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A method to detect the characteristics of intermittent generalized synchronization based on calculation of probability of the synchronous regime observation

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A method to define the characteristic phases in the behavior of unidirectionally coupled systems located near the boundary of the generalized chaotic synchronization regime onset based on calculating the probability of the synchronous regime observation in an ensemble of coupled systems is proposed. Using the example of unidirectionally coupled Rössler systems in the band chaos regime, we have shown its efficiency in comparison with other known methods for detecting the characteristics of intermittent generalized synchronization.

Keywords: generalized synchronization, intermittent behavior, unidirectionally coupled systems, multistability, laminar phases, probability of the laminar phase observation.

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The intermittent behavior is characteristic of systems of various natures and is a universal phenomenon [1]. Intermittency is one of the classical scenarios of transition from periodical oscillations to chaotic ones; it is also observed near the boundaries of synchronous modes. In this connection, the following synchronization types are distinguished: intermittent total synchronization, intermittent lag synchronization, intermittent generalized synchronization, intermittent noise–induced synchronization, intermittent phase synchronization, and time scale intermittent synchronization [2–9].

Each of these types of the intermittent synchronous behavior possesses its own onset mechanisms and its own duration characteristics of the laminar (synchronous) phase. Moreover, in some cases the intermittency type depends not only on the synchronization type but also on the mismatch between the interacting systems or on the topology of attractors of these systems. For instance, intermittency of the „eyelet"type is observed near the phase synchronization boundary in the case of a relatively weak mismatch of the systems, while intermittency of the „ring"type is observed in the case of a great mismatch [6,7]. In the systems with a relatively simple attractor topology, the intermittency of the „*on-off*" type [3] occurs at the generalized synchronization boundary, while in the systems of a relatively complex (two-sheeted) structure the jump intermittency is observed [9]. Onset mechanisms of all the above mentioned intermittency types, as well as their statistical characteristics, also appear to be different.

In defining the intermittency onset mechanisms and in calculating statistical characteristics of the intermittent behavior, a significant role is played by the methods for distinguishing characteristic phases of the system dynamics. For the mode of the intermittent generalized synchronization, there are known a few such methods based on calculating local Lyapunov exponents [4], applying continuous wavelet transform [10] or analyzing the arrangement of points on attractors of the interacting systems [9]. For the mode of the intermittent generalized synchronization, there are known a few such methods based on calculating local Lyapunov exponents [4], applying continuous wavelet transform [10] or analyzing the arrangement of representative points on attractors of the interacting systems [9]. At the same time, the most widely used and efficient method for analyzing intermittent generalized synchronization is the auxiliary system approach [11]. This method may be easily applied to unidirectionally coupled dynamic systems, while for mutually coupled systems it gives incorrect results [12].

The auxiliary system method [11] consists in the following: along with the response system, an auxiliary system is considered which is identical to the response system and is under the action of the same drive system. The initial conditions of the auxiliary system are to differ from the initial state of the response system but are to lie in the attraction basin of the same chaotic attractor. In the mode of the generalized synchronization, a functional coupling (composite function) [13,14] arises between the states of the drive and response systems, as well as between the states of the drive and auxiliary systems, which makes identical the states of the response and auxiliary systems on the completion of the transient process. In the absence of generalized synchronization, states of the response and auxiliary systems always prove to be different, while in the intermittent generalized synchronization mode these states coincide with each other only in certain time intervals referred to as laminar (synchronous) phases, and at the remaining time moments asynchronous phases (turbulent surges) take place [3].

To obtain the intermittency characteristics, it is necessary to analyze the signal that is a time dependence of the difference in the response and auxiliary system states [3]. Time intervals during which the modulus of this difference is lower than a certain preset low quantity Δ correspond to the phases of laminar behavior, the remaining time moments correspond to the turbulent phases. Based on the statistics of duration of laminar phases (distributions of laminar phase durations at fixed values of control parameters and dependences of average laminar phase duration on the coupling parameter) it is possible to unambiguously define the intermittency type realized in the system.

In analyzing the intermittent behavior near the generalized synchronization boundary by using the auxiliary system approach, the probem of selecting initial conditions for the response and auxiliary systems is of great importance. Notice that this problem has not been discussed in literature so far (except for the statement that initial conditions for those systems are to be different but belonging to the same attraction basin). As shown below, the selection of initial conditions for those systems appears to be quite significant for the mode detected at a fixed time moment. In other words, in the mode of the intermittent generalized synchronization of unidirectionally coupled systems under one and the same state of the drive system either synchronous or asynchronous dynamics may be detected depending on the response system initial conditions, i. e., multistability takes place.

The presence of multistability near the generalized synchronization boundary raises the question of improving the methods for distinguishing characteristic phases of the system behavior taking into account this specific fact. This paper proposes a method for distinguishing laminar phases near the generalized synchronization boundary, which is based on calculating the probability of observing the synchronous mode in an ensemble of coupled systems. The method is based on considering time dependence of the drive oscillator and relatively large ensemble of response systems unidirectionally coupled with it, and on calculating the probability of observing the laminar phase of behavior (synchronous section of behavior).

As the research object, an ensemble was selected of unidirectionally coupled weakly−nonidentical Rössler systems staying in the band chaos mode:

$$\begin{aligned}
\dot{x}_d &= -\omega_d y_d - z_d,\\
\dot{y}_d &= \omega_d x_d + a y_d,\\
\dot{z}_d &= p + z_d(x_d - c),\\
\dot{x}_r^i &= -\omega_r y_r^i - z_r^i + \varepsilon\big(x_d - x_r^i\big),\\
\dot{y}_r^i &= \omega_r x_r^i + a y_r^i,\\
\dot{z}_r^i &= p + z_r^i\big(x_r^i - c\big),
\end{aligned} \tag{1}$$

where indices d and r relate to the drive and response systems, respectively, index $i = 1, \dots, N$ indicates the response system number, initial conditions for these systems are to be selected so as to be different, $a = 0.15$, $p = 0.2$, $c = 10$, $\omega_d = 0.93$ and $\omega_r = 0.95$ play the role of control parameters of the interacting systems, ε is the coupling parameter. At the selected values of control parameters and $\varepsilon \geqslant 0.178$, the generalized synchronization mode is observed in system (1) [15]. As discussed above, intermittent behavior takes place near the synchronous mode boundary.

To distinguish characteristic phases in system (1) and determine the intermittency characteristics, let us introduce into consideration the probability of observing the turbulent phase

$$P_a = 1 - \sum_{i=1}^{N} \frac{n(\mathbf{x}_r^i)}{N(N-1)}, \tag{2}$$

(where $n(\mathbf{x}_r^i)$ is the number of systems staying in the mode synchronous with the i-th oscillator, $N = 1000$ is the number of oscillators in the ensemble) and consider its time dynamics. If P_a is close to unity, the turbulent phase is observed in the system under study. If P_a is close to zero, the laminar phase is detected. Evidently, P_a may get intermediate values within the (0, 1) range. Thus, when the threshold for distinguishing the laminar and turbulent phases is set, it becomes possible to distinguish the characteristic phases of the interacting systems.

Fig. 1, *a* presents the $P_a(t)$ dependence for system (1) at $\varepsilon = 0.17$. Fig. 1, *b* shows the difference between the states of two response systems (one of them plays the role of the auxiliary system) $\xi(t) = |x_r^1 - x_r^2|$ at the same coupling parameter. Comparing Figs. 1, *a* and *b*, we can see that characteristic phases of the systems distinguished by two different methods are somewhat different though concentrated in the vicinity of the same time interva detection, the transition from the asynchronous state to synchronous one is accompanied by a quite drastic rise in the probability, which makes it possible to avoid detection of ultrashort laminar phases (which is typical of the auxiliary system method) and, hence, ensures more accurate determination of durations of characteristic phases of the systems.

Fig. 2 presents one of statistical characteristics of the laminar phase durations, i. e., distributions of the laminar phase durations calculated in two different ways at coupling parameter $\varepsilon = 0.17$, namely, by the auxiliary system method and probabilistic method; the figure also presents theoretical approximation of the distributions with the power function

$$p(\tau) = k\tau^{-3/2} \tag{3}$$

(where k is a positive constant), characteristic of the „*on-off*“ intermittency [3]. One can see that in both cases power function (3) fits well the numerical simulation data but, in case the method proposed here is used, standard deviation of the numerical simulation data from the theoretical ones appears to be essentially lower (see the Fig. 2 caption). The above indicates the possibility of using the proposed

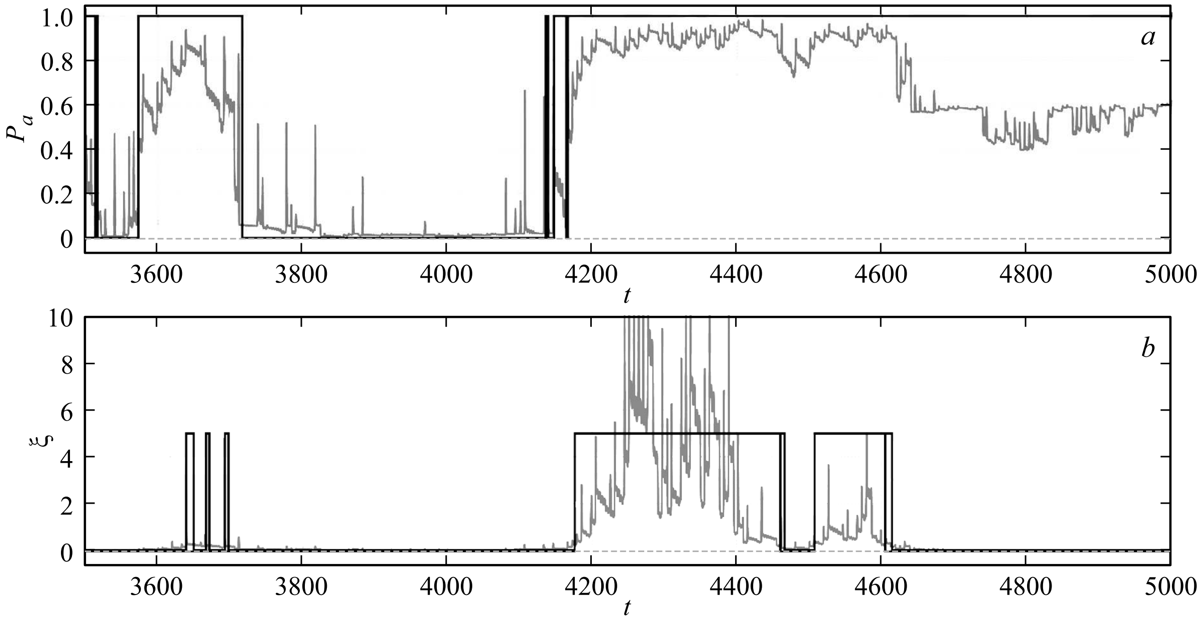

Figure 1. Time dependence of the probability of observing the asynchronous behavior phase P_a for the ensemble of unidirectionally coupled Rössler systems (1) at $\varepsilon = 0.17$ (a) and time dependence of the difference between the states of two response (response and auxiliary) systems $\xi(t) = |x_r^1 - x_r^2|$ at the same coupling parameter (b).

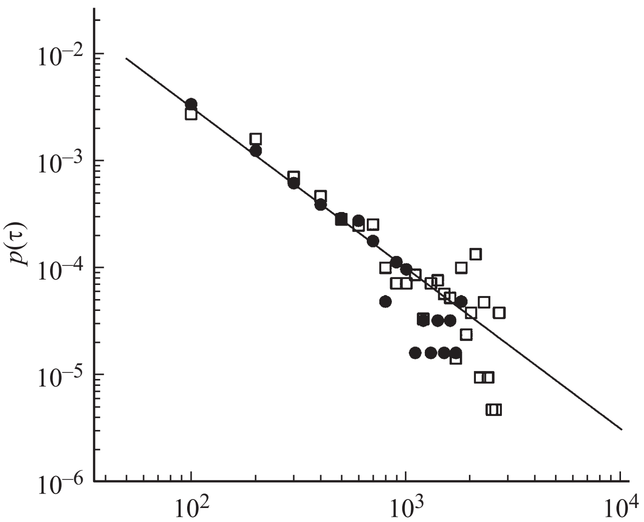

Figure 2. Distributions of durations of laminar phases of unidirectionally coupled Rössler systems (1) obtained at $\varepsilon = 0.17$ by two methods: the auxiliary system method (squares) and probabilistic method proposed in this study (circles), as well as their theoretical approximation by power function (3), $k = 3$. Standard deviations of the numerical simulation data from the theoretical ones are $\sigma_1 = 4.9 \cdot 10^{-9}$ and $\sigma_2 = 1.6 \cdot 10^{-9}$, respectively.

approach to determine the intermittency characteristics also in more complex systems where conventional methods fail to unambiguously identify whether the power or exponential law is observed in the considered case; this opens wide opportunities for practical application of this method.

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Conflict of interests

The authors declare that they have no conflict of interests.

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