Low temperature thermal magnetoconductance of metals

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A low temperature transverse thermal magnetoconductance of metals and semimetals is treated theoretically. It is shown that its high magnetic field behavior is determined by the characteristic time τ_ε of jumps from one cyclotron circle to another due to electron-phonon collisions rather than by the transport time $\tau_{\rm tr}$ that determines the conductance. The phonon-electron drag contribution to the magnetoconductance is also discussed.

1. Introdiction

During recent years, various transport phenomena of quantum wells were investigated both theoretically and experimentally. One can list such properties as electric conductance, thermoelectric and thermomagnetic phenomena. The topic of the present paper is theoretical consideration of the heat magnetoconductance of 2D and 3D samples of metals and semimetals. It is important to monitor the distribution of heat fluxes during operation of nanoelectonic devices as their heating may influence their functioning. The rate of the Joule heat generation by electric current in nanostructures has been studied by one of the authors in Ref. [1] and also by Muradov and Gurevich in Ref. [2] but it is often important to know the spatial distribution of the generated heat fluxes. Moreover, more often than never it is essential to know which part of the heat is transferred by the conduction electrons rather than by phonons. Indeed, with the conduction electrons, one can have ways to redirect the heat fluxes but this is more difficult to do with fluxes transferred by the phonons. Therefore, it is important to know in each case which part of the heat flux is transferred by the conduction electrons. Investigation of the heat magnetoconductance may help here as the external magnetic field influences the electron part of the heat conductance and leaves intact the phonon part. So, it allows us to discern the electron and phonon parts of thermal conductance (the latter has been investigated in Ref. [3]).

The purpose of the present paper is a consideration of the heat conductance in magnetic field in the low temperature region where the electron-phonon interaction is essentially inelastic and the Wiedemann–Franz law does not hold [4]. The opposite, hight temperature region, has been investigated previously [5] within framework of the Wiedemann–Franz law. A special attention is payed to consideration of the electron-phonon drag effects.

We will assume that the magnetic field $\mathbf{B}$ is parallel to the z axis, the temperature gradient ∇T is oriented along the x axis and will discuss the heat conductance $\varkappa_{xx}(B)$. We will also assume that in external magnetic field the electron trajectories in the crystal momentum space are closed. This condition is automatically satisfied for all the metals with closed Fermi surfaces and is valid for a number of solid angular intervals of $\mathbf{B}$ directions for the metals with open Fermi surfaces. It is also satisfied for almost all the semimetals and semiconductors. The influence of the open Fermi surfaces on the thermomagnetic effects have been investigated both theoretically [6] and experimentally (see, for example, [7]). Their role is usually drastic and requires a special treatment.

2. Preliminary considerations

There are two relaxation times that determine transport phenomena in pure metals at low temperature region, i.e. τ_ε and $\tau_{\rm tr}$. The first one is determined by the rate of energy transfer of the order of T in the course of electron-acoustic phonon (e-ph) collisions (we assume $k_{\rm B} \equiv 1$ throughout the paper). Importance of the e-ph interaction for the transport phenomena in pure metals has been investigated experimentally in Ref. [8] and in a great number of other papers. The second one is determined by the rate of electron crystal momentum relaxation in the course of e-ph collisions. Indeed, the electron crystal momentum variation as a result of each such collision is of the order

$$\Delta p \sim \frac{T}{\hbar s} \ll p_F, \tag{1}$$

where s is the sound velocity. Δp is much smaller than the Fermi quasimomentum p_F. As a result, one has as a relaxation mechanism a sort of electron diffusion over the Fermi surface, so that

$$\frac{1}{\tau_{\rm tr}} \propto \overline{(\Delta p)^2}, \tag{2}$$

where the symbol on the right-hand side means an average over many collisional events (see, for instance Ref. [3] or [9]).

In 3D and 2D cases, the first relaxation time determines the conductivity σ that can be estimated as

$$\sigma \sim \frac{ne^2\tau_{\mathrm{tr}}}{m}, \quad \text{where} \quad \tau_{\mathrm{tr}} \sim \tau_0 \left(\frac{\Theta}{T}\right)^5. \tag{3}$$

Here m is the electron effective mass while $\tau_0 \sim \hbar/\Theta$, Θ being the Debye temperature. The second one determines the thermal conductance κ as

$$\varkappa \sim \frac{p_F^2}{\hbar^3} \frac{T\tau_\varepsilon}{m}, \quad \text{where} \quad \tau_\varepsilon \sim \tau_0 \left(\frac{\Theta}{T}\right)^3. \tag{4}$$

For $\Theta/T \gg 1$, these two relaxation times can differ by orders of magnitude.

In the present paper we wish to elucidate the following point. As is well known, because of the helicoidal motion of electrons in the magnetic field B we have for the transverse magnetoconductivity $\sigma_{xx}(B)$

$$\sigma_{xx}(B) \sim \frac{\sigma}{1 + (\Omega\tau_{\mathrm{tr}})^2}, \tag{5}$$

where

$$\Omega = \frac{eB}{mc} \tag{6}$$

is the cyclotron frequency. Or, more rigorously, for closed Fermi surfaces one can indicate two types of asymptotic behavior, i. e.

$$\sigma_{xx}(B) \sim \frac{\sigma}{(\Omega\tau_{\mathrm{tr}})^2} \quad \text{for} \quad (\Omega\tau_{\mathrm{tr}})^2 \gg 1 \tag{7}$$

and $\sigma_{xx} = \text{const}$ for $(\Omega\tau_{\mathrm{tr}})^2 \ll 1$.

Turning to thermal magnetoconductance $\varkappa_{xx}(\mathbf{B})$, one should expect a relation of the type (5). A noteworthy problem is as to which of the two times of relaxation, τ_ε or τ_{tr}, enters the equation for $\varkappa_{xx}$. At a first sight, the variation of a heat flux in a strong magnetic field takes place because of bending the electron trajectories in the real space, i. e. because of a helical motion of electrons that shows itself in the $\mathbf{B}$ dependence of $\sigma_{xx}(\mathbf{B})$. It would have meant that the characteristic time monitoring the $\varkappa_{xx}(\mathbf{B})$ variation were τ_{tr}. We will see below, however, that this illustrative reasoning is wrong. We will demonstrate that the decrease of the thermal flux is determined by the probability of the jumps from one cyclotron circle to another due to e-ph collisions, i. e. by $1/\tau_\varepsilon$. Also, we will give estimations for the drag effects. According to this estimations one can observe a contribution to $\varkappa$ due to this effects in a semimetals and semiconductors.

We would like to mention that for the semimetals and degenerate semiconductors the estimates Eqs. (3) and (4) should be modified. Namely, Θ in these equations should be replaced by a smaller quantity sp_F that depends on the electron concentration (it is assumed that still the electron concentration is so high that sp_F is bigger than T). In view of the aforementioned we would like to indicate also the following. In 3D samples, the carriers are usually provided by donors that are within the same structure, and naturally the temperature dependence of the thermal conductance discussed in the present paper is observable only provided the electron-phonon scattering is predominant over the electron-impurity (donor) scattering. For 2D samples where the carriers may be provided by a (relatively remote) spacer, so that the impurity scattering may be less intensive, the situation can be more favorable for observation of the discussed effects. One should keep in mind, though, that for 2D case our results are valid provided the effects of lateral quantization are of no importance (a thick plate). Otherwise the e-ph interaction would be modified because of the quantum effects.

3. Electron-phonon collisions and thermal conductance

To begin with, we write the Boltzmann equation for the electron distribution function

$$F = F_0 + f,$$

where F_0 is the equilibrium distribution function where f is its nonequilibrioum part

$$-v_x \frac{\partial F_0}{\partial \varepsilon} \frac{\varepsilon - \mu}{T} \nabla T + \mathbf{v} \frac{\partial f_{\mathbf{p}}}{\partial \mathbf{r}} + \frac{e}{c} [\mathbf{v} \times \mathbf{B}] \frac{\partial f_{\mathbf{p}}}{\partial \mathbf{p}} = St_{\text{e-ph}}[f, N]. \tag{8}$$

Here $St_{\text{e-ph}}p[f, N]$ is the term describing the electron-phonon collisions.

We assume that the magnetic field $\mathbf{B}$ is parallel to the z-axis, while the temperature gradient ∇T is oriented along the axis x. Further on we will introduce instead of p_x, p_y and p_z the variables ε, p_z, ϕ. Here the dimensionless variable ϕ is a trajectory time of the electron multiplied by Ω (see for instance Abrikosov [3], §5.1). We have

$$v_x \frac{\partial f_{\mathbf{p}}}{\partial x} + \Omega \frac{\partial f_{\mathbf{p}}}{\partial \phi} + \left[\frac{\partial f_{\mathbf{p}}}{\partial t}\right]_{\text{coll}} = St_{\text{e-ph}}[f, N] + v_x \frac{\partial F_0}{\partial \varepsilon} \frac{\varepsilon - \mu}{T} \nabla T. \tag{9}$$

To gain understanding of the case of closed electron trajectories in magnetic field it is sufficient to consider the simplest form of the electron spectrum

$$\varepsilon_{\mathbf{p}} = p^2/2m \tag{10}$$

and

$$v_x = v_\perp \cos\phi, \quad v_y = -v_\perp \sin\phi, \quad v_\perp^2 + v_z^2 = v^2, \quad v = p/m, \tag{11}$$

where m is the effective mass.

We will study the asymptotic behavior of the thermal conductance for $\Omega\tau \gg 1$. For this purpose, one can iterate the Boltzmann equation in powers of the small parameter $(\Omega\tau)^{-1}$. The first order term term is proportional to $1/\Omega$

and independent of τ (a collisionless term). It is responsible for the flux in the y-direction. To calculate the heat flux in the x-direction we need the next approximation. To get it, we insert the first approximation

$$f_{\mathbf{p}}^{(1)} = -v_y \frac{\partial F_0}{\partial \varepsilon} \frac{\varepsilon - \mu}{T} \nabla T$$

into the collision term.

3.1. Electron contribution to thermal conductance. To calculate the electron contribution, we will assume the phonons to be in equilibrium at temperature T. The linearized collision term is

$$St_{\text{e-ph}}[f^1, N_0] = -A(\varepsilon, p_\perp) \sin\tau,$$

$$A(\varepsilon, p_\perp) = \frac{\alpha \nabla T}{\Omega} \frac{T^3}{s^3} \frac{p_\perp}{|p|} I\left(\frac{\varepsilon_p - \mu}{T}\right). \qquad (12)$$

Here we have assumed the matrix element of electron-phonon (e-ph) interaction to have the form

$$W = \alpha|k|, \quad \text{where} \quad \alpha \sim \frac{\Theta a_0^2}{m}. \qquad (13)$$

Here k is the phonon wave vector and α_0 is the lattice constant. This estimate is valid for typical metals [9]. (Below we will use the system of units with $\hbar = 1$ and restore it in resulting equations.) Now,

$$I(\eta) = \frac{1}{2(\pi)^2} \int_0^\infty dx x^3 \frac{\exp x}{(\exp x - 1)^2} \left[\frac{2}{\exp\eta + 1} - \frac{1}{\exp(\eta - x) + 1} - \frac{1}{\exp(\eta + x) + 1}\right].$$

Further on we will calculate the flux averages over the sample cross section. This means that we will look for a spatially homogeneous solution. Thus, we can write the Boltzmann equation for $f^{(2)}$ as sollows

$$v_x \frac{\partial f_{\mathbf{p}}^{(2)}}{\partial x} + \Omega \frac{\partial f_{\mathbf{p}}^{(2)}}{\partial \phi} + \frac{1}{\tau^{(0)}} f_{\mathbf{p}}^{(2)} = -A(\varepsilon, p_\perp) \sin\phi. \qquad (14)$$

We have introduced an infinitesimal damping $1/\tau^{(0)}$ for intermediate calculations; it will not appear in the final equations. The damping for $\phi \to +\infty$ solution of Eq. (14) does not depend on x and is given by

$$f^{(2)}(\tau) = \frac{1}{\Omega} A(\varepsilon, p_\perp) \frac{v_x(\tau)}{v_\perp}. \qquad (15)$$

The x-component of the heat flux is

$$Q = \int \frac{d^3 p}{(2\pi)^3} v_x(\varepsilon_p - \mu) f^{(2)}(\mathbf{p}).$$

As a result,

$$Q = -\frac{1}{3\pi^2} \frac{\alpha p_F T^4}{\Omega^2 s^3} \int d\eta\eta \left(-\frac{\partial f_0}{\partial \eta}\right) I(\eta) \nabla T \qquad (16)$$

and making use of this equation one can easily get the following order-of-magnitude estimate for the thermal conductance $\varkappa$

$$\varkappa_{xx} \sim \varkappa_0 \frac{1}{(\tau_\varepsilon \Omega)^2}. \qquad (17)$$

Here $\varkappa_0 \sim \varepsilon_F p_F \Theta$, is the thermal conductance for $B = 0$ [9], and τ_ε^{-1} given by Eq. (4) is the low temperature rate of electron-phonon scattering [3]. Physically, this means that it is the reciprocal probability of the e-ph scattering rather than the time τ_{tr} that determines the field-dependent variation of thermal conductance.

3.2. Drag of electrons by phonons. To treat the phenomenon of electron-phonon drag one should take into consideration that the phonon distribution function deviates from equilibrium and has the form

$$N(\mathbf{k}) = N_0(T) + \Delta N, \quad \Delta N = -(\partial N_0/\partial \omega)\chi(\mathbf{k}), \qquad (18)$$

where $N_0(T)$ is the equilibrium Bose function while the second term on the right-hand side is a small nonequilibrium part. We will present the nonequilibrium part of the electron distribution function in an analogous form

$$f_{\mathbf{p}} = -(\partial f_0/\partial \varepsilon)\varphi(\mathbf{p}). \qquad (19)$$

To find ΔN one should solve the Boltzmann equation for the phonons with a collision term describing the phonon distribution function variation due to the phonon emission and absorption

$$St_{\text{ph-e}}[F, N] = \int \frac{d^3 p}{(2\pi)^3} W\big[F_{\mathbf{p}}(1 - F_{\mathbf{p-k}})(1 + N_{\mathbf{k}}) - F_{\mathbf{p-k}}(1 - F_{\mathbf{p}})N_{\mathbf{k}}\big]\delta(\varepsilon_{\mathbf{p-k}} + \omega - \varepsilon_{\mathbf{p}}). \qquad (20)$$

The linearized collision term is [9]

$$St_{\text{ph-e}}[f, N] = \frac{\partial N_0}{\partial \omega} \int \frac{d^3 p}{(2\pi)^3} W\left(\varphi_{\mathbf{p-k}}^1 - \varphi_{\mathbf{p}}^1 + \chi(\mathbf{k})\right) \times \big(F_0(\varepsilon_{\mathbf{p-k}}) - F_0(\varepsilon_{\mathbf{p}})\big)\delta(\varepsilon_{\mathbf{p-k}} + \omega - \varepsilon_{\mathbf{p}}). \qquad (21)$$

The terms with φ^1 in Eq. (21) are responsible for the so-called mutual electron-phonon drag (see, for instance, Ref. [10]) or, in other words, for exchange of the crystal momentum between the electrons and phonons. As for the term with χ, it makes a contribution to the collision term of the Boltzmann equation for the phonons

$$\alpha m^2 T I^+(\omega)\chi(\mathbf{k}) = (\nabla T, \mathbf{s})\frac{\omega}{T} + \frac{\alpha m T k_y}{\Omega} \nabla T \left(I^-(\omega) + \frac{m s^2 - \omega}{2T}\right), \qquad (22)$$

where

$$\mathbf{s} = \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{k}}$$

is the sound velocity. The first term on the right-hand side of Eq. (22) is the simple, or direct, drag effect [11] — the phonon flux is brought about by the temperature gradient, and the phonons transfer their crystal momentum to the electrons. Here

$$I^-(\omega/T) = \frac{1}{4\pi^2}\int d\eta\eta\left[F_0\left(\eta - \frac{\omega}{T}\right) - F_0(\eta)\right], \qquad (23)$$

while

$$I^+(\omega/T) = \frac{1}{4\pi^2}\int d\eta\left[F_0\left(\eta - \frac{\omega}{T}\right) - F_0(\eta)\right]. \qquad (24)$$

For ω/T of the order of 1, both I^- and I^+ are also of the order of 1.

In the course of derivation of Eq. (22) we had to integrate over all directions of the electron crystal momentum. There is a cosine of the angle between $\mathbf{k}$ and $\mathbf{p}$ in the argument of the δ-function. To do the integration it is convenient to choose the coordinate system where the azimuthal angle is counted off from the vector $\mathbf{k}$ while the distribution function depends on the momentum projection on the plane $\mathbf{B}\otimes\nabla T$.

We begin with discussion of a mutual drag contribution into the thermal megnetocunductance. It corresponds to the last term on the right-hand side of Eq. (22). One should insert this nonequilibrium part of the phonon distribution function into the linearised collision term $St_{\text{e-ph}}$ of the Boltzmann equation for the electrons that is given by

$$St^{(1)}_{\text{e-ph}} = -\int d^3kW(\partial N_0/\partial\omega)\left[F_0(\varepsilon_{\mathbf{p}-\mathbf{k}}) - F_0(\varepsilon_{\mathbf{p}})\right]\chi(\mathbf{k})$$
$$\times\left[\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{k}} - \omega_k) + \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{k}} + \omega_k)\right]. \qquad (25)$$

After integration over the phonon wave vectors we get a mutual drag term on the right-hand side of the Boltzmann equation for electrons

$$-\frac{\alpha T^5}{\Omega s^5 p^2}\,\frac{p_y}{p}\,\nabla T G(\varepsilon_{\mathbf{p}}/T). \qquad (26)$$

Here

$$G(\varepsilon_{\mathbf{p}}/T) = \frac{1}{8\pi^2}\int d\eta\big(\partial N_0(\eta)/\partial\eta\big)\eta^4\left[\frac{I^-(\eta)}{I^+(\eta)} + \frac{ms^2}{2T} - \eta/2\right]$$
$$\times\left\{F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} + \eta\right) + F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} - \eta\right) - 2F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}\right)\right.$$
$$\left. + \frac{2ms^2}{T\eta}\left[F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} - \eta\right) - F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} + \eta\right)\right]\right\}. \qquad (27)$$

This term is to be compared with the right-hand side of Eq. (12). We get, as a result, the ratio of the mutual drag contribution into the thermal magnetoconductance ($\varkappa_{\text{md}}$) to the ordinary electron one

$$\frac{\varkappa_{\text{md}}}{\varkappa} \sim \left(\frac{T}{s p_F}\right)^2 \sim \left(\frac{T\hbar}{p_F a_0 \Theta}\right)^2. \qquad (28)$$

In metals we have considered so far $p_F a_0 \sim 1$ so that one can discard the mutual drag terms. However, in semimetals and degenerate semiconductors one can have $p_F a_0 \ll 1$. Traditionally, for this case the matrix element of electron-phonon interaction is expressed *via* deformation potential constant(s) (for instance, [3]) but is cancelled out in the ratio (3.21) and this estimate still holds. This means that in semiconductors, both 3D and 2D, this contribution may become predominant due toe the $p_F a_0 \ll 1$ relation.

We will not write down explicitly an expression for $\varkappa_{\text{md}}$; it can be, in principle, obtained making use of Eqs. (26) and (27). As for comparison with experiment the main problem will be to single out the magnetic field dependent electron contribution (in comparison with the lattice one) and to take into consideration the realistic form of electron spectrum. We would like to mention that Eq. (17) still remains valid provided one inserts there the appropriate for semimetal and semiconductor expressions for the e-ph scattering probability and for $\varkappa_0$.

Now we will turn discussion of an ordinary drag contribution. It is described by the first term on the right-hand side of Eq. (22). This part of the nonequilibrium phonon distribution function is proportional to k_x, i.e. to the x-component of the phonon wave vector. Incerting it into the linearised collision term (25), we get after solution of the electron transport equation the following contribution into the electron distribution function

$$\delta\phi^{(\text{dr})} = -\frac{T^3}{\Omega p^2 s^3 m}\,\frac{p_y}{p}\,\nabla T G_1\left(\frac{\varepsilon_{\mathbf{p}}}{T}\right), \qquad (29)$$

$$G_1\left(\frac{\varepsilon_{\mathbf{p}}}{T}\right) = \frac{1}{8\pi^2}\int d\eta\big(\partial N_0(\eta)/\partial\eta\big)\eta^4\,\frac{1}{I^+(\eta)}$$
$$\times\left\{F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} + \eta\right) + F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} - \eta\right) - 2F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}\right)\right.$$
$$\left. + \frac{2ms^2}{T\eta}\left[F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} - \eta\right) - F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T} + \eta\right)\right]\right\}. \qquad (30)$$

It generates a flux in the Hall direction rather than along ∇T. The corresponding contribution is of the first order in $1/\Omega$. To get the x-component of the current, one should perform with $\delta\phi^{(\text{dr})}$ the transformation described above while treating the mutual drag. It can be seen that the part of the phonon distribution function $\delta\chi^{(\text{dr})}$ generated by the nonequilibrium electron distribution (29) is

$$\delta\chi^{(\text{dr})} = \frac{T^2}{\Omega m^2 s^3}\,\frac{k_y}{p_T}\,\nabla T J(\omega/T), \qquad (31)$$

where $p_T = \sqrt{mT}$ and

$$J(\omega/T) = \int \frac{d\eta}{\eta^{3/2}}\,G_1(\eta)\left[F_0\left(\eta - \frac{\omega_k}{T}\right) - F_0(\eta)\right].$$

This item generates the following term on the right-hand side of the electron Boltzmann equation

$$-\left\{\frac{T^2}{ms^3 p_T}\right\}\frac{\alpha T^5}{\Omega s^5 p^2}\,\frac{p_y}{p}\,\nabla T G_2(\varepsilon_{\mathbf{p}}/T), \qquad (32)$$

$$G_2(\varepsilon_{\mathbf{p}}/T) = \frac{1}{8\pi^2}\int d\eta \big(\partial N_0(\eta)/\partial\eta\big)\eta^4 J(\eta)$$
$$\times\left\{F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}+\eta\right)+F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}-\eta\right)-2F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}\right)\right.$$
$$\left.+\frac{2ms^2}{T\eta}\left[F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}-\eta\right)-F_0\left(\frac{\varepsilon_{\mathbf{p}}}{T}+\eta\right)\right]\right\}. \tag{33}$$

Comparing Eqs. (32) and (26) one can see that within a factor of the order of unity, they differ only by the factor in the braces in Eq. (32). This permits one, with the help of (28), to obtain the ratio of the drag contribution ($\varkappa_{\mathrm{dr}}$) to the ordinary electron one

$$\frac{\varkappa_{\mathrm{dr}}}{\varkappa} \sim \frac{T^2}{ms^3 p_T}\left(\frac{T\hbar}{p_F a_0 \Theta}\right)^2. \tag{34}$$

Thus, the drag effect can be predominant in the thermal conductance in a strong magnetic field for not too low temperatures. This can be true for metals, to say nothing about semiconductors. One should, however, keep in mind that the electron part of thermal conductance in a semiconductor may be small due to a small electron concentration. However, it can still be observed as a difference effect in zero and in a very strong magnetic field where the electron contribution is suppressed. The main scattering process determining the elctron thermal conductance may be the elastic scattering, in 3D samples this is in the first place the impurity scattering.

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