

Optical Fiber Communications: Group of the Nonlinear Transformations

© S. Lekić, S. Galamić*, Z. Rajilić*

"Kosmos", Cetinjska 1,
78000 Banja Luka, Republic of Srpska, Bosnia and Herzegovina

*Physics Department, Faculty of Science,

M. Stojanovića 2, 78000 Banja Luka, Republic of Srpska, Bosnia and Herzegovina

E-mail: slekic@urc.bl.ac.yu

A new method for finding solutions of the nonlinear Shrödinger equation is proposed. Comutative multiplicative group of the nonlinear transformations, which operate on stationary localized solutions, enables a consideration of fractal subspaces in the solution space, stability and deterministic chaos. An increase of the transmission rate at the optical fiber communications can be based on new forms of localized stationary solutions, without significant change of input power. The estimated transmission rate is 50 Gbit/s, for certain available soliton transmission systems.

The propagation of pulsed light in an optical fiber can be described by the nonlinear Schrödinger equation,

$$i \frac{\partial q(\xi, \tau)}{\partial \xi} + \frac{1}{2} \frac{\partial^2 q(\xi, \tau)}{\partial \tau^2} + |q(\xi, \tau)|^2 q(\xi, \tau) = 0, \quad (1)$$

where $q(\xi, \tau)$ is a complex envelope function of the effective electric field amplitude and

$$\xi \propto x, \quad \tau \propto \left(t - x \frac{\partial k}{\partial \omega} \right). \quad (2)$$

The higher order dispersion and the effect of fiber loss are neglected here [1]. We take

$$q = q_0 e^{i \frac{q_0^2}{2} \xi} y(\tau), \quad (3)$$

where $y(\tau)$ is a real function, and get

$$y - \frac{1}{q_0^2} \frac{d^2 y}{d\tau^2} - 2y^3 = 0. \quad (4)$$

The solution of this equation [2]

$$y_0(\tau) = \frac{1}{\cosh q_0 \tau} \quad (5)$$

describes the optical soliton. Its unchangeable shape is a property that makes it attractive for applying to ultra high speed optical communications [3,4]. The equation (1) is completely integrable one. The inverse scattering transformation method [5] yields the general solutions of such nonlinear partial differential equations. Our aim is to propose here an alternative approach to the nonlinear Schrödinger equation and discuss applicability of the obtained results to optical fiber communications.

The equation (4) describes a stationary pulse in optical fiber. We take a localized solution $y(\tau)$ of this equation and define the nonlinear operator H_{c_1}

$$H_{c_1} y = \sum_{j=1}^{\infty} c_j y^j, \quad (6)$$

where c_j are real coefficients. Does $H_{c_1} y$ satisfy the equation (4)? The case $y = y_0$ is considered yet and the

answer is positive [6]. Putting $H_{c_1} y$ into the equation (4), we find that $H_{c_1} y$ is actually a solution of this equation if

$$c_{2j} = 0, \quad (7)$$

while c_{2j+1} satisfy the recursive relation

$$c_{2j+1} = \frac{1}{2j(j+1)} \left\{ j(2j-1)c_{2j-1} - \sum_{n=2}^{2j} c_{2j+1-n} \sum_{k=1}^{n-1} c_{n-k} c_k \right\}, \quad (8)$$

where c_1 is an arbitrary coefficient. Using the relations (6)–(8), with $c_1 = 1$, we get

$$H_1 y = y. \quad (9)$$

In the following text, H_{c_1} will mean both the series (6) and the recursion (8) with (7). For a localized $y(\tau)$ and a finite c_1 , convergence of the series (6) can be numerically tested. Our calculations yield that $H_{c_1} y$ is localized too. Therefore, using different values of c_1 , we are able to get uncountable many new localized solutions of the equation (4) from only one known localized solution (Fig. 1). In the following text "the solution" will mean "the localized solution of the equation (4)". The solution value preciseness will be limited only by the number of calculated coefficients. The solution in form different from (6) does not exist. Each solution pair $z(\tau)$ and $y(\tau)$ must be in a relation $z = H_{c_1} y$, with specific value of c_1 :

$$c_1 = \lim_{\tau \rightarrow \pm\infty} \frac{z(\tau)}{y(\tau)}. \quad (10)$$

Starting with a solution $y(\tau)$ we can construct the complete solution space. There is an analogy to the superposition principle from linear theory. According to the relation (10), a solution is determined by its asymptotics.

The nonlinear Schrödinger equation has infinite number of symmetries corresponding to the conserved quantities: total energy, momentum, Hamiltonian etc. [1]. We find that there are actually uncountable conserved quantities. Let us

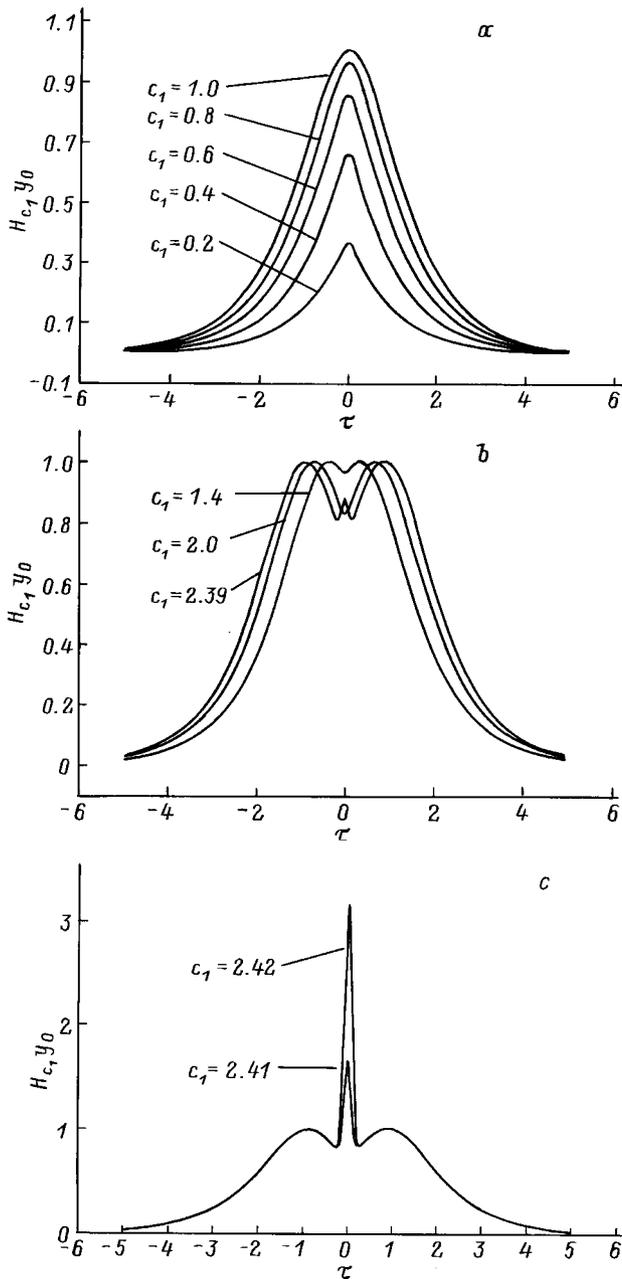


Figure 1. Solutions $H_{c_1} y_0$. *a* — stable solutions with $c_1 = 0.2, 0.4, 0.6, 0.8, 1.0$; *b* — stable solutions with $c_1 = 1.4, 2.0, 2.39$; *c* — unstable solutions with $c_1 = 2.41, 2.42$.

consider the total energy only (for $H_{c_1} y$):

$$q_0^2 \int_{-\infty}^{\infty} (c_1 y + c_3 y^3 + c_5 y^5 + \dots) d\tau. \quad (11)$$

We can choose uncountable different values of c_1 and use the relations (7) and (8).

The relations (6)–(8) yield

$$H_{a_1} H_{b_1} = H_{a_1 b_1}. \quad (12)$$

Hence

$$\{H_{c_1}; c_1 \neq 0\} \quad (13)$$

is the commutative multiplicative group of the nonlinear transformations (GNT). Group properties of the GNT originate from group properties of real numbers $c_1 \neq 0$. For example,

$$H_{c_1} H_{1/c_1} = H_1. \quad (14)$$

For definite coefficient c_1 and solution $y(\tau)$, we can construct a fractal subspace of the solution space. The fractal subspace covers solutions of form

$$H_{c_1} H_{c_1} \dots H_{c_1} y. \quad (15)$$

In the phase plane, a fractal subspace is represented by a geometrical fractal (Fig. 2).

For optical fiber communications it is an important question whether small disturbances will destroy the information-carrying pulses. Solution parameters, amplitude (pulse width) and velocity (frequency), are affected by various perturbations: outside-produced noise, incoherence of the light source, fiber inhomogeneities, absorption, amplifier noise, soliton interactions. . . It is an experimental fact that optical solitons (equation (5)) are unlikely to be destroyed by perturbations — they are very robust. We expect that at least the part of new solutions we have expressed here are actually stable. We are going to consider this problem theoretically, although it will be open until an experimental verification. The GNT method enables the following statement: the stability of a solution $y(\tau)$ is equivalent to the relation

$$\lim_{\varepsilon \rightarrow 0} H_{1+\varepsilon} y = y. \quad (16)$$

The relations (6)–(8) and (16) yield

$$|y(\tau)| \leq 1. \quad (17)$$

A localized solution of the equation (4) is stable one if and only if the relation (17) holds (Fig. 1, *a, b*). As well as for the *KdV* soliton [7], the classical argument about the counterbalance between nonlinearity and dispersion is not sufficient to explain the stability. Consideration of the Lyapunov exponent,

$$\lambda(c_1) = \lim_{j \rightarrow \infty} \frac{1}{j} \ln \left| \frac{dc_{2j+1}}{dc_1} \right|, \quad (18)$$

yields that deterministic chaos will appear at close packing of solitons, when c_1 is large enough (Fig. 3, *a*). The deterministic chaos we can expect for $c_1 > 2.4$. Near $c_1 = 1$, stability is exceptional (Fig. 3, *b*).

New forms of localized stationary solutions of the nonlinear Schrödinger equation enable an increase of the transmission rate at the optical fiber communications, without significant change of input power. An information may be contained in the special form of soliton (Fig. 1, *a, b*). The

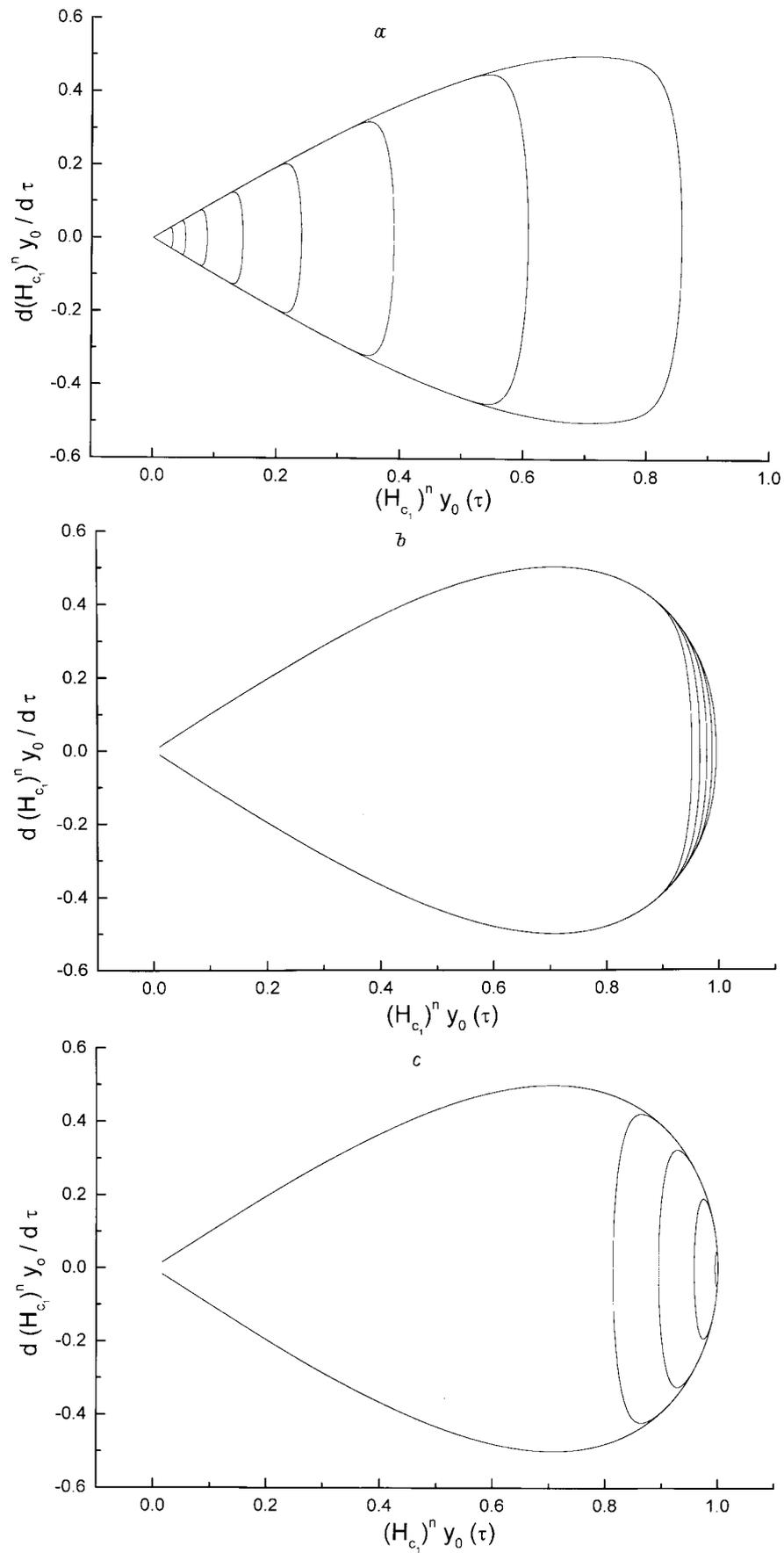


Figure 2. Phase diagrams of the solutions $(H_{c_1})^n y_0$. *a* — $c_1 = 0.6$, $n = 1$ to 8 ; *b* — $c_1 = 0.95$, $n = 1$ to 5 ; *c* — $c_1 = 1.2$, $n = 1$ to 5 .

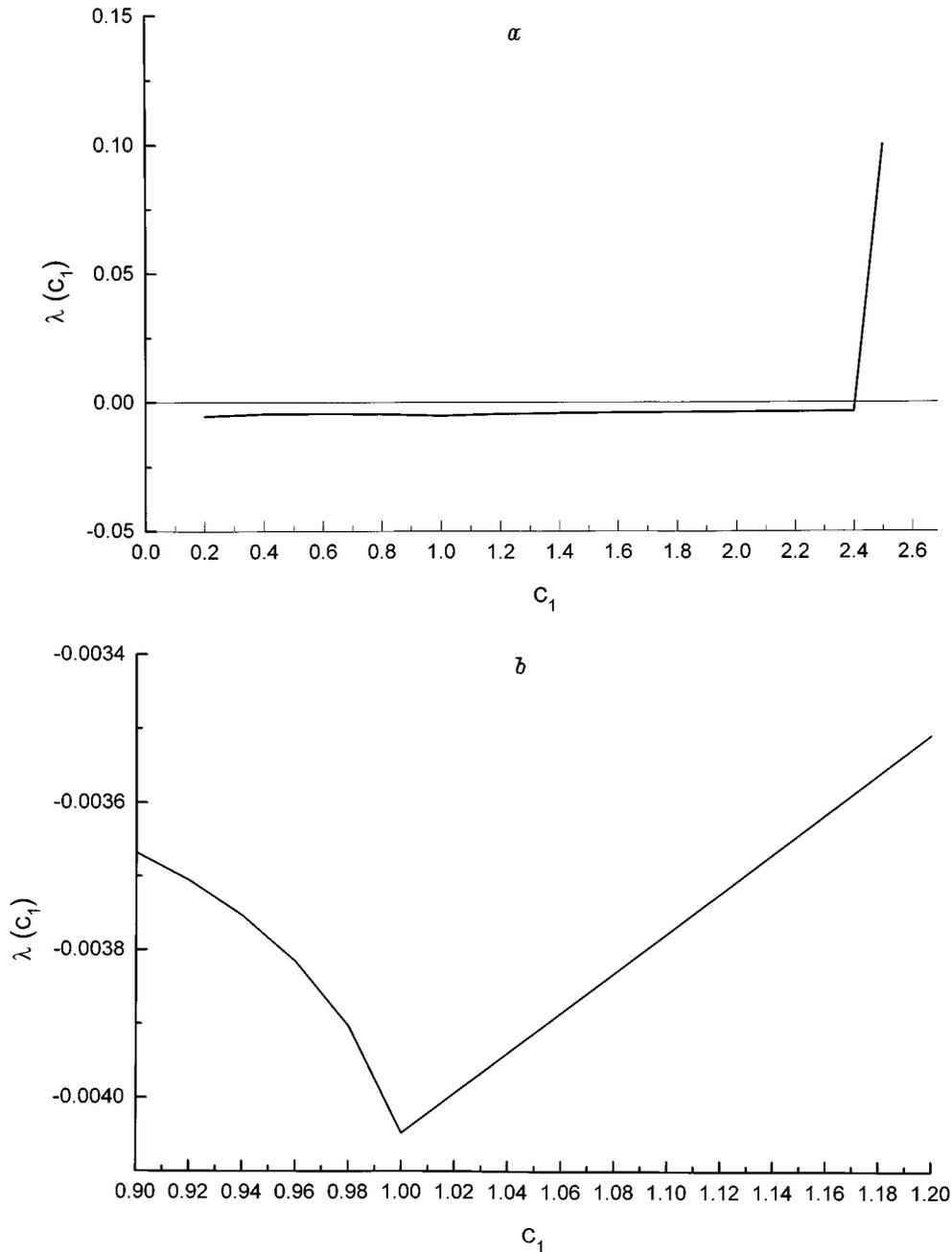


Figure 3. *a* — lyapunov exponent; *b* — exceptional stability near $c_1 = 1$.

known optical soliton, described by (5), is one of many possible stationary pulses. Let us consider an available soliton transmission system. If the fiber core cross sectional area is $S = 60 \mu\text{m}^2$, the carrier wavelength is $\lambda = 1.55 \mu\text{m}$, the soliton pulse (equation (5)) width is $\tau_s = 25 \text{ ps}$, the peak power is $P_m = 2.1 \text{ mW}$, and the separation between two adjacent solitons is $3\tau_s$, then the transmission rate is 10 Gbit/s [8]. In the same transmission system, using stable pulses of form H_{c_1} with different c_1 (Fig. 1, *a*), the transmission rate will be greater. It becomes equal to 50 Gbit/s, at 40 photons resolution of energies. The new

forms of stable solutions (Fig. 1, *b*) make possible increase of the transmission rate in the same system.

In conclusion, we have proposed the GNT method for solving of the nonlinear Schrödinger equation. New forms of the stationary localized solutions, usable for an improvement of the optical fiber communications, are obtained.

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