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Two-dimensional low density electrons in high magnetic field

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The photoluminescence spectrum from the two-dimensional low density electrons with the localized valence-band holes in magnetic field is studied. The ground state is considered as Wigner crystal ore the strongly correlated electron system. For the quantum Wigner crystal the Landau levels for vacancions (quasiholes appearing in the process of photoluminescence) are calculated in the quasiclassical approximation. The spectrum of single-particle excitations for a triangular lattice in the nearest-neighbor approximation is used. It is found that Landau levels for vacancions depend unusually on magnetic field. For the electron system with strong Coulomb interaction the Mahan exciton effect in the photoluminescence for the two-dimensional electrons in magnetic field is considered.

Keywords: Wigner crystal, magnetic field, Mahan exciton, correlated electrons.

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Interest to study of highly correlated low-dimensional systems is maintained at a high level due to the appearance of new research targets and extension of experimental techniques. Thus, the effects of energy renormalization not only at the Fermi level, but also in the entire region of two-dimensional electrons' spectrum are studied using the method for analysis of radiation recombination spectra of electrons with photoexcited holes bound on remote acceptors [1–3]. Papers [2,3] studied the low-temperature luminescence spectra for two-dimensional electron gas in the MgZnO/ZnO heterojunction. The luminescence band width, studied in detail in [3], is related by the authors to a renormalized value of the weight of optical density of states. The weight defined in this way varies from 0.6 to $0.3m_0$ ($m = 0.3m_0$ — value of effective weight for ZnO in the volume) upon a change in the parameter r_s from 6.5 to 2.4 ($r_s = (\pi n_s)^{-1/2}/a_B$), n_s — density of two-dimensional electrons, $a_B = \epsilon\hbar^2/(me^2)$ — effective Bohr radius for ZnO parameters). The perpendicular magnetic field distinctly shows luminescence lines for individual Landau levels, which makes it possible to assume the presence of quasi-holes as well-defined quasi-particles for all energy values, and not only near the Fermi level. Samples with a low electron density also have unusual behavior in the magnetic field: the fan of Landau levels looks inverted, as stated by the authors of paper [3]. An inverted fan, apparently, means a decrease of Landau level energy with magnetic field growth in the magnetic field range $5-8T$ studied in the experiment. Unfortunately, the paper does not give the results of study of quasi-holes' Landau level behavior. Nevertheless, it is necessary to consider theoretically the behavior of Landau levels in a highly correlated system of two-dimensional electrons.

However, most theoretical methods work well with high electron densities (small values of the parameter r_s), when electron kinetic energy dominates over their interaction energy. The multi-particle problem for intermediate values of r_s does not have a correct theoretical description both for determination of the main state and for calculation of excitation energy. The main state of a two-dimensional electron system depending on r_s may be considered as an electron gas, an electron Fermi liquid or a Wigner crystal ([4,5]). Electrons in the crystal form a triangular lattice, while the system's spin state can be either ferromagnetic or antiferromagnetic. Electron density for crystallization in an ideal system shall be very low — $r_s = 37 \pm 5$ [6], however, with consideration of impurities, the liquid – crystal transition shifts to a more realistic value of $r_s = 7$ [6], thus making it possible to consider the quasi-holes from the experimental work [3] as quasi-particles in a Wigner crystal — vacansions [7]. The author's previous paper [8] considered the shape of the luminescence band for a recombination of an electron and a localized hole with the formation of a vacansion, assuming that two-dimensional electrons form a Wigner crystal.

This paper presents the obtained Landau levels for vacansions in a quasi-classical approximation, using the dispersion law in a tight-binding approximation for the triangular lattice of the Wigner crystal. The vacansion dispersion law $E(\mathbf{k})$ in a tight-binding approximation for the triangular lattice with interatomic distance a while considering tunneling only between the nearest neighbors has the form

$$E(\mathbf{k}) = \mathbf{2t}\left[\cos(\mathbf{k_x a}) + \mathbf{2}\cos(\mathbf{k_x a/2})\cos(\sqrt{\mathbf{3}}\mathbf{k_y a/2})\right], \quad (1)$$

where t is the hopping integral (tunneling parameter). Band width is $D = 9|t|$. The sign of t depends on spin ordering. With $t < 0$ (the ferromagnetic case), the minimum of $E(\mathbf{k})$ is in the center of the Brillouin zone at the point

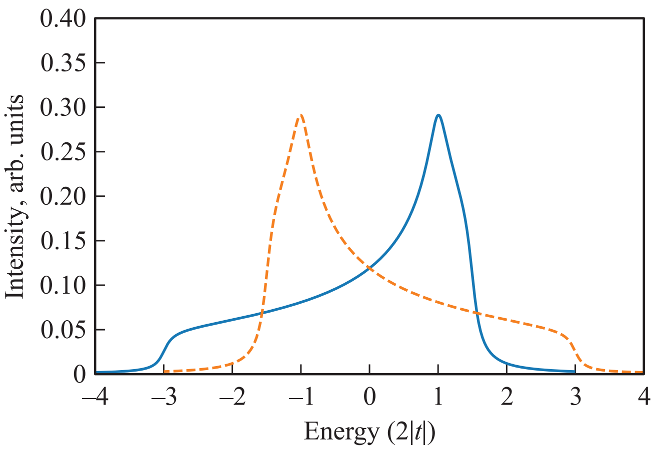

Figure 1. Luminescence spectra for recombination of 2D-electrons and localized holes with formation of vacansions. Solid line:$t < 0$, dashed line: $t > 0$. Attenuation 0.2, energy unit $2|t|$.

Γ $(E(0) = 6t)$, the maximum of $E(\mathbf{k})$ is at the point K $(E(K) = -3t)$, with $t > 0$ the maximum and the minimum change places.

Density of states with $E \to -2t$ turns into ∞ under the logarithmic law (a van Hove singularity emerges). As already noted [8], the point of singularity corresponds to the luminescence intensity maximum. Therefore, a change in the sign of the hopping integral causes a change in the position of the luminescence band maximum. The spectrum transformation is shown in Fig. 1.

In a quasi-classical approximation, the Bohr-Sommerfeld quantization rule can be used to express the Landau level energy E_n as follows:

$$S(E_n) = \left(\frac{2\pi eH}{\hbar c}\right)(n + \gamma). \quad (2)$$

Here, $S(E)$ is the surface area of a closed race-track in the $\mathbf{k}$-space, determined by isoenergetic curve $E(\mathbf{k}) = E$. Energy, corresponding to the point of singularity, in the magnetic field also has a peculiarity: a closed race-track does not exist for it. Accordingly, Landau levels are located separately for one valley with a center at point Γ and for two other valleys with a center at point K. The dispersion law in the vicinity of the extreme values is quadratic, and Landau level energies can be easily found. The quantization constant $\gamma = 1/2$ can be determined from here.

The value of $S(E)$ can be expressed through the density of states $N(E)$ (using the expression for $N(E)$, e.g., from [9]). Thus, for $t < 0$

$$S(E) = S_0 \int_{E_{min}}^{E} N(E)dE \quad (E < -2t) \quad (3)$$

$$S(E) = 0.5 \times S_0 \int_{E}^{E_{max}} N(E)dE \quad (E > -2t). \quad (4)$$

$S_0 = (8\pi^2)/(\sqrt{3}a^2)$ is the area of the Brillouin zone. Magnetic field H can be conveniently expressed in dimensionless units H/H_0. With $H = H_0$, the filling factor is $\nu = 1$, the corresponding magnetic length a_H

$$a_{H_0} = sqrt\frac{\hbar c}{eH_0} = a\left(\sqrt{3}/(4\pi)\right)^{1/2}. \quad (5)$$

The equations for Landau level energies take the form

$$\int_{E_{min}}^{E} N(E)dE = \frac{H}{H_0}\left(n + \frac{1}{2}\right), \quad (6)$$

$$\int_{E}^{E_{max}} N(E)dE = \frac{2H}{H_0}\left(n + \frac{1}{2}\right). \quad (7)$$

The position of Landau levels depending on magnetic field is shown in Fig. 2.

The figure shows unusual behavior of Landau levels. With $t < 0$ for a valley with a center at Γ-point, their energies increase as the field grows, for K-valley the energies decrease. Most of the Landau levels are of the Γ-type. However, there are two K-valleys, capacitance of Landau levels is twice higher, and their manifestation can be expected to be more intensive in luminescence. The magnetic field stabilizes the Wigner crystal; decrease of the tunneling parameter with magnetic field growth must enhance the effect of Landau levels' motion from different valleys toward each other. Nevertheless, a change in the tunneling parameter need not be considered in quasi-classical consideration, applicable in weak magnetic fields only.

The obtained dependence of Landau level energy is due to the existence of a limited energy band and may mean possible existence of a Wigner crystal or Wigner glass. If a Wigner crystal is characterized by strict ordering, long-range order for Wigner glass (or Wigner liquid) is broken, but the short-range order remains, which is possible for the case with impurities. Density decrease must cause breakdown of the ordering, it will be simply a system of localized electrons [6], also characterized by a limited energy band.

Dependence of ground-state energy depends little on spin polarization both for a pure Wigner crystal and with account of impurities [10,11]. However, it is more probable that the ground state of a Wigner crystal with impurities at densities close to the crystallization threshold is antiferromagnetic (frustrated antiferromagnet) and becomes ferromagnetic only at very low densities or with very strong magnetic fields ($\nu \ll 1$) [6,11,12].

In the experiment [3], the formation of an intensity peak for low-density samples ($r_s \sim 5{-}6.5$) on the edge of the luminescence band from the side of high energies and unusual behavior of Landau levels were explained by a phenomenon called „Mahan exciton".

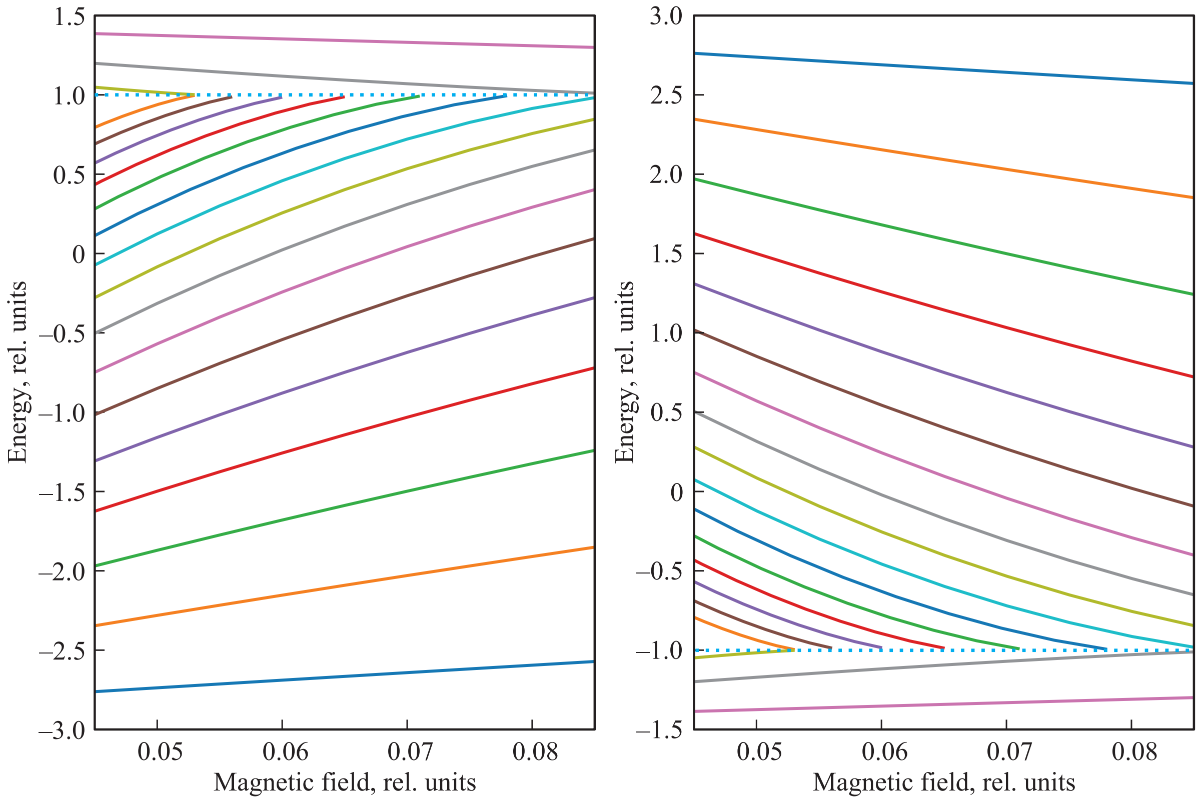

Figure 2. Landau levels for vacansions depending on magnetic field. On the left — for $t < 0$ (ferromagnet), on the right — for $t > 0$ (antiferromagnet). Energy unit is $2|t|$, unit of magnetic field — H_0, dashed line — singularity energy.

The „Mahan" exciton manifests itself as a peak of photoluminescence intensity below the Fermi energy E_F for an electron system at a low temperature and a relatively low density. Intensity increase that occurs in absorption and luminescence at energies close to the Fermi energy E_F, due to multiple electron-hole scattering near E_F, but without the formation of a true exciton, has been theoretically shown for the three-dimensional [13] and two-dimensional [14] cases. Such scattering processes for $E \ll E_F$ are suppressed due to the Pauli exclusion principle. Mahan's paper [13] theoretically considers the interband optical absorption in a direct-gap semiconductor in the case when one band is degenerate and is considered as Fermi gas. It was shown that exciton effects due to the electron-hole Coulomb attraction cause a logarithmic singularity on the absorption band edge even when real bound electron-hole states do not form. This peculiarity („Mahan exciton") exists at intermediate densities of particles in the degenerate band ($r_s \sim 2$) and disappears at high densities ($r_s \ll 1$).

The paper by Schmitt–Rink with co-authors [14] considered the absorption and luminescence spectra for quasi-two-dimensional electron-hole plasma depending on carrier concentration and temperature. Intensity increase arising due to effects of intersubband absorption and luminescence was theoretically shown at energies close to the Fermi energy, due to an increased role of interparticle scattering at a low temperature and a relatively low density of particles ($r_s \sim 1{-}3$). Own-energy corrections and multiple electron-hole scattering in the Bethe–Salpeter equation were taken into account for the statically-screened Coulomb interaction. The obtained spectra significantly differed from the single-particle ones due to the enhancement of pair correlations in the two-dimensional case. Experimental observation of the peculiarity of „Mahan exciton" in low-temperature luminescence spectra for InGaAs-InP quantum wells [15] agrees well with the calculations [14].

It could be expected that this peculiarity in luminescence spectra will also manifest itself in intermediate magnetic fields. Possible manifestation of the peculiarity of type „Mahan exciton" in the magnetic field in luminescence for a model of two-dimensional electron gas with strong Coulomb interaction is considered in this paper similarly to the case without the magnetic field, considered in the paper [14].

The problem was solved in single-exciton approximation. In the presence of electron gas with N filled Landau levels, the latter of which is partially filled, we considered a state representing the superposition of pairs, consisting of an electron on a free or partially filled Landau level M ($M \geq N$) and a hole from the valence band with the same Landau level number. The hole mass was assumed to be infinite. Statically-screened electron-electron and electron-hole interaction V_s was used for weak magnetic fields

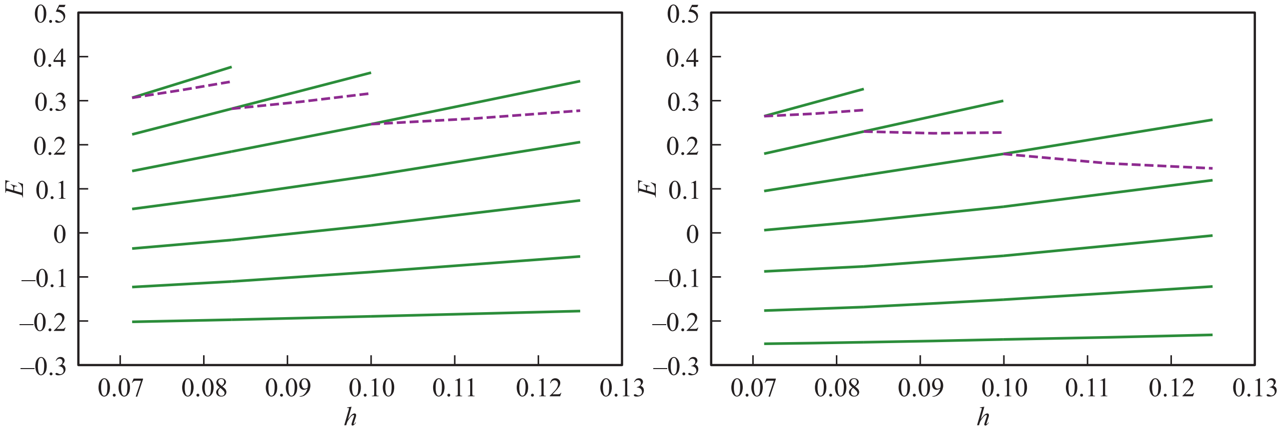

Figure 3. Landau level energies (solid lines) and „Mahan exciton" (dashed lines) in RPA approximation depending on magnetic field. On the left, for $r_s = 2$, on the right for $r_s = 7$. Energy unit is $E_c(H_0)$, magnetic field energy unit is H_0.

($\nu \gg 1$), obtained in the paper [16]:

$$V_s(q) = \frac{2\pi e^2}{\epsilon q}\frac{1}{\varepsilon(q)}, \quad \varepsilon(q) = 1 + \frac{2}{a_B q}\left[1 - J_0^2(q a_H)\right], \quad (8)$$

J_0 is the Bessel function.

Exciton energy was found by numerically solving the Bethe—Salpeter equation taking into account the electron-hole interaction and own-energy corrections, similarly to the calculation of intersubband excitations in the magnetic field, the Hartree–Fock approximation [17–19] was used for partially filled levels. The typical energy scale for screened Coulomb interaction (8) E_s is somewhat lower than for the unscreened one, but significantly higher than the electron's cyclotron energy, that's why the Landau levels not only with $M \sim N$, but with $M \gg N$ were taken into account when considering the „Mahan exciton".

In the magnetic field (exclusive of attenuation due to spectrum discreteness), the bound electron-hole states, formed by electrons of the conductivity band and holes of the valence band with identical Landau levels, exist always and are located near the Fermi energy. Starting from $r_s = 2$, the energy of this state stops rising with growth of the magnetic field. For $r_s \geq 4$, energy of the „Mahan exciton" starts decreasing, as opposed to energies of Landau levels, dependence on electron concentration is negligible, qualitative pattern of behavior in the magnetic field is the same. Energies of Landau levels and the level of the „Mahan exciton" for $r_s = 2$ and $r_s = 7$ are shown in Fig. 3.

For $r_s = 2$, the statically-screened Coulomb interaction can still be used both for the case of a two-dimensional electron gas in a semiconductor without magnetic field [14], and for the case, considered here, in the magnetic field with $\nu \sim 5{-}7$. With greater r_s, such screening cannot be used, but one may expect retention of the qualitative pattern with growth of r_s, along with manifestation of Coulomb interaction screening. Thus, it has been showed [20] in superstrong magnetic fields ($\nu = 1$) that the contribution of exchange energy at very low electron densities ($7 < r < 11$) of the order of cyclotron energy $\hbar\omega_c$, which is lower than the usually considered typical scale of Coulomb energy $E_c = e^2/(\epsilon a_B)$; renormalization of exchange energy has been shown numerically for a finite system of particles and experimentally by the method of inelastic light scattering.

Energy decrease with growth of the magnetic field for the effect of the „Mahan exciton" manifests itself only for one line, while for an antiferromagnetic Wigner crystal the fan of lines has inverted behavior. The abnormal behavior of Landau levels in two-dimensional electron systems of a very low density can more likely be explained by the transition to the Wigner crystal state, possibly, in the form of separate domains, without establishing of a long-range order throughout the sample. Unfortunately, the lack of experimental data prevents from making of unambiguous conclusions in favor of a particular model.

Conflict of interest

The author declares that he has no conflict of interest.

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