device weighing $18 \cdot 10^{-3}$ kg contains a FT-35T-2.8BL piezoelectric converter, a slab waveguide 45 mm in length, the free butt end whereof is 2 mm wide, and a U-shaped resonator of 10×5 mm. When the device is placed on the water surface and 20 V 58 Hz alternating voltage is supplied to the piezoelectric converter electrodes, the device velocity is 10 mm/s, velocity of the surface flow in the channel is 30 mm/s.

Practical applications of the phenomenon of generation of a surface liquid flow in channels with a source and a resonator of capillary oscillations due to a small capillary wave intensity can be limited by small liquid volumes. These phenomena can be of interest as a basis for the creation of technical devices to generated controlled surface liquid flows. The operating principle of movers with a capillary-wave accelerator of flow in the channel, where the free liquid surface for capillary waves is a waveguide, will be possibly used in the development of technical devices containing waveguides with resonators and sources of acoustic and electromagnetic oscillations, built into their cavities.

Conclusion

Thus, generation of a directed flow on a liquid surface in channels with resonators and sources of capillary oscillations can be explained by a local deformation of the free liquid surface by a capillary oscillation source, when a deformed excess surface forms, and by the carry-over of the said surface by waves. Capillary waves with a transverse component in the form of a standing wave, which progress along the liquid surface in the channel, carry over (together with the energy flux) the excess surface whose flux density is equal to the flux of surface deformation in the wave. As a result, an averaged flux originates on the liquid surface in the channel.

Conflict of interest

The author declares that he has no conflict of interest.

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