Technical Physics Letters, 2024, Vol. 50, No. 6

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Laminar chaos in an experimental system with quasiperiodic modulation of the delay time

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Received May 10, 2023
Revised March 6, 2024
Accepted March 9, 2024

For the first time, the possibility of laminar chaos existence in a time-delay radioengineering generator with quasiperiodic delay time modulation has been experimentally shown. The laminar chaos regimes are compared when the delay time of the generator is modulated by a single-frequency and a two-frequency signal.

Keywords: Time-delay systems, Quasiperiodic delay time modulation, Laminar chaos, Radio physical experiment.

DOI: 10.61011/TPL.2024.06.58475.19624

Equations with a time delay are commonly used to model physical processes that may be characterized only with an allowance made for a finite signal propagation velocity. Model equations for time-delay systems are studied in a number of scientific fields and engineering applications [1–3]. In the general case, the introduction of a time delay into a system makes its dynamics more complex. Even first-order delay-differential equations may then exhibit high-dimensional chaotic behavior [4].

It should be noted that time-delay systems with a constant delay time have been studied the most extensively. If the delay time in a system varies, its dynamics normally becomes even more complex [5–7]. Regimes characterized by strong and rapid fluctuations of the dynamic variable are often observed in both constant-delay and variable-delay systems. These high-dimensional oscillation regimes are referred to as turbulent chaos. However, a time-delay system with certain parameters of modulation of the delay time by an external signal may exhibit a simpler behavior with significantly smaller chaotic attractor dimensions. This type of chaotic behavior, which is observed in systems with a periodically varying delay time, has been identified relatively recently and called laminar chaos [8]. It is characterized by extended laminar phases with an almost constant value of the dynamic variable that are interspersed periodically with short irregular bursts. The value of the variable changes chaotically in transition from one laminar phase to the other, and the duration of laminar phases varies periodically and is specified by a circle map.

The phenomenon of laminar chaos has been examined numerically and experimentally in various systems with a periodically varying delay time [9–12]. The possibility of synchronization of interacting systems of this kind in the laminar chaos regime has been reported [13,14]. It has recently been demonstrated theoretically for the first time in [15] that laminar chaos may also be observed in time-delay systems with quasiperiodic modulation of the delay time. In the laminar chaos regime, the dynamic variable in such systems varies chaotically in transition from one laminar phase to the other, and the duration of laminar phases varies quasiperiodically and may be characterized by a torus map.

The present study is the first to demonstrate experimentally the presence of laminar chaos in a system with quasiperiodic modulation of the delay time. A radioengineering generator with delayed feedback and modulation of the delay time by an external two-frequency signal was chosen as the object of study. The dynamics of this ring generator formed by a delay line with a variable delay time, a nonlinear element, and a low-pass first-order RC filter is characterized by the following equation:

$$RC\dot{V}(t) = -V(t) + f\Big(V\big(t - \tau(t)\big)\Big), \tag{1}$$

where $V(t)$ and $V\big(t - \tau(t)\big)$ are the voltages at the input and output of the delay line, respectively; $\tau(t)$ is the variable delay time; R and C are the resistance and the capacitance of filter elements; and f is the transfer characteristic of the nonlinear element. The cases of periodic variation of the delay time

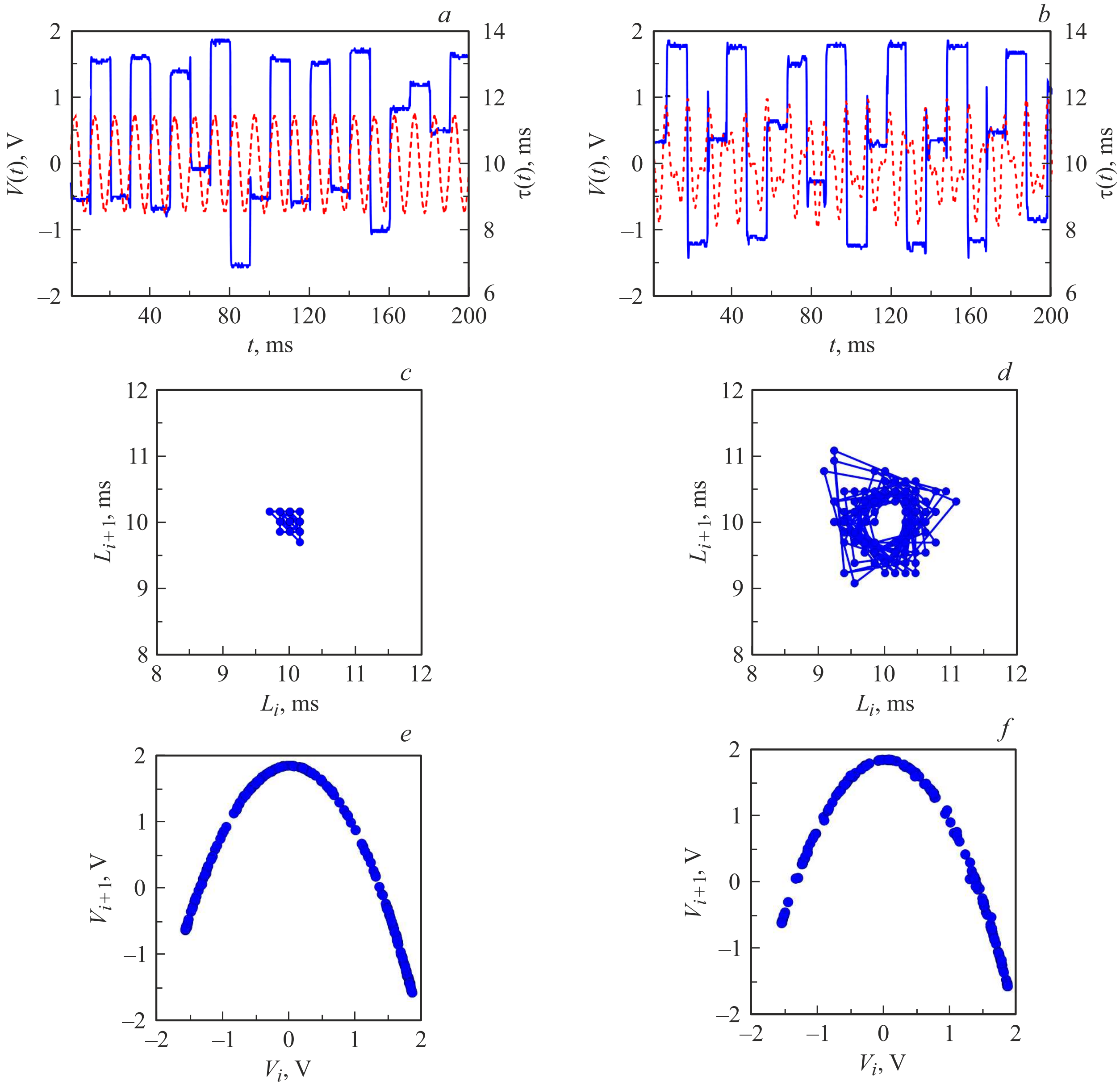
$$\tau(t) = \tau_0 + \tau_1 \sin(2\pi\nu_1 t) \tag{2}$$

and its quasiperiodic variation with two incommensurate frequencies

$$\tau(t) = \tau_0 + \tau_1 \sin(2\pi\nu_1 t) + \tau_2 \sin(2\pi\nu_2 t), \tag{3}$$

where τ_0 is the mean generator delay time, τ_1 and τ_2 are the modulation amplitudes, and ν_1 and ν_2 are the incommensurate modulation frequencies, were examined.

The low-pass filter of the generator was an analog element in our experiment, while the nonlinear element and the generator delay line were digital. The schematic diagram of the experimental setup agrees in general with the diagram presented in [10] and was used to implement harmonic



a, *b* — Temporal realizations of signals $V(t)$ (solid curve) and $\tau(t)$ (dashed curve); *c*, *d* — dependences of the duration of the subsequent laminar phase on the duration of the previous one; *e*, *f* — dependences of the level of the subsequent laminar phase on the level of the previous one. Panels *a*, *c*, and *e* illustrate the case of periodic modulation of the delay time, while panels *b*, *d*, and *f* correspond to quasiperiodic modulation of the delay time.

modulation (2) of the delay time. The sole modification made to the setup was the addition of a summing amplifier that provided an opportunity to apply a two-frequency external signal for control over the delay time modulation.

The control parameters of the generator were as follows: $RC = 0.5$ ms; $f(V) = \lambda - V^2$, where nonlinearity parameter $\lambda = 1.85$; and $\tau_0 = 10$ ms. The frequencies of the external signal, which modulated the delay time, had incommensurate values of $\nu_1 = 99$ Hz and $\nu_2 \approx \nu_1\sqrt{\pi} = 175.47$ Hz. The amplitudes of τ_1 and τ_2 modulation were varied from 0.5 to 2 ms. Signal $V(t)$ and the external delay time modulation signal were digitized with an oscilloscope at a rate of 6.5 kHz. Delay time τ_0 then corresponded to 65 discrete sampling steps.

The results of experimental examination of dynamics of the radioengineering generator with delay time modulation are presented in the figure. For ease of comparison, plots obtained with periodic variation (2) of the delay time are shown on the left, and plots corresponding to its quasiperiodic modulation (3) are shown on the right.

Panel *a* presents temporal realizations of signal $V(t)$ in the laminar chaos regime and signal (2) at $\tau_1 = 1.5$ ms. The dynamics of variable $V(t)$ is the one typical of laminar chaos that is observed when period $T = 1/\nu_1$ of the external influence is close to τ_0. The durations of laminar phases are roughly the same, and their levels, which are specified by $V(t)$, change chaotically in transition from one laminar phase to the other. Note that laminar phases modeled numerically are virtually flat horizontal plateaus [8], while the experimental plots (see panel *a*) feature weak $V(t)$ oscillations within laminar sections. This is attributable to experimental noise. Panel *b* presents temporal realizations of system (1) under quasi-harmonic variation of $\tau(t)$ and signal (3) at $\tau_1 = 1.3\,\text{ms}$ and $\tau_2 = 0.7\,\text{ms}$. Laminar phases have different durations in this case; just as in panel *a*, their levels change chaotically.

It is convenient to analyze durations L_i of laminar phases by plotting dependences $L_{i+1}(L_i)$ (i.e., dependences of the duration of the subsequent laminar phase on the duration of the previous one). In panel *c*, dots in plane (L_i, L_{i+1}) are grouped in the center of the figure and form a small cloud. This is indicative of the fact that the durations of laminar phases remain approximately constant in the system with periodic variation of the delay time. In panel *d*, dots in plane (L_i, L_{i+1}) form a circular figure in the center of the figure, indicating that the duration of laminar phases in the system with two-frequency modulation of the delay time varies quasiperiodically. At a sampling rate of 6.5 kHz used in the experiment, dots in planes (L_i, L_{i+1}) cannot be spaced by less than 0.154 ms. For illustration purposes, we drew lines between successive dots in panels *c* and *d* of the figure.

Dependences $V_{i+1}(V_i)$ (i.e., dependences of level V_{i+1} of the subsequent laminar phase on level V_i of the previous one) are presented in panels *e* and *f*. In both cases (with periodic and quasiperiodic modulation of the delay time), the obtained curves agree well with true transfer characteristic f of the nonlinear generator element. Note that the experimental data presented in panels *d* and *f* agree qualitatively with the numerical results reported in [15].

Thus, it has been demonstrated experimentally for the first time that laminar chaos may be observed in a system with quasiperiodic variation of the delay time. The nonlinear function of a radioengineering generator in the laminar chaos regime with quasiperiodic modulation of the delay time was reconstructed, and it was revealed that the duration of laminar phases varies quasiperiodically in this generator.

Funding

This study was carried out under the state assignment of the Saratov Branch of Kotelnikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by D.Safin