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Synchronization and motion control of an ensemble of mobile agents

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This article several ways to implement sequential, parallel and in the form of a given configuration of the motion of an ensemble (swarm) of mobile agents using the effect of chaotic phase synchronization. The possibility of controlling the motion of the ensemble is also shown.

Keywords: Mobile agent, synchronization, ensemble motion control, Rossler oscillator.

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The ensemble of mobile agents to study and analysis of the collective dynamics during last years is widely used in different areas of science and technic [1,2]. Synchronization is the main object of the major of studies relating the collective dynamic [3], the synchronization greatly depends on topology of the ensemble couplings [4]. We can emphasize three main types of couplings in the ensembles: local (coupling with adjacent neighbors), non-local (coupling not only with adjacent neighbors), global (coupling as per principle „each with each" [5]). Most frequently such couplings in studied mathematical models have stationary nature, i.e. topology and force of couplings do not change with time. But actually the topology of major of structures is not permanent, couplings between elements can appear and disappear, strengthen or weaken [6]. Systems, where additionally to the coupling force the nodes position can vary, can be easy studied as the ensemble of mobile agents [7]. Thus, it was possible to study, for example, the synchronization of mobile robots [8], localization of objects of distributed tracking system [9]. Paper [10] presents the results of behaviour control of the ensemble of mobile agents on plane, reviews some configurations of motion (sequential and parallel motion of mobile agents on plane).

As the mobile agent we consider a point moving in three-dimensional space (x, y, z), so its trajectory completely coincides with the trajectory of corresponding to it chaotic oscillator. As a candidate for the chaotic trajectory generation any dynamic system can be taken, which can generate chaotic oscillations, for example, systems of Lorenz [10], Rossler [10], Chua et al. In present paper, without loss of generality, let us consider the Rossler oscillator

$$\begin{cases} \dot{x}_i = -w_i y_i - z_i, \\ \dot{y}_i = w_i x_i + a_i y_i, \qquad i = \overline{1, N}, \\ \dot{z}_i = b_i + z_i(x_i - c_i), \end{cases} \tag{1}$$

where a_i, b_i, c_i — positive parameters. In further experiments assume $a_i = 0.22$, $b_i = 0.1$, $c_i = 8.5$, parameter characterizing time scale of oscillations, $w_i \in$ [0.93,1.07]. The system parameters are chosen such that, for initial conditions sufficiently close to zero (in this paper, we consider a cube with an edge length of 10 with a center at the origin of coordinates), the phase trajectories do not go to infinity, but are attracted to a chaotic quasi-attractor (see, for example, [11]). Formally, the modeling and the indicated bifurcation scenario give only a chaotic set.

Organization of motion control of the ensemble of mobile agents in space can be divided into two stages: determining definite configuration of agents and bringing agents to a given trajectory of motion.

To solve the set objectives we used the methods of chaotic phase synchronization (to provide the ensemble of agents with definite configuration of their location in three-dimensional space) and forced synchronization (to ensure the achievement of given law of motion in space). The chaotic phase synchronization means the process of achievement, under rather strong coupling between the interacting chaotic oscillators, of same averaged frequency of oscillations and limited n magnitude phase difference (phase shift) of chaotic oscillations [12].

Our study objective is creation of such ensemble of agents, in which the agent interactions with neighbours start only at their sufficient (preset) proximity, so at any configuration the coupling between i-th and j-th agents will meet the condition

$$d = \begin{cases} d' & \text{for } (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 < r^2, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $d' = \text{const}$ — coupling parameter. In our study we take $d' = 0.2$, $r = 4$, i.e. agents start interaction upon entering the ball with radius r.

1. Sequential motion of agents

Let's add to system (1) „attractive" coupling along the coordinate y as follows:

$$\begin{cases} \dot{x}_i = -w_i y_i - z_i, \\ \dot{y}_i = w_i x_i + a_i y_i + \sum_j d(y_j - y_i), \quad i = \overline{1, N}, \\ \dot{z}_i = b_i + z_i(x_i - c_i). \end{cases} \tag{3}$$

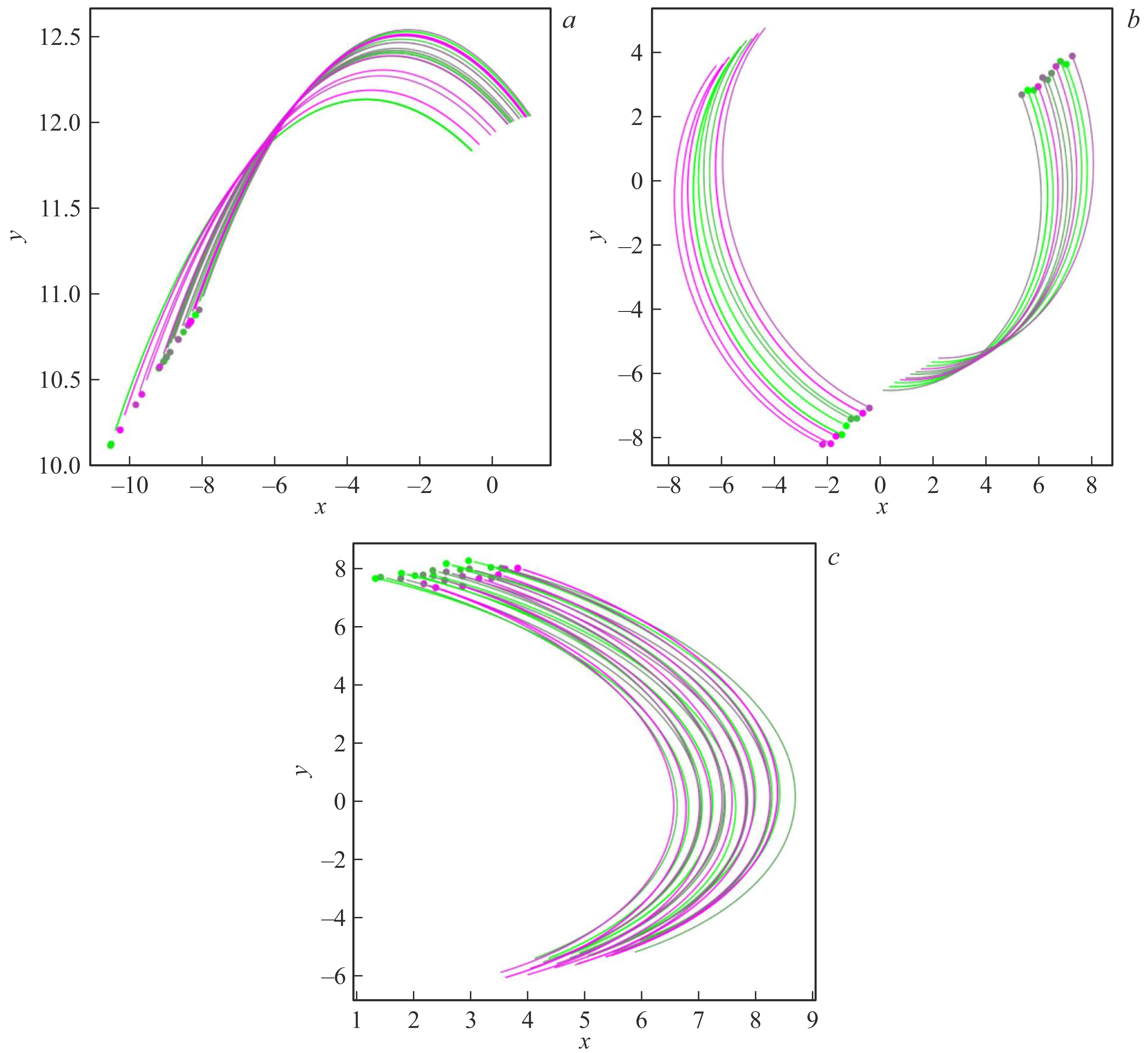

Figure 1. Synchronization of ensemble of mobile agents. a — complete synchronization during implementation of sequential motion, b — cluster synchronization of the swarm during parallel motion (later two clusters will merge into one), c — giving the swarm a structure of square of 5×5 elements.

Initial conditions for all oscillators are different. With time and due to chaotic motions the depicting point sooner or later come closer to a distance less r. Interaction between agents appears, and, if the coupling is rather strong, the agents are synchronized. Further more and more agents come close to distance r and are synchronized. Clusters of synchronized agents appear. At that the phase trajectories are close, but phase shift is observed. As a result the global chaotic phase synchronization is achieved, and as result of this the agents move one after another „along chain“. Result of numerical experiments (global synchronization of agents) is presented in Fig. 1, a.

2. Parallel motion of agents

Besides the sequential motion of agents it is possible to realize their parallel motion — motion in the form of a „single formation“. The parallel motion is achieved by addition of „repulsive“ coupling along coordinate x

$$\begin{cases} \dot{x}_i = -w_i y_i - z_i + \sum_j d(x_j - x_i) + \sum_j \frac{d}{x_i - x_j}, \\ \dot{y}_i = w_i x_i + a_i y_i + \sum_j d(y_j - y_i), \quad i = \overline{1, N}, \\ \dot{z}_i = b_i + z_i(x_i - c_i). \end{cases} \tag{4}$$

Similarly to the case of sequential motion with time pairs, triples etc. are formed in parallel to the moving agents. In contrast to the sequential motion during approach of i-th and j-th agents the introduced by the above method coupling results in occurrence of opposite directed

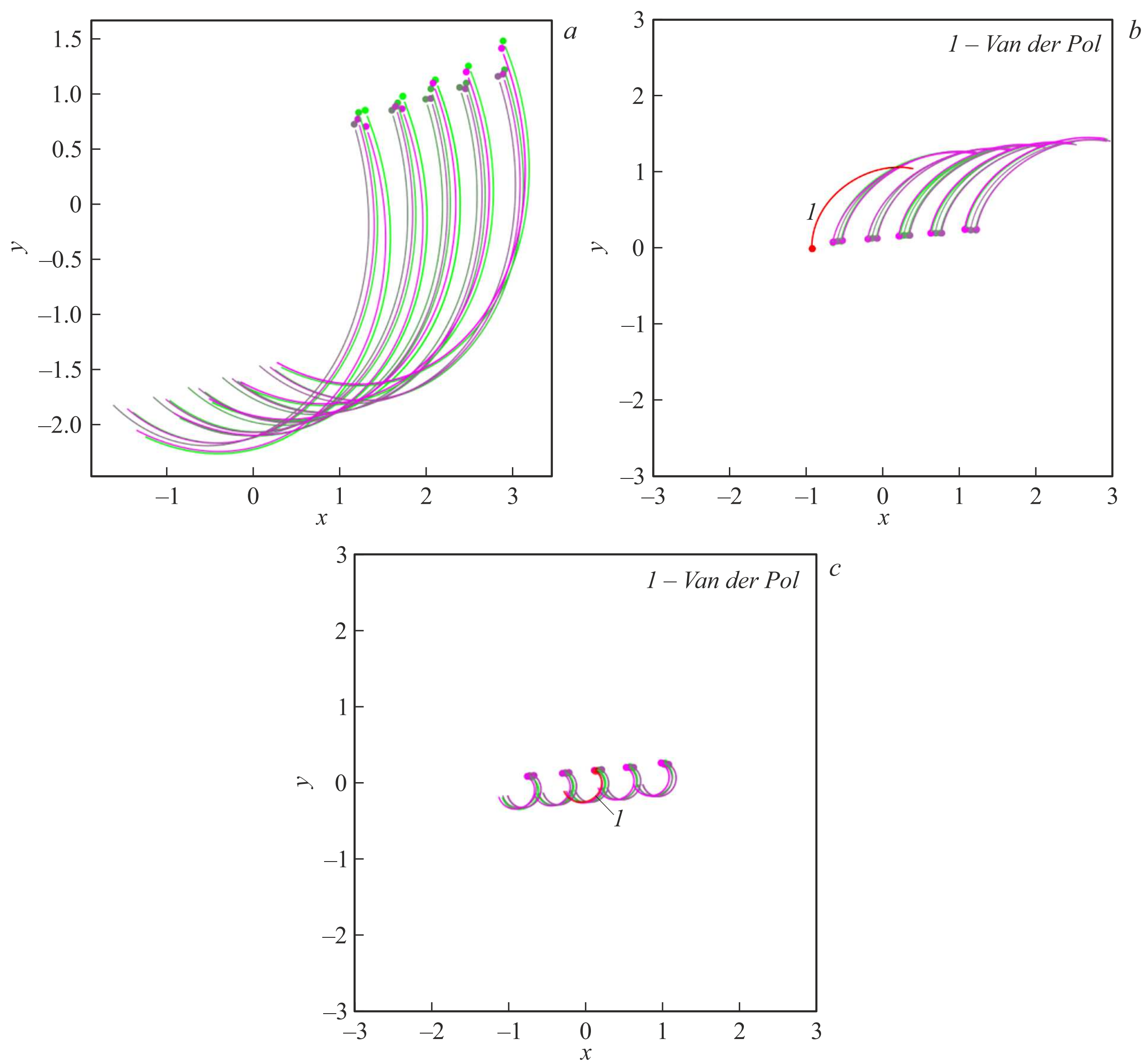

Figure 2. Ensemble catching by the agent set by Van der Pol equation. *a* — swarm state before new agent addition, *b* — new agent start all agents attraction, *c* — swarm moves along the focus trajectory to equilibrium state of agent, set by Van der Pol equation. Parameter $\mu = -0.2$.

interaction forces, which forces the agents to be at a certain distance perpendicular to the motion, i.e. due to the „repulsive" coupling the agents begin to move in a single row, parallel to each other. The elements behaviour during this coupling is illustrated by Fig. 1, *b*.

3. Swarm of mobile agents provision with structure having different geometric forms

In this case we use definite combinations of couplings of sequential and parallel motion to obtain structures of different geometric forms (rectangle, circle, triangle, etc.) Without loss of generality, let us consider the configuration „rectangle".

To organize such motion we introduce parameters k_{str} and k_{col} – number of elements in one line and one column respectively. Then for each separate line we need to add „repulsive" coupling. At the same time we need to coupling the line with neighboring lines by „attractive" coupling:

$$\begin{cases} \dot{x}_i = -w_i y_i - z_i + \sum\limits_{j=s(i)}^{s(i)+k_{str}-1} \left[d(x_j - x_i) + \frac{d}{x_i - x_j} \right] = f_{grid}, \\ \dot{y}_i = w_i x_i + a_i y_i + \sum\limits_{j=\max(0;s(i)-k_{str})}^{\min(N;s(i)+2k_{str}-1)} d(y_j - y_i) = g_{grid}, \\ \dot{z}_i = b_i + z_i(x_i - c_i) = h_{grid}, \end{cases} \quad (5)$$

where $i = \overline{1, N}$; $k_{str}k_{col} = N$; $s(i) = i - i\mathrm{mod}(k_{str}) + 1$ — first element of current line for element i; $i\mathrm{mod}(k_{str})$ — remainder of i division by k_{str}.

With time due to synchronization the agents, couplinged by both „attractive", and „repulsive" couplings, start formation of groups of agents moving sequentially and in parallel, and are synchronized into groups. Further agents are combined into single cluster as structure given in Fig. 1, c.

Now, let's consider the objective of all agents deactivation — their motion into set space point. For this additionally to agents set by the system (1) we introduce one more agent moving along set trajectory. As such agent we take Van der Pol oscillator

$$\begin{cases} \dot{X} = -Y, \\ \dot{Y} = WX + \mu(1 - X^2)Y, \end{cases} \tag{6}$$

where μ — negative parameter, $W = 1$. At given set of parameters the system (6) has a single attractor — stable equilibrium state in point (0,0). Just into this point the entire ensemble shall come. The larger (by magnitude) μ is the quicker ensemble will enter the set point. Obviously, the control trajectory can be any. This can be set regular or chaotic motion.

For the rest agents we take equation (5) and add to all elements the coupling with Van der pol equation as follows:

$$\begin{cases} \dot{x}_i = f_{grid}, \\ \dot{y}_i = g_{grid} + D(Y - y_i), \quad i = \overline{1, N}, \\ \dot{z}_i = h_{grid}, \end{cases} \tag{7}$$

where coupling D works similarly to coupling d, but upon mobile agent approaching the agent moving according to Van der Pol equation. As result all agents enter the vicinity of equilibrium state (0,0) (Fig. 2).

As a result of the study of synchronization and control of the collective dynamics of swarm of mobile chaotic agents — the Rössler oscillator — it turned out to be possible to obtain specified types of motions of the mobile agents in three-dimensional space: sequential (one after another at a certain distance, which can be controlled), parallel („single front") and motion in the form of specified geometric structures. All suggested methods of structure formation of the swarm of mobile agents can be considered as self-organization processes. It was demonstrated that using „external" agent we can specify the required motion of swarm. For example, „sit" swarm in specified point.

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Conflict of interest

The authors declare that they have no conflict of interest.

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