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Self-Generation of Dark and Bright Envelope Pulses in Bidirectionally Coupled Vyshkind—Rabinovich Parametric Oscillators

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The generation modes of dark and bright envelope pulses are obtained in the system of two bidirectionally coupled Vyshkind—Rabinovich parametric oscillators. Each oscillator describes the temporal dynamics of a quadratic medium with inertialess amplification at a given point in space. Oscillations of an unstable wave amplify linearly, while oscillations of parametrically excited waves attenuate in a linear approximation. It is shown that a sequence of dark envelope pulses is formed on an unstable wave and is parametrically coupled with a sequence of bright envelope pulses generated on parametrically excited waves only at certain values of the bidirectional coupling.

Keywords: parametric instability, coupled parametric oscillators, dark and bright pulses.

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At present, one of the urgent tasks in the field of radiophysics and microwave electronics is the creation of sources of ultrashort pulses. Such pulsed sources are of interest for information transmission systems, for the study of fast processes, and for micromachining of various materials and biological tissues [1]. Some of the first works on the generation of relatively short envelope pulses in the microwave range are those of Professor B.A. Kalinikos. In his work, he used the optical method of mode synchronization on the Kerr nonlinearity to obtain periodic sequences of „light" and „dark" solitons of nanosecond duration envelope [2–4]. A film of yttrium-iron garnet was used as a medium with cubic (Kerr) nonlinearity, in which magnetostatic spin waves (MSWs) with different types of (normal or anomalous) waveguide dispersion propagated. Such a dispersing nonlinear medium was in the feedback circuit of the ring resonant cavity together with an amplifier performing linear amplification of the MSW. Impulse signals formed in such strongly non-equilibrium nonlinear systems with gain and loss belong to dissipative solitons [5]. In the last few years, a group of Nizhny Novgorod researchers headed by the Corresponding Member of the Russian Academy of Sciences N.S. Ginzburg proposed to use another optical method (the method of passive mode synchronization) to obtain dissipative envelope solitons in vacuum ring oscillators of the microwave range. Here, sequences of powerful short envelope pulses of sub-nanosecond duration were obtained by using a TWT suppressor (TWT — travelling wave tube) as a saturable absorber [6,7].

In the microwave range, the generation of pulse signals in active ring resonant cavities with yttrium-iron garnet films can be realized not only due to the cubic nonlinearity of the ferromagnetic medium, but also due to the quadratic nonlinearity resulting from the parametric decay of the long-wave MSW into short-wave thermal spin waves. In this case, a periodic sequence of relaxation envelope pulses of microsecond duration is formed on the parametrically unstable MSW, which is linearly amplified. As shown in [8], such a pulse sequence on an unstable wave is well reproduced in a numerical experiment using the Vyshkind—Rabinovich [9] model, which describes the temporary dynamics of dissipative media with amplification under conditions of parametric decay. From the results of numerical modelling, it followed that on parametrically excited waves at a given point of space are generated periodic sequences of bell-shaped envelope pulses, the duration of which is much shorter than the duration of relaxation pulses of the envelope on an unstable wave. In experiment, pulse sequences at short-wavelength spin waves could be registered only by Mandelstam-Brillouin spectroscopy because of their strong attenuation [10].

In the last few years, the authors of the present work have experimentally obtained hyperchaotic multisoliton complexes consisting only of dark dissipative envelope solitons of nano- and even sub-nanosecond duration [11,12]. Such short envelope pulses were generated in a microwave active ring resonant cavity operating in multimode mode. The feedback circuit of the ring resonant cavity contained a bent ferromagnetic microwave conduit supporting not only three- and four-wave nonlinear spin-wave interactions, but also MSW dispersion control, and an amplifier operating in the output power saturation mode.

In the present paper we reveal one of the mechanisms for the formation of short dark envelope pulses observed in the experiment, using a two-mode oscillator model in the form of two bidirectionally coupled parametric Vyshkind—Rabinovich oscillators.

The two-mode model of the ring oscillator, in which the parametric decay is performed on each of the two

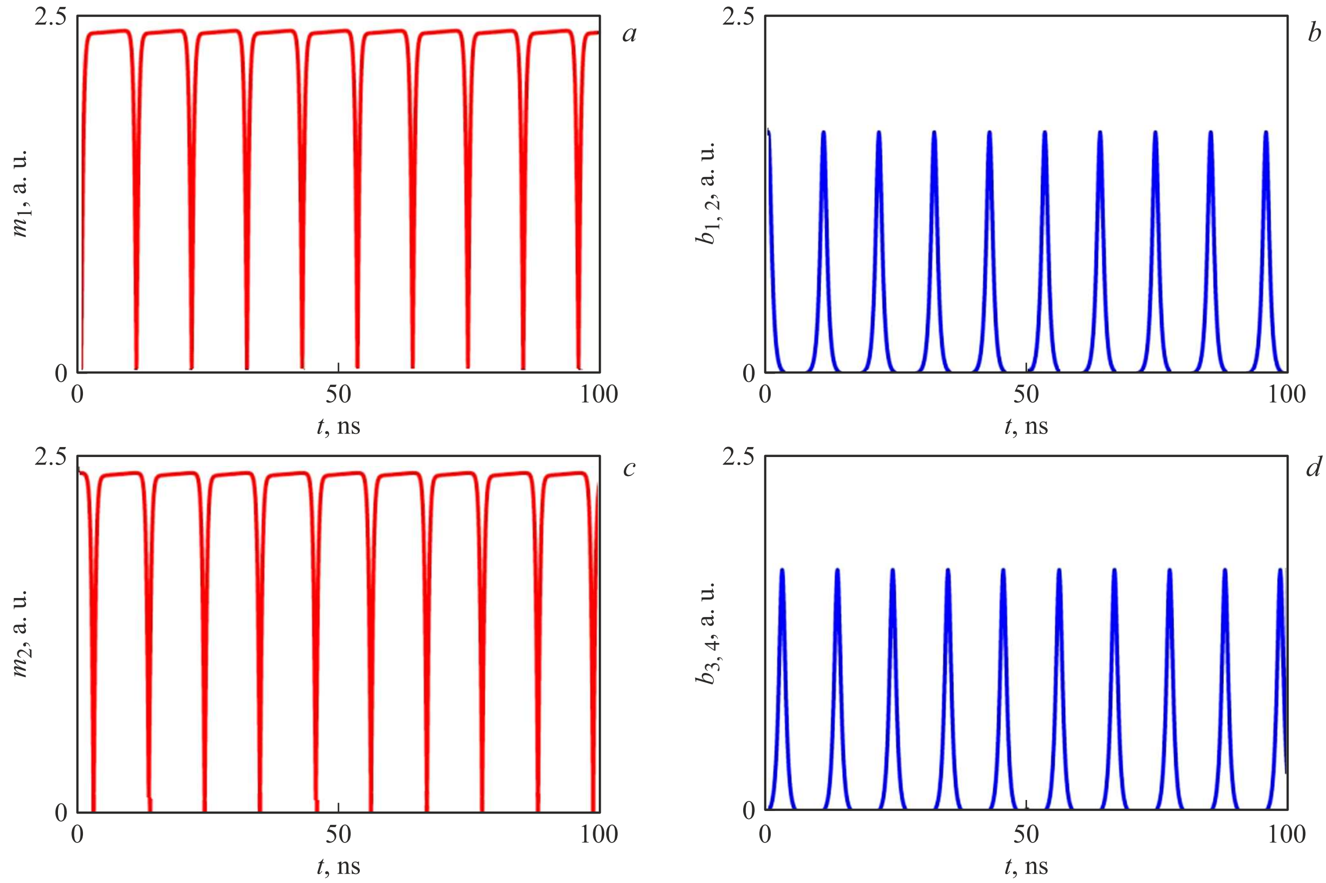

Figure 1. Impulse sequences on unstable (a, c) and parametrically excited (b, d) waves calculated for the first (a, b) and second (c, d) bidirectionally coupled parametric Vyshkind–Rabinovich oscillators. The calculations were performed for μs^{-1}, $\gamma = 0.7\,\mu s^{-1}$, $\delta = 0.01\,\mu s^{-1}$, $c_0 = 1497\,\mu s^{-1}$, $c = 745\,\mu s^{-1}$, $K_1 = 0.595$, $K_2 = 0.6$.

ring modes, and the modes themselves are connected by bidirectional coupling, has the following form:

$$\frac{\partial m_1(t)}{\partial t} = -c_{01} b_1(t) b_2(t) \exp(-i\delta_1 t) + \gamma_1 m_1(t) + K_1 m_2,$$
$$\frac{\partial b_1(t)}{\partial t} = c_{11} m_1(t) b_2^*(t) \exp(i\delta_1 t) - \nu_1 b_1(t),$$
$$\frac{\partial b_2(t)}{\partial t} = c_{21} m_1(t) b_1^*(t) \exp(i\delta_1 t) - \nu_1 b_2(t),$$
$$\frac{\partial m_2(t)}{\partial t} = -c_{02} b_3(t) b_4(t) \exp(-i\delta_2 t) + \gamma_2 m_2(t) + K_2 m_1,$$
$$\frac{\partial b_3(t)}{\partial t} = c_{12} m_2(t) b_4^*(t) \exp(i\delta_2 t) - \nu_2 b_3(t),$$
$$\frac{\partial b_4(t)}{\partial t} = c_{22} m_2(t) b_3^*(t) \exp(i\delta_2 t) - \nu_2 b_4(t), \qquad (1)$$

where $m_{1,2}(t)$ — complex amplitude of the oscillation envelope of the parametrically unstable wave, $b_{1,2,3,4}(t)$ — complex amplitudes of the oscillation envelopes of the parametrically excited waves, $\delta_{1,2}$ — frequency detuning from the parametric resonance in the first and second parametric oscillators, $\gamma_{1,2}$ — increments in the first and second parametric oscillators, $\nu_{1,2}$ — decrements in the first and second parametric oscillators, c_{01}, c_{11}, c_{21}, c_{02}, c_{12}, c_{22} — arbitrary constants in the first and second parametric oscillators, $K_{1,2}$ — bidirectional coupling coefficients. It follows from (1), that in the absence of coupling between the modes ($K_{1,2} = 0$), each is a Vyshkind–Rabinovich [9] parametric oscillator in which the fluctuations in the amplitude of the parametrically unstable wave at a given point in space experience inertia-free linear amplification, and the fluctuations in the amplitudes of the parametrically excited waves are damped. It should be noted that the proposed model (1) can describe the modes of generation of pulse signals realized in the system of two bidirectionally coupled single-mode oscillators, in each of which a parametric decay is realized.

Fig. 1 shows the results obtained from the numerical solution of the system of ordinary differential equations (1) using the Runge–Kutta method of fourth order. The calculations are performed assuming equality of complex amplitudes of oscillations of two parametrically excited waves in each parametric oscillator ($b_1 = b_2$ and $b_3 = b_4$), decrements of unstable waves ($\gamma = \gamma_1 = \gamma_2$), decrements of parametrically excited waves ($\nu = \nu_1 = \nu_2$), frequency detunings from parametric resonance ($\delta = \delta_1 = \delta_2$), and

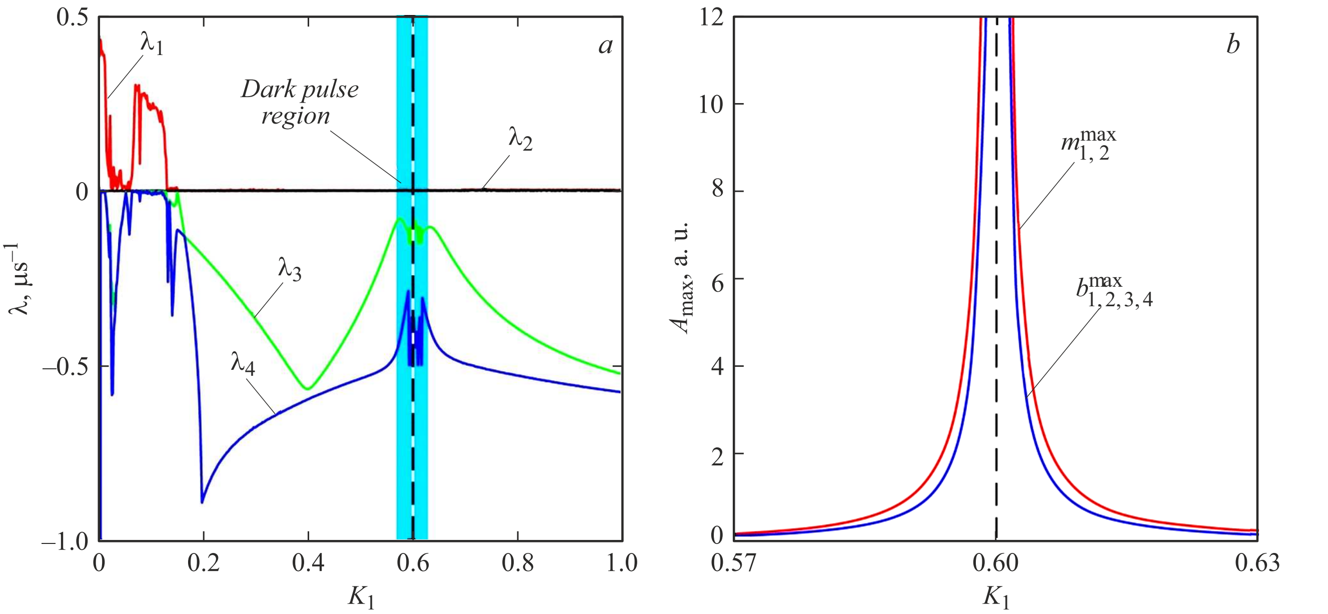

Figure 2. Dependences of the three senior Lyapunov indices $\lambda_{1,2,3}$ (a) and peak values of the oscillation amplitudes of the six waves $m^{\max}_{1,2}$, $b^{\max}_{1,2,3,4}$ (b) on the variation of the coupling coefficient K_1. The calculations were performed for $\nu = 9\,\mu\text{s}^{-1}$, $\gamma = 0.7\,\mu\text{s}^{-1}$, $\delta = 0.01\,\mu\text{s}^{-1}$, $c_0 = 1497\,\mu\text{s}^{-1}$, $c = 745\,\mu\text{s}^{-1}$ and $K_2 = 0.6$. The dashed line shows the instability region of the numerical scheme ($K_1 = K_2$).

arbitrary constants for unstable waves ($c_0 = c_{01} = c_{02}$) and parametrically excited waves ($c = c_{11} = c_{12} = c_{21} = c_{22}$). For the calculations, the values of ν, δ, c_0 and c were taken from the work [8]. It follows from the calculation results presented in Fig. 1 that if the bidirectional coupling coefficients are not significantly different from each other ($K_1 \cong K_2$), then periodic sequences of dark envelope pulses on unstable waves are formed in both parametric oscillators (Fig. 1,a,c) and light envelope pulses on parametrically excited waves (Fig. 1,b,d). The duration of the dark envelope pulses for the selected system parameters is ~ 4 ns, and the duration of the light envelope pulses — is ~ 0.5 ns. The calculated durations of the dark envelope pulses are in almost perfect agreement with the experimental values ($\sim 1-2$ ns) [11]. Since in numerical simulations parametrically coupled sequences of dark and light envelope pulses are formed in a dissipative nonlinear system with gain due to the balance between gain and loss, by analogy with the work [11] such pulses can be classified as dissipative solitons. The narrow region of values of coupling parameters, in which this mode is realized, indicates the experimental difficulties of its realization in a two-mode system with identical parameters. Calculations show that this effect is also realized in a system of two coupled non-identical parametric oscillators, and an increase in the number of parametric oscillators to three (such a system is closer to the experimental case described in [11]) leads to an expansion of the region of values of the coupling parameters at which sequences of dark and light envelope pulses are observed.

To detect the chaotic dynamics of the system of two bidirectionally coupled parametric Vyshkind–Rabinovich oscillators, the Lyapunov exponent spectrum was calculated using the proposed model (1). The calculation was based on a well-known algorithm using equations in variations and Gramm's orthogonalization method–Schmidt [13]. Fig. 2,a shows the results of calculating the three senior Lyapunov indices $\lambda_{1,2,3}$. The calculations are performed for the case when one of the two coupling coefficients is constant (K_2), and the other varies its value (K_1). The results presented in Fig. 2,a show that the chaotic dynamics is observed at relatively small values of K_1, belonging to the interval [0, 0.15]. In the specified interval $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_3 < 0$. In this parameter region, the modes of generation of relaxation pulses of envelope on unstable waves are realized. Generation of dark envelope pulses on unstable waves is observed at large values of K_1, belonging to the two regions $0.57 \leqslant K_1 < 0.6$ and $0.6 < K_1 \leqslant 0.63$ (shown by the fill). Since in the indicated regions $\lambda_{1,2} = 0$, and $\lambda_3 < 0$, these pulse sequences are periodic. At $K_1 = K_2$, the numerical scheme is unstable. It should be noted that in the case of unidirectional coupling, when $K_1 = 0$, $K_2 \neq 0$ (Fig. 2,a) or $K_2 = 0$, $K_1 \neq 0$, the generation of dark envelope pulse sequences is not observed.

Fig. 2,b shows the results demonstrating the variation of peak amplitude values of dark and light envelope pulses depending on the coupling coefficient K_1. It can be seen that these dependencies are resonant, and at $K_1 = K_2$, the peak values of the amplitudes of the light and dark envelope pulses go to infinity.

The obtained results may be of interest for developers of short pulse sources not only in the microwave but also in the optical range.

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Conflict of interest

The authors declare that they have no conflict of interest.

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