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Anishchenko-Astakhov quasiperiodic generator excited by external harmonic force

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A harmonic effect on a modified Anishchenko-Astakhov generator capable of demonstrating two-frequency quasi-periodic oscillations in the autonomous mode is considered. The possibility of doubling the three-frequency tori in a non-autonomous system is shown. The possibility of the effect of chaos suppression by an external signal is demonstrated, which leads not only to periodic, but also to quasi-periodic modes when the influence amplitude exceeds a certain threshold.

Keywords: torus doublings, suppression of chaos, quasiperiodic generator

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A radiophysical generator proposed by Anishchenko and Astakhov may be considered as one of the basic models manifesting deterministic chaos [1]. This generator is a three-dimensional dynamic system and has been thoroughly examined both theoretically and experimentally (see monographs [2,3] and references therein). Its modification supporting autonomous quasi-periodic oscillations in addition to periodic and chaotic regimes has been proposed in [4]. An oscillation circuit in the feedback loop, which provides a new additional frequency, is used for this purpose. The end result is an autonomous four-dimensional model that is convenient for the study of quasi-periodic oscillations. This generator has been studied in [5], and the possibility of doubling of a two-frequency torus upon an increase in the excitation parameter has been demonstrated. The problem of synchronization of a resonance limit cycle on a torus, the emergence of resonance two- and three-frequency tori on the surface of a four-frequency torus, the influence of noise on a four-frequency torus, and other problems arising in the case of two coupled generators have been discussed [5–7]. The emergence of hyperchaos via secondary Neimark–Sacker bifurcation has also been examined [8,9]. At the same time, the influence of a harmonic signal on the modified generator has remained understudied. This problem appears significant in the context of formulating a sufficiently complete description of synchronization of quasi-periodic oscillations.

The equations of the modified Anishchenko–Astakhov generator are as follows [4]:

$$\dot{x} = mx + y - x\varphi - dx^3,$$
$$\dot{y} = -x,$$
$$\dot{z} = \varphi,$$
$$\dot{\varphi} = -\gamma\varphi + \gamma\Phi(x) - gz, \tag{1}$$

where

$$\Phi(x) = I(x)x^2, \quad I(x) = \begin{cases} 1, x > 0, \\ 0, x \leqslant 0. \end{cases} \tag{2}$$

Here, m is the generator excitation parameter, d is the nonlinear dissipation parameter, γ is the attenuation parameter, and g is the inertia parameter of a filter providing the second independent frequency. We use the following parameter values: $d = 0.001$, $\gamma = 0.2$, and $g = 0.5$.

Let us now add an external harmonic influence:

$$\dot{x} = mx + y - x\varphi - dx^3 + a\cos\omega t,$$
$$\dot{y} = -x,$$
$$\dot{z} = \varphi,$$
$$\dot{\varphi} = -\gamma\varphi + \gamma\Phi(x) - gz, \tag{3}$$

where a and ω are its amplitude and frequency.

When excitation parameter m increases, doubling of a three-frequency torus (instead of a two-frequency one) may be observed in this case. This is illustrated by Fig. 1. Portraits of attractors in a double Poincaré section are shown in the insets of this figure. Let us explain how such a section is plotted. The result of a common Poincaré section for a system subjected to external harmonic influence is a set of points obtained by way of a stroboscopic section. In order to plot a double section, we considered only those points from the mentioned set that fall within a certain thin phase-space layer defined, e.g., by condition $|x| \leqslant 0.005$. The result of a double section (i.e., stroboscopic section and section by plane $x = 0$) of the phase space of system (3) is presented in Fig. 1. In a double section, a three-frequency torus looks like two smooth ovals. When m increases, doubling of this torus

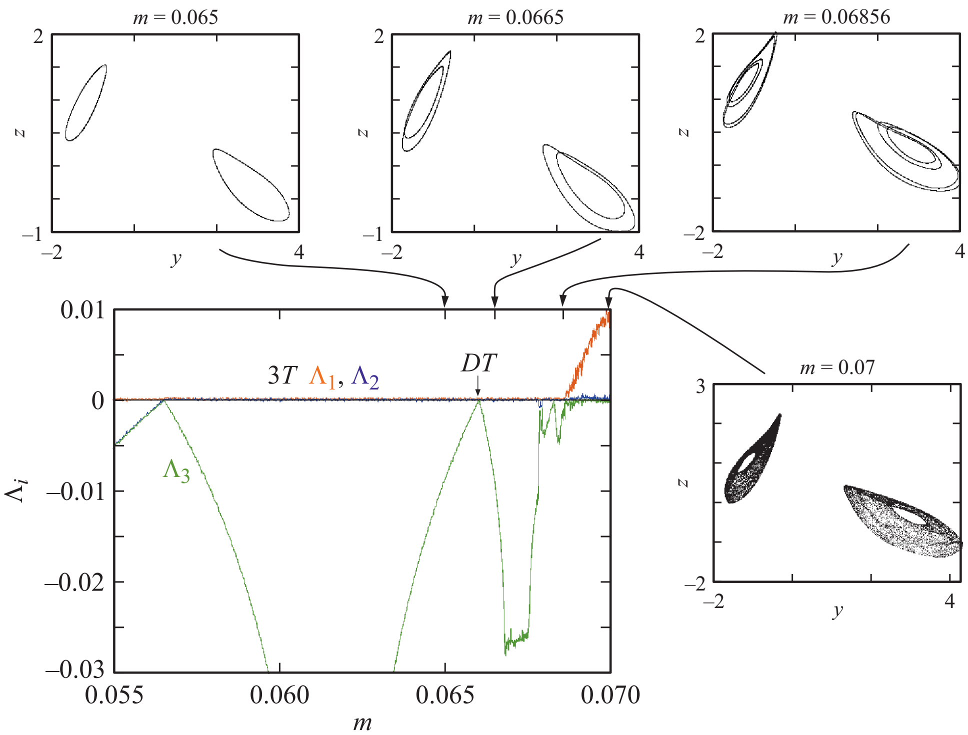

Figure 1. Portraits of three-frequency tori $3T$ in a double Poincaré section (insets) and dependences of Lyapunov exponents Λ_i of system (3) on excitation parameter m. $a = 0.03$, $\omega = 4$. DT is the point of doubling of a three-frequency torus.

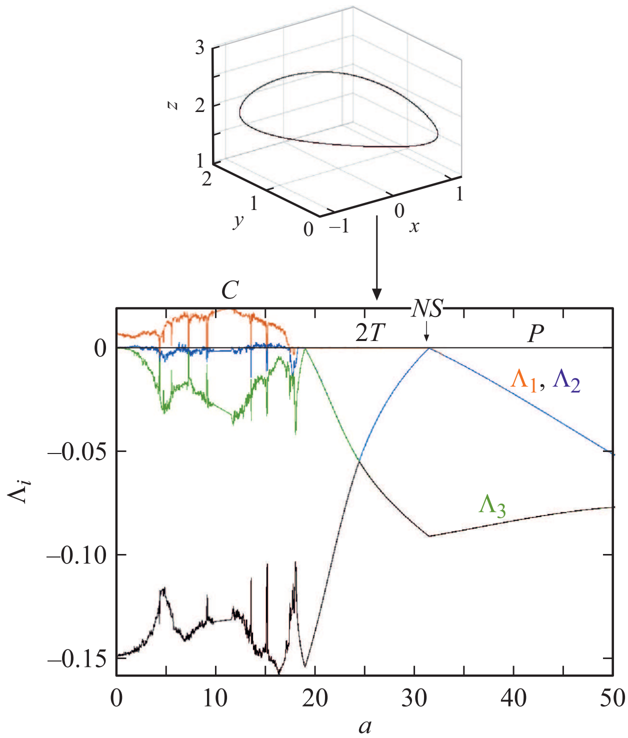

Figure 2. Portrait of system (3) in a stroboscopic section (inset) and plots of its Lyapunov exponents Λ_i. $m = 0.07$, $\omega = 6$. P is the region of periodic regimes, $2T$ is the region of two-frequency tori, C is the chaos region, and NS is the Neimark–Sacker bifurcation point.

occurs at point DT; as m grows further, the torus gets destroyed.

The main part of Fig. 1 shows the dependences of the three largest Lyapunov exponents of system (3) on excitation parameter m. Note that one exponent is always equal to zero in flow systems. Since we calculate the exponents in a stroboscopic section, this zero exponent is dropped. Thus, zero values of two exponents $\Lambda_1 = \Lambda_2 = 0$ correspond to a three-frequency torus (a similar pattern is seen for discrete maps [10]). The presented plots also confirm the nature of bifurcation: exponent Λ_3 goes to zero at the bifurcation point and remains negative in its vicinity. This is the sign of torus-doubling bifurcation [11,12].

Let us now consider the changes in behavior of the system induced by the variation of input amplitude a (note that chaos is observed at $a = 0$). We fix the value of parameter $m = 0.07$ corresponding to the destruction of a torus. The dependences of Lyapunov exponents on input amplitude a are shown in Fig. 2. It can be seen that, as expected, chaotic or hyperchaotic regimes with one or two positive Lyapunov exponents are established at low amplitudes. Periodic regime P with all the exponents being negative, however, emerges at large amplitudes. Thus, the effect of suppression of chaos by an external periodic force is observed in the system [13]. Two Lyapunov exponents are equal in this case ($\Lambda_1 = \Lambda_2$) and go to zero at point NS. This is the point of Neimark–Sacker bifurcation that induces two-frequency quasi-periodic regime $2T$ with $\Lambda_1 = \Lambda_2 = 0$. The corresponding attractor in a stroboscopic

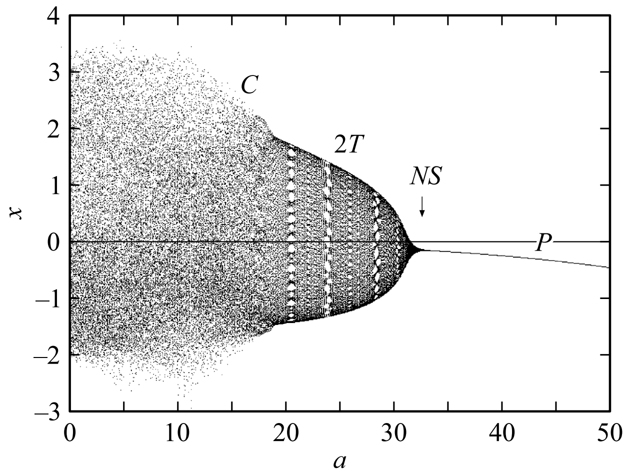

Figure 3. Bifurcation tree of system (3) plotted using a stroboscopic map. $m = 0.07$, $\omega = 6$.

section is presented in the inset of Fig. 2. This attractor is a closed invariant curve. Thus, owing to the suppression of chaos, a quasi-periodic regime, which occupies an extensive area in terms of the input amplitude, emerges in this system in addition to a periodic regime similar to the one reported in [13]. As the input amplitude decreases further, the torus undergoes doubling and then gets destroyed.

The bifurcation tree for $\omega = 6$ is presented in Fig. 3. Neimark–Sacker bifurcation point NS and two-frequency quasi-periodic regime $2T$ are seen.

Thus, new effects may be observed if a quasi-periodic Anishchenko–Astakhov generator is subjected to the influence of a harmonic signal. At small input amplitudes, this new effect is the doubling of a three-frequency torus. At large amplitudes, the effect of chaos suppression, which induces both periodic and quasi-periodic regimes, manifests itself.

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Conflict of interest

The authors declare that they have no conflict of interest.

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