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Quantization of the spectrum of spinon pairs in a magnetic field and formation of bosons in the form of fragments of 1D-waves of charge/spin density in SmMnO$_{3+\delta}$

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Quantum oscillations of the temperature dependences of magnetization of SmMnO$_{3+\delta}$ in magnetic fields $H = 100$ and 350 Oe, 1 and 3.5 kOe are studied within a temperature interval of 4.2–12 K. The continuum of thermal excitations of spinon pairs in magnetic field $H = 100$ Oe is divided into three overlapping Landau bands with energies E_n ($n = 1, 2, 3$) and fractional band filling factors ν. The symmetric intense „supermagnetization“doublet consisting of two overlapping peaks around mean excitation temperature $T_{\text{spinon}} \cong 8$ K produces the primary contribution to magnetization. As the field increases to $H = 1$ kOe, the spinon excitation spectrum transforms into a broad sinusoid peak with its apex around $T_{\text{spinon}} \cong 8$ K, which is typical of the excitation continuum of spinon pairs with strong dispersion in the regime of their confinement. The thermal excitation spectrum of spinons changes significantly in magnetic field $H = 3.5$ kOe: a new type of quantization of the spinon spectrum in the form of magnetization features shaped like narrow steps (plateaus), which correspond to integer filling with spinons of three Landau bands with a finite gap, emerges in Landau bands with $n = 1, 2$, and 3. The evolution of the boson spectrum in the form of thermal excitations of gapless quasi-one-dimensional waves of charge/spin density in the Luttinger liquid at temperatures below 60 K is examined. A continuous quantum phase transition of the quantum spin liquid into the Luttinger liquid state induced by an increase in the magnetic field intensity is found at temperatures around $T = 0$. This transition has features typical of the formation of 1D-waves of charge/spin density induced by the confinement of spinon pairs and is accompanied by strong field hysteresis.

Keywords: quantum spin liquid, Luttinger liquid, confinement of spinon pairs, spinon — gauge field system, landau quantization, 1D-waves of charge/spin density, quantum oscillations.

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1. Introduction

The Tomonaga–Luttinger liquid (or Luttinger liquid, LL) model has attracted much interest in recent years. This model characterizes well the interaction of electrons or other fermions in a one-dimensional conductor. A model like this is required, since the commonly used Fermi liquid model is inapplicable in the one-dimensional case. The LL theory characterizes low-energy collective excitations in a one-dimensional electron gas as bosons. The free-electron Hamiltonian is divided with respect to electrons having opposite (left–right) directions of motion. Conduction electrons in the regime of fractional or integer quantum Hall effect and 1D-chains of half-integer spins characterized by the Heisenberg model are among the physical systems to which the LL model is applicable. The LL theory characterizes well the low-energy properties of a broad class of gapless one-dimensional interacting fermion systems [1,2]. The restriction to low excitation energies is normally mitigated by linearization of the spectrum of physical fermions around the right and left Fermi points $\varepsilon(k) \approx v_{\text{F}}(\pm k - k_{\text{F}})$. A system may be characterized rigorously within this approximation even if nonzero interactions between particles are present. The theory of interacting fermions transforms into a theory of noninteracting bosons, and all correlation functions may be calculated accurately. It is assumed that many-particle excitations in the form of charge and spin density waves, which follow the Bose–Einstein statistics, generally substitute for elementary fermion excitations in the one-dimensional case. The spin and charge separation is one of the singular features of one-dimensional conducting systems of fermions in the LL state. The Hamiltonian of an interacting system is divided into two commuting terms that act in different Hilbert spaces and characterize the charge and spin degrees of freedom separately. This unusual state of one-dimensional conducting systems is characterized completely by the velocities of spin and charge density waves (v_s and v_c, respectively) and Luttinger parameter K that depends on the strength of interparticle interaction. These velocities for nonzero interactions differ. The collective nature of fundamental LL oscillations and the presence of spin and charge separation are manifested clearly in various dynamic response functions. Structural charge $S(k, \omega)$ and spin $S^{\pm}(k, \omega) = S^x \pm iS^y$, $S^{zz}(k, \omega)$ density factors measure the linear response of a system of particles with momentum k and energy ω to an excitation that alters the charge and spin density in the system. In a spin LL, the dynamic structural charge density factor is $S(k,\omega)\propto\delta(\omega-\nu_c|k|)$, while the dynamic structural spin density factor is $S^{zz}(k,\omega)=1/2S^{\pm}(k,\omega)\propto\delta(\omega-\nu_s|k|)$. The presence of a δ-like Dirac peak of these functions is reflective of the fact that charge and spin waves are fundamental LL oscillations, and a certain excitation energy corresponds to each wavenumber. This result is related directly to linearization of the fermion spectrum in the linear LL theory. It should be noted that charge and spin density waves are uncoupled completely in this model. Away from the Fermi points, the curvature of the physical fermion spectrum $\varepsilon(k)$ cannot be neglected: for example, the fermion spectrum curvature leads to coupling of charge and spin density waves [3–5].

The dynamic response of strongly correlated one-dimensional Mott insulators with spectral gap M with half and quarter filling of the conduction band was examined in [6,7]. It was found that the functions of dynamic response of charges and spins differ greatly in these two cases. According to [6], both systems exhibit spin–charge separation in the limit of low excitation energies. This made it possible to calculate exactly their dynamic spectral charge and spin density function $A(\omega, k_{\mathrm{F}}+q)$. It was established that the creation and annihilation operators for right- and left-moving fermions in a one-dimensional Mott insulator with half filling at temperature $T=0$ factorize into spin and charge fragments upon bosonization in the form of spin and charge density waves. A series of spectral functions $A_R(\omega, k_{\mathrm{F}}+q)$ calculated for $\nu_s=0.8\,\nu_c$, $-4M\le q\le 4M$ was presented in [6]. This series clearly reveals the presence of two peaks corresponding to the excitation of a charge density wave (CDW) and a spin density wave (SDW).

Most of the spectral weight is concentrated in these features, which are a direct manifestation of spin–charge separation. The peak feature of the spectral function with a lower (higher) energy corresponds to the case when all fermions with momentum q forming the collective excitation continuum are spin (charge) carriers. It should be noted that both peaks have a significant width. At low temperatures $T<M$, the influence of the temperature increase on the correlation function of charges is weak, but the spin part of the spectral function may be altered considerably, since the spinon spectrum is gapless. It was found that the spectral function of a one-dimensional Mott insulator with quarter filling does not feature two clear peaks associated with separate excitation of charge and spin density waves. In contrast to half filling, only one asymmetric and very broad disperse peak of the density of states $A(\omega, k_{\mathrm{F}}+q)$ located around k_{F} is expected.

The results of examination of quasi-one-dimensional properties of Hubbard chains weakly coupled by interchain tunneling in [6] are of the utmost interest. The case when the interchain interaction is long-ranged only in the direction perpendicular to chains was considered for simplicity. Just as in the case of a coupled LL, interchain tunneling $t_\perp$ in a one-dimensional Mott insulator is regarded as an excitation that allows one to extend the region of applicability of the method proposed in [6] to a quasi-one-dimensional system of weakly coupled chains. It was found that Green's function $G_{3\mathrm{D}}(\omega, q, \mathbf{k})$ of the excitation density for a system of chains has a pole at $q=k_{\mathrm{F}}$ that corresponds to a bound state of an antiholon and a spinon with the quantum numbers of an electron. With an infinitely small value of interchain tunneling $t_\perp\neq 0$, this bound state has a gap with an energy lower than the one of the Mott gap. At excitation energies above the Mott gap, a continuum of excitation states (similar to the one obtained for uncoupled chains) is present. The gap of the bound state contracts with increasing $t_\perp$. The most intriguing case is the one when this gap becomes very small. In view of the authors of [6], transverse interchain hops of fermions are the primary factor governing the formation of closed trajectories (loops) in this case. As $t_\perp$ increases, the gap of the bound state contracts until a Fermi surface typical of a metallic state emerges in the form of coupled electron and hole regions. Thus, an increase in $t_\perp$ in a system of weakly coupled chains may induce an insulator−metal phase transition in a quasi-one-dimensional Mott insulator, which transforms into a metallic Fermi liquid.

According to various theoretical models, the presence of interchain coupling in a system of antiferromagnetic (AFM) spin chains with effective spin $S=1/2$ grants fundamentally new physical properties to such systems. This is the reason why these systems have attracted much attention of experimenters [8–14]. The results of examination of a system of AFM chains with effective pseudospin $S=1/2$ in $\mathrm{Yb_2Pt_2Pb}$ by high-resolution neutron scattering in strong magnetic fields were reported in [8]. Spinons (free fractional quasiparticles) are the fundamental excitations of such chains. Being paired, spinons form a triplet excitation continuum. In the strong Ising anisotropy limit $\Delta\gg 1$, spinons may be regarded as domain walls in an antiferromagnetically ordered state of a chain. Angular momentum conservation mandates that spinons are always created in pairs. Therefore, each spinon carries a fraction $(\pm 1/2)$ of the angular momentum change $\Delta S^z=0,\pm 1$ required to initially introduce the domain walls in an infinite chain. Since moving these domain walls is an energy and angular momentum conserving process, the walls will propagate freely, carrying the quanta of energy E and linear momentum q introduced into the system by their creation. This leads to the separation of spin and electron degrees of freedom, which is typical of a Luttinger liquid. The spinon excitation continuum in chains of spins with the Ising exchange interaction has an energy gap, and the ground state of the spin system is ordered. According to the results of experiments performed in [8], an external magnetic field closes this gap in $\mathrm{Yb_2Pt_2Pb}$ and stimulates the transition of AFM spin chains into an LL-like state. It was demonstrated that the interchain coupling restricts the motion of spinons (i.e., establishes confinement), and a gapless mode of fundamental longitudinal oscillations of the system of spin chains arises from this confinement.

We have already determined [15] the Landau quantization of the spectrum of pairs of low-energy magnetic excitations of Z_2 quantum spin liquid (QSL) in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the spinon–gauge field system within a temperature interval of 4.2−12 K. It was demonstrated that the formation of a continuous spectrum of QSL thermal excitations in the regime of „weak magnetic fields“ $H = 100$, 350, and 1 kOe results in quantum oscillations of „supermagnetization“ of samples within a temperature interval of 4.2−12 K in the form of three narrow paired peak features, which are superimposed on the exponential decay of the gap QSL magnetization at $T \to 0$. The emergence of an unusual spectrum of low-energy QSL excitations was interpreted within the models of Landau quantization of the spectrum of composite quasiparticles with fractional filling factors ν of three overlapping Landau bands. In the regime of „strong external magnetic field“ $H = 3.5$ kOe, quantum oscillations of the temperature dependences of magnetization of an incompressible spinon liquid were detected in the form of three narrow steps (plateaus) corresponding to the complete filling with spinons of non-overlapping Landau bands with integer filling factors. In the present study, we examine the features of the temperature dependences of „supermagnetization“ of $SmMnO_{3+\delta}$ ($\delta \sim 0.1$) measured in fields $H = 100$ and 350 Oe and 1 and 3.5 kOe in the field-cooled (FC) regime at temperatures $4.2 \le T \le 80$ K and the field dependences of magnetization around $T = 0$ measured in the FC and zero−field−cooled (ZFC) regimes. The obtained experimental data are interpreted using the available models.

2. Experimental procedure

Samples of self-doped $SmMnO_{3+\delta}$ ($\delta \sim 0.1$) manganites were grown from high-purity lanthanum, samarium, and electrolytic manganese oxides taken in a stoichiometric ratio. Synthesized powder was pressed under 10 kbar into disks 6 mm in diameter and 1.2 mm in thickness and sintered in air at a temperature of 1170°C for 20 h with subsequent temperature reduction at a rate of 70° C/h. According to the results of X-ray studies, the obtained pellets were single-phase ceramics. X-ray diffraction studies were carried out at 300 K using a DRON-1.5 diffractometer in $NiK_{\alpha 1+\alpha_2}$ radiation. The lattice symmetry and parameters were determined by the position and the nature of splitting of reflections of the perovskite-type pseudo-cubic lattice. The temperature dependences of the sample magnetization were measured using a VSM EGG (Princeton Applied Research) vibration magnetometer and a noncommercial magnetometer in constant magnetic fields of 100 and 350 Oe, 1 and 3.5 kOe. The temperature dependences of magnetization were obtained in the FC measurement regime: the samples were cooled in advance in the measurement field to 4.2 K and then heated to 100 K.

3. Experimental results

It can be seen from Fig. 1, *a* that the well-known ground state of a QSL with energy gap Δ_s between the ground and excited states is established in the studied $SmMnO_{3+\delta}$ samples in magnetic field $H = 100$ Oe. Commonly used models of the resonating valence bonds (RVB) state of low-dimensional frustrated antiferromagnets suggest that the presence of low-energy magnetic excitations of the ground nonmagnetic state of the system of spins, which consists of singlet pairs of spins of nearest neighbors, in the form of pairs of neutral quasiparticles with spin $S = 1/2$ (spinons) with various wavevectors is typical of the gap phase of a spin liquid. It is demonstrated in Fig. 1, *a* that the thermal excitation of spinon pairs proceeds within a fairly wide temperature interval of 4.2−12 K. Spinon pairs with collinear spins $S = 1/2$ are excited in magnetic field $H = 100$ Oe

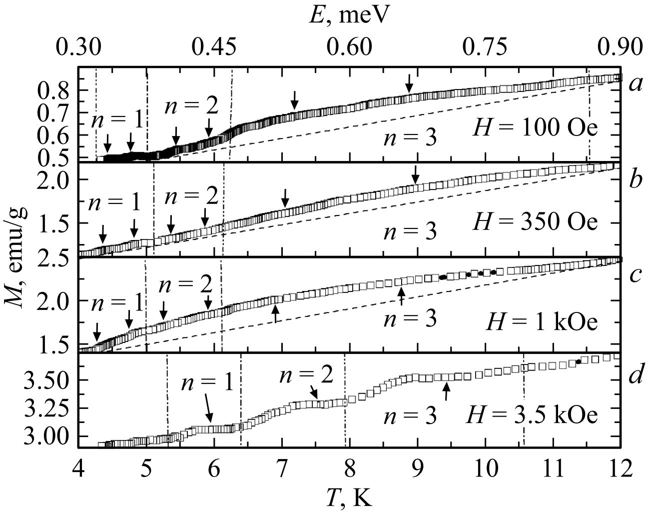

Figure 1. Quantum oscillations of the temperature dependence of magnetization of $SmMnO_{3+\delta}$ in magnetic field H. The continuum of thermal excitations of „supermagnetization“ of spinon pairs is divided into three overlapping Landau bands ($n = 1, 2, 3$). (*a*) $H = 100$ Oe. Oscillations of magnetization $M(T)$ correspond to the spectrum (quantized by the gauge field) of low-energy excitations of gap QSL (spinons) in the form of periodic paired peak „supermagnetization“features, which are superimposed on the reduction of $M(T)$ at $T \to 0$. (*b*) Formation of an intense asymmetric peak of $SmMnO_{3+\delta}$ „supermagnetization“ in magnetic field $H = 350$ Oe with its apex around $T_{\text{spinon}} \cong 8$ K. It is evident that the relative contribution of the excitation of spinon pairs in narrow Landau bands with energies $E_{1,2} < \Delta_s \approx 0.4$ meV to the sample „supermagnetization“ decreases. (*c*) Formation of an intense sinusoid peak of $SmMnO_{3+\delta}$ „supermagnetization“ in magnetic field $H = 1$ kOe with its apex around $T_{\text{spinon}} \cong 8$ K. This is the result of further enhancement of „giant“ fluctuations emerging in the ground QSL state in the process of H growth and leads to further smearing of the broadened spectrum of low-energy spinon excitations in the Landau band with number $n = 3$. The relative contribution of the excitation of spinon pairs in narrow Landau bands with numbers $n = 1$ and 2 to the sample „supermagnetization“ decreases further. (*d*) $H = 3.5$ kOe.

primarily within a temperature interval of 6−10 K in the form of a doublet of two broad and almost overlapping peaks of the sample „supermagnetization“ around mean excitation temperature $T_{\text{spinon}} \cong 8\,\text{K}$, which corresponds to thermal spinon excitation energy $E_{\text{spinon}} \cong 0.6\,\text{meV}$. Two weak doublets of magnetization of spinon pairs are observed at temperatures below 6 K. Their excitation energy E_{spinon} is smaller than low-energy gap $\Delta_s \approx 0.4\,\text{meV}$ that exists in the spinon excitation spectrum in a 2D system of spins between nonmagnetic singlet states of spins in the ground RVB QSL state and the excited magnetic QSL state in the form of spinons with collinear orientations of spins $S = 1/2$. The doublet excitation of spinon pairs around T_{spinon} proceeds within a temperature interval that overlaps with the depairing temperatures of pairs of 2D vortices of a superconducting liquid around temperature $T_{\text{KT}} \cong 12\,\text{K}$ of the topological Kosterlitz–Thouless phase transition of depairing of 2D vortex−antivortex pairs in a coherent superconducting state. This is indicative of the fact that their excitation energies are virtually the same. According to Fig. 1, *a*, a continuous spectrum of thermal excitations of spinon pairs in the form of periodic paired narrow peak features of the temperature dependences of supermagnetization around mean temperatures $T_1 \cong 4.6\,\text{K}$, $T_2 \cong 5.6\,\text{K}$, and $T_3 \cong 8\,\text{K}$, which are superimposed on the quasi-linear decay of the sample magnetization at $T \rightarrow 0$, is seen in the $\text{SmMnO}_{3+\delta}$ sample in magnetic field $H = 100\,\text{Oe}$ within a temperature interval of 4.2−9.5 K.

This allowed us to divide the continuum of thermal excitations of magnetization into narrow bands $n = 1$, 2, and 3 with certain magnetization features characterizing each one of them. The likely basis for this is quantization of the spectrum of spinons with fractional factors ν of filling of three overlapping Landau bands in the regime of a weak external magnetic field. Figures 1, *b* and 1, *c* show that the spectrum of spinon pairs in narrow Landau bands with $n = 1$, 2, and 3 becomes smeared noticeably as the external magnetic field increases to $H = 1\,\text{kOe}$. This results in the formation of a single symmetric sinusoid peak of „supermagnetization“ of $\text{SmMnO}_{3+\delta}$ within a fairly wide temperature interval of 4.2−12 K with its apex around $T_{\text{spinon}} \cong 8\,\text{K}$. The obtained result is indicative of the emergence of „giant“ fluctuations that form in the ground QSL state in an increasing magnetic field and induce smearing of the spectrum of low-energy spinon excitations. Thus, the spectrum of low-energy excitations of spinon pairs in $\text{SmMnO}_{3+\delta}$ in weak magnetic fields $H \leq 1\,\text{kOe}$ differs considerably both in shape and in the response to weak external influences (magnetic field, temperature) from the spectrum of spinon excitation in $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$ manganite of a close composition that was studied in [15]. The specific feature of the temperature dependences of „supermagnetization“ $M(T)$ obtained in an external magnetic field increasing to 3.5 kOe is the presence of well-marked step oscillations of magnetization of the spinon gas. Figure 1, *d* shows that supermagnetization oscillating within a temperature interval of 4.2−11 K is characterized by the emergence of equidistant narrow threshold features with width $\Delta T \cong 1{-}2\,\text{K}$ in the form of three narrow steps (plateaus) in bands $n = 1$, 2, and 3 of the dependences of „supermagnetization“ of the 2D spinon gas. The height of thresholds and the width of steps (plateaus) increase with T. As was demonstrated in [15], new quantum oscillations of dependences $M(T)$ correspond to integer filling with spinons of three Landau levels with a finite gap in the regime of a strong external magnetic field. Thus, even a relatively minor increase in the external field intensity, which reached $H = 3.5\,\text{kOe}$, resulted in a transition from a continuous spectrum of pair excitations to a discrete one.

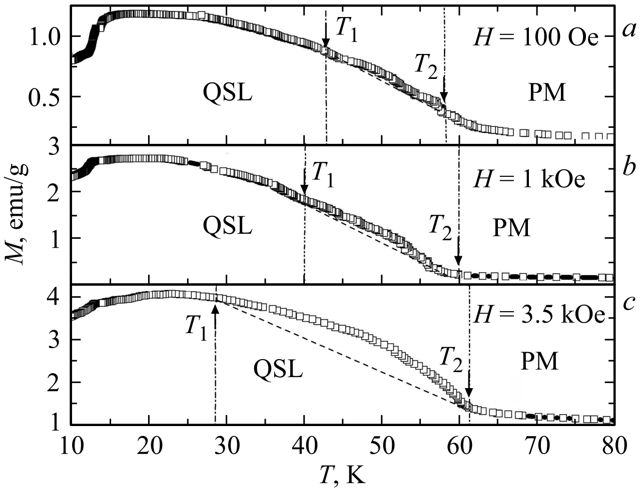

Figure 2. Temperature dependence of magnetization $M(T)$ in magnetic field H with a temperature interval of 10−80 K. (*a*) Spin–charge separation in $\text{SmMnO}_{3+\delta}$ in magnetic field $H = 100\,\text{Oe}$ at temperatures below 60 K. A symmetrical feature of the temperature dependence of the sample magnetization in the form of two „supermagnetization“ peaks of the same intensity around the temperatures of 48 and 56 K is formed within the $T_1 < T < T_2$ temperature interval with a width on the order of 15 K. These peaks are induced by the thermal excitation of fluctuating quasi-one-dimensional fragments of separate charge and spin density waves in the Luttinger liquid. (*b*) $H = 1\,\text{kOe}$. (*c*) $H = 3.5\,\text{kOe}$. (QSL stands for a quantum spin liquid, and PM is a paramagnetic state.)

The formation of charge/spin fragments in the form of fluctuating CDW/SDW LL states was observed in the FC regime of measurement of the temperature dependences of magnetization in fields $H = 100\,\text{Oe}$, 1 and 3.5 kOe at temperatures below 60 K (Figs. 2−5). Figures 2, *a* and 3 show that a symmetrical double-peak feature of the sample „supermagnetization“ in the form of two well-marked magnetization $M(T)$ peaks of the same intensity around the temperatures of 48 and 56 K is formed within the $T_1 < T < T_2$ temperature interval with a width around 15 K in magnetic field $H = 100\,\text{Oe}$. These peaks are induced by the thermal excitation of fluctuating quasi-one-dimensional fragments of charge and spin density waves. We have studied similar peak features of magnetization

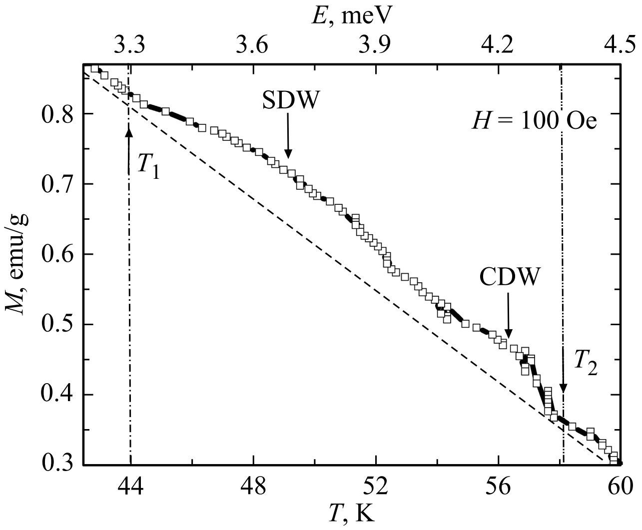

Figure 3. Formation of bosons in $SmMnO_{3+\delta}$ in magnetic field $H = 100$ Oe at temperatures below 60 K. A symmetrical feature of the temperature dependence of the sample magnetization in the form of two „supermagnetization" peaks of the same intensity around the temperatures of 48 and 56 K is formed within the $T_1 < T < T_2$ temperature interval with a width around 15 K. These peaks are induced by the thermal excitation of fluctuating quasi-one-dimensional fragments of charge (CDW) and spin (SDW) density waves in the Luttinger liquid.

$M(T)$ earlier [15] in different magnetic fields H and measurement regimes. According to [15], the emergence of two peak features of different width and intensity at temperatures below 60 K is attributable to the existence of a small pseudogap Δ_e in the electron spectrum, which is typical of a weak Mott insulator with a spinon Fermi surface. The pattern changed drastically when the magnetic field intensity increased to $H = 1$ kOe (Figs. 2, *b* and 4).

Figure 4 shows that an asymmetric „supermagnetization" peak with a width of 16 K forms around $T = 50$ K. We associate this peak with the thermal excitation of a fluctuating coupled state of charge and spin density waves in the Luttinger liquid. When the external magnetic field increases further to $H = 3.5$ kOe, the low-energy thermal LL excitation changes even more significantly: a considerably broader sinusoid „supermagnetization" peak forms around $T = 50$ K (Figs. 2, *c* and 5).

In the present study, confinement of low-energy thermal excitations of spinons in $SmMnO_{3+\delta}$ was also observed in the form of excitations similar to nanofragments of 1D charge and spin density waves, which are typical of low-energy excitations of a metallic one-dimensional quantum LL. According to the field dependences of magnetization $M(H)$ measured in different regimes at temperatures around $T = 0$ (Figs. 6, 7), an increasing intensity of the external magnetic field eventually induces a quantum second-order phase transition to the LL state in QSL. Peak features of „supermagnetization" with different degrees of overlapping, which are typical of the excitation of gapless uncoupled charge and spin density weaves in an LL, form as a result of this transition around $H = 0$. Figures 6 and 7 present the field dependences of magnetization of $SmMnO_{3+\delta}$ around $T = 0$ in the FC and ZFC measurement regimes plotted as isotherms *1* and *2* of remagnetization of samples with different directions of growth of intensity of the external magnetic field. It can be seen that two peak features of magnetization $M(H)$ form around $H = 0$ within a field interval of ±500 Oe in the process of remagnetization in two measurement regimes. According to [6], spin–charge separation occurs in the limit of low excitation energies in

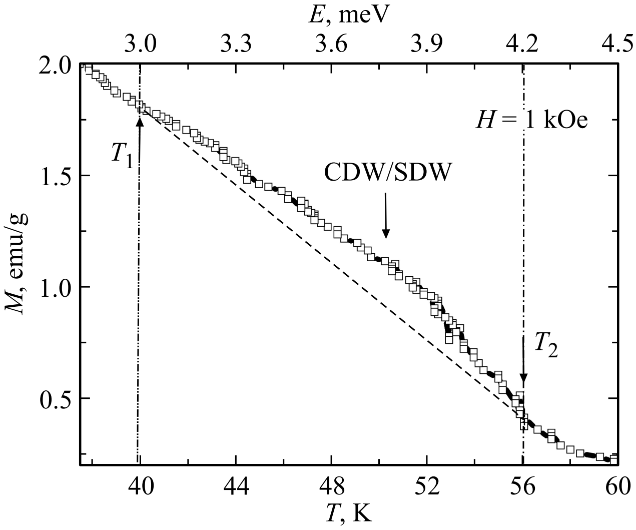

Figure 4. Asymmetric „supermagnetization" peak with a width of 16 K forms in the temperature dependence of magnetization $M(T)$ around $T = 50$ K when the magnetic field intensity increases to $H = 1$ kOe. We associate this peak with the thermal excitation of a fluctuating coupled LL state in the sample.

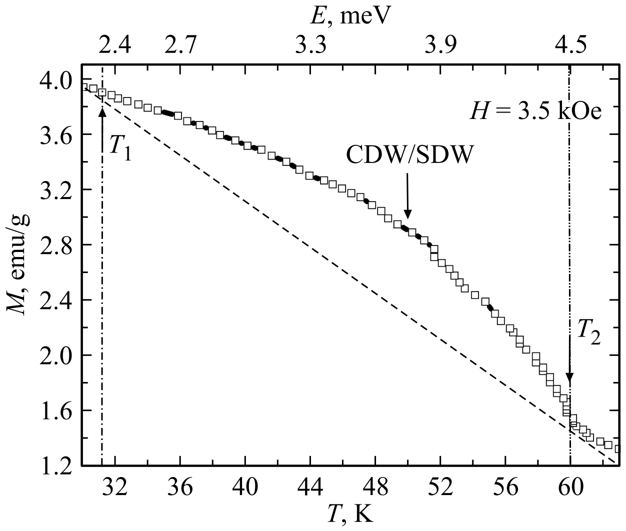

Figure 5. When the external magnetic field increases to $H = 3.5$ kOe, the low-energy thermal LL excitation changes significantly: a considerably broader sinusoid „supermagnetization" peak forms around 50 K.

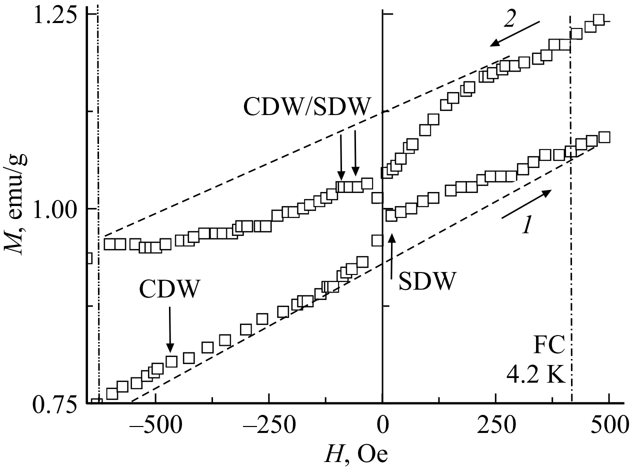

Figure 6. Field dependences of magnetization of $SmMnO_{3+\delta}$ at $T = 4.2\,K$ in the FC measurement regime plotted as isotherms *1* and *2* of remagnetization of samples with different directions of growth of intensity of the external magnetic field. Two peak features of magnetization form around $H = 0$ within a field interval of $\pm 500\,Oe$ in the process of sample remagnetization in the FC measurement regime. (FC — field-cooled.)

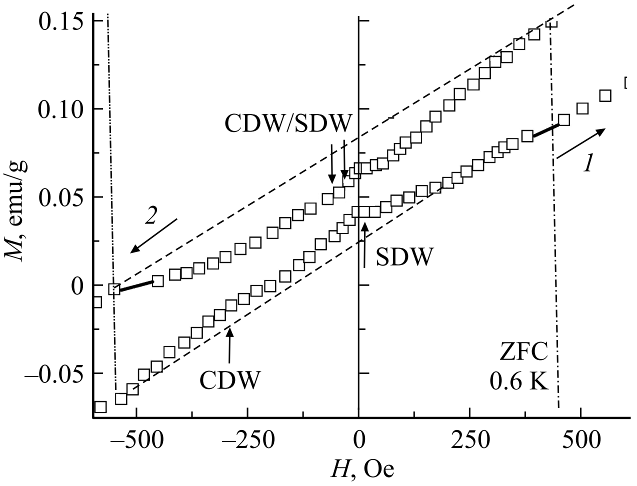

Figure 7. Field dependences of magnetization of $SmMnO_{3+\delta}$ at $T = 0.6\,K$ in the ZFC measurement regime plotted as isotherms *1* and *2* of remagnetization of samples with different directions of growth of intensity of the external magnetic field. It can be seen that two peak features of magnetization form around $H = 0$ within a field interval of $\pm 500\,Oe$ in the process of sample remagnetization in the ZFC measurement regime. (ZFC — zero-field-cooled.)

a one-dimensional Mott insulator. This made it possible to calculate exactly their dynamic spectral charge and spin density function $A(\omega, k_F + q)$. It was established that the creation and annihilation operators for right- and left-moving fermions in a one-dimensional Mott insulator at temperature $T = 0$ factorize into spin and charge regions upon bosonization in the form of fragments of separate one-dimensional spin and charge density waves incommensurate with the crystal lattice.

We believe that the emergence of two unusual peak features of isotherms *1* and *2* of $SmMnO_{3+\delta}$ remagnetization in near-zero magnetic field is a corollary of spin–charge separation typical of a one-dimensional Mott insulator in the quantum LL state, which has already been thoroughly examined theoretically [6,7].

Most conspicuous is the fact that two peak features of isotherms *1* and *2* of remagnetization around $H = 0$ differ depending on the direction of growth of intensity of the external magnetic field. It can be seen from Figs. 6 and 7 that uncoupled CDW and SDW states form when the magnetic field increases in the positive direction (isotherm *1*). At the same time, when the direction of growth of field intensity is reversed (isotherm *2*), coupled CDW/SDW states are seen around $H = 0$.

We believe that this behavior of magnetization in the low-temperature region is attributable to the fact that the transverse component of the external magnetic field closes spinon gap Δ_s in $SmMnO_{3+\delta}$ and facilitates the transition of a system of zigzag AFM spin chains weakly coupled by the exchange interaction to a state similar to a metallic LL. The spatial separation of charge and spin excitations is a specific feature of this state. Many-particle excitations in the form of 1D charge and spin density waves, which follow the Bose–Einstein statistics, are dominant in this quasi-one-dimensional case and generally substitute for elementary excitations in the form of spinon pairs.

Intriguing is the result that uncoupled charge/spin density waves form in isotherms *1*, while a coupled 1D CDW/SDW collective excitation of the quantum spin liquid emerges in isotherms *2* in near-zero magnetic field. The excitation of collective 1D states of this kind in an external magnetic field has been observed earlier in systems of weakly exchange-coupled Heisenberg and Ising AFM spin chains of various configurations and degrees of anisotropy. According to these studies, even a slightly increased transverse component of an external magnetic field may induce further confinement of spinon pairs that is accompanied by the emergence of gapless modes of fundamental longitudinal oscillations of a system of spin chains.

4. Discussion of experimental results

4.1. Landau quantization of the spinon spectrum by the gauge field in gap Z_2 quantum spin liquid with a magnetic flux

We have already provided a detailed qualitative explanation for the evolution of quantum oscillations of „supermagnetization“ which were measured for the first time in self-doped $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ manganite within a temperature interval of 4.2–100 K, in [15] using the available mean-field models of Landau quantization of the spectrum of low-energy excitations of Z_2 quantum spin liquid with flux [16–34]. The quantization of spinons in QSL with a spinon Fermi surface in a state close to the Mott insulator–metal transition was examined within the Hubbard model in [17]. An effective spin Hamiltonian on the triangular lattice was constructed for the four-spin exchange interaction in an external magnetic field. The first term of the Hamiltonian corresponds to the exchange between pairs of nearest-neighbor spins with energy J_2, while the second term corresponds to the ring exchange interaction for four neighboring spins with energy J_4. An additional term emerges in the spin Hamiltonian in an external magnetic field. This term is the sum of three-spin exchange interactions associated with the motion of spins in individual triangles. It is proportional to a small flux $\Phi^{\mathrm{ext}}_{\Delta} \ll 1$ of the external magnetic field through a triangle with coupling constant J_3. It was demonstrated that the external magnetic field is related linearly to the chirality of spins in the triangular lattice. The spin chirality corresponds to the flux of the internal gauge field within the gauge field theory that is commonly used to characterize spin liquid. According to the proposed model, the external magnetic field induces a static internal flux in a system of spins with four-spin exchange interaction. The results of quantitative evaluation of this effect revealed that the magnitude of the produced magnetic orbital field acting on spinons is comparable to (and even exceeds) the magnitude of the external magnetic field. It was argued that since the stiffness of the internal gauge field is very small, the homogeneous state of the spinon–gauge field system is unstable at low temperatures due to strong Landau quantization of the energy spectrum of spinons. This instability is reminiscent of the strong magnetic field regime in metals, but, according to the results of quantitative evaluation, the temperature–magnetic field range of existence of instability of the homogeneous state is significantly wider in the spinon–gauge field system. The response of the spinon–gauge field system changes dramatically at low temperatures: the Landau quantization of spinons in the static internal field at temperatures below the critical value is not smeared by temperature. It was found that the homogeneous state for $k-(\mathrm{ET})_2\mathrm{Cu}_2(\mathrm{CN})_3$ QSL with a continuously varying internal field becomes unstable at temperatures below several Kelvin in moderate external magnetic fields. A similar effect has earlier been studied extensively for magnetic oscillations in common metals. The instability regime of the homogeneous state in the spinon–gauge field system is significantly wider than in common metals, since the stiffness of the internal gauge magnetic field is much lower than the stiffness of the external field. Roughly speaking, spinon states with integer filling of Landau levels are more stable in the spinon–gauge field system than states with continuously varying filling. Therefore, it is energetically favorable for the internal gauge field to adjust its discretization to achieve this. An instability of this kind allows one to examine the properties of the spinon Fermi surface directly with the use of experimental data on oscillations of the sample magnetization. The mechanism of Landau quantization of the spinon–gauge field system spectrum in an increasing external magnetic field and at increasing temperature of the sample was discussed in detail in this study. An inhomogeneous state of a system of spinons at temperature $T = 0$ was considered first. In zero external magnetic field $(\Phi^{\mathrm{ext}} = 0)$ with weak ring exchange, the flux through each triangle is $\Phi_{\Delta} = \pi/2$. Just as in the case of strong ring exchange, the state with zero flux is then the optimum one. It was found that states with flux have higher values of absolute susceptibility $|\chi_{\Phi}| \leq |\chi_0|(1 + c\Phi^2_{\Delta})$ at $\Phi^{\mathrm{ext}} = 0$ than the state without flux. In the case of a small external flux $\Phi^{\mathrm{ext}} \neq 0$, static flux $\Phi^{\mathrm{int}} \equiv \gamma\Phi^{\mathrm{ext}}$, where $\gamma \gg 1$, of the internal gauge field is generated in the system of spins with a triangular lattice. Thus, the effective orbital field acting on spinons is comparable in magnitude to the applied magnetic field. The dependence of magnetic susceptibility $|\chi_{\Phi}|$ of the spinon–gauge field system on filing factor ν of Landau levels was calculated numerically within the mean-field model. If static internal flux $\Phi^{\mathrm{int}}_{\Delta}$ is present in elementary triangles, the spinon spectrum consists of Landau bands. The filing factor of Landau levels is $\nu = \pi/(2\Phi^{\mathrm{int}}_{\Delta})$ for both spin orientations; the internal magnetic flux states become special if ν assumes integer values. It was determined within the mean-field model that magnetic susceptibility $|\chi_{\Phi}| = |\chi_0|(1 + c\Phi^2_{\Delta}) - \Xi^{\mathrm{osc}}_{\Phi}$ of the spinon–gauge field system ($\Xi^{\mathrm{osc}}_{\Phi}$ is the oscillating part of magnetic susceptibility) near integer values of the filing factor of Landau levels has the form of a series of sharp upward-pointing peaks. They rapidly decrease in size as ν increases, since the following is true for the oscillating part of magnetic susceptibility: $\Xi^{\mathrm{osc}}_{\Phi} \propto \nu^{-2}(\nu - k)(k + 1 - \nu)$. Quantum oscillations of $|\chi_{\Phi}|$ emerge due to the discretization of spinon Landau levels in the form of a series of stepwise first-order transitions of the spinon–gauge field system through these states. In the regime of measurements of magnetic susceptibility of the system with a continuously varying intensity of the external magnetic field, internal flux Φ^{int} then undergoes a series of stepwise transitions through a discrete set of values corresponding to integer filling of Landau levels with spinons. The spinon spectrum in this case has a gap corresponding to cyclotron frequency $\hbar\omega_c \approx 3.57 t_{\mathrm{spinon}}/\nu$. This state of spinons is essentially the same as the well-known chiral state of a spin liquid with an internal gauge field [20]. Oscillating part $\varepsilon^{\mathrm{osc}} = 12 t_{\mathrm{spinon}} \Xi^{\mathrm{osc}}_{\Phi}$ (where $t_{\mathrm{spinon}} = 2g_2 J_2 |\chi_0| + 32 g_4 J_4 |\chi_0|^3$) of the spinon energy may be interpreted as the oscillating part of the effective spinon kinetic energy. A finite temperature smears the effects of discretization of Landau levels; the instability of flux states with continuously varying Φ_{int} becomes weaker. The emergence of a series of magnetization plateaus of frustrated Heisenberg AFMs and a chiral spin liquid in an external magnetic field is closely related to the Landau quantization of the spinon spectrum by the Chern–Simons gauge field [35–37].

According to [20], the ground state of the gapless phase of Z_2 QSL with flux is characterized by Hamiltonian H_{mean} that is equivalent to the one used for electron hopping between the sites of a crystal lattice with „magnetic" flux π

through a site. The Fermi surface for a system of fermions in this state with half filling of the conduction band contains singular points at $(k_x, k_y) = (0, 0)$ and $(0, \pi/a)$. Gapless excitations of spins through singular points of the Fermi surface correspond to the state of a spin liquid with flux. The Z_2 gap spin liquid phase with topological order parameter $\chi_{i,j}$, which generates flux, may also exist within the mean-field model. Fermions characterized by Hamiltonian H_{mean} behave as though they move in a magnetic field. When the flux is strictly commensurate with the fermion density (i.e., one fermion per site), an integer number of Landau levels (bands) is filled completely. The fermion gas then becomes incompressible, since a finite gap between Landau levels forms. Fluctuations of the fermion density are lacking in this case. The emergence of this quantized fermion state is accompanied by spontaneous breaking of time reversal and parity symmetry. The corresponding states of the spin system are called chiral spin states. Coupling a_μ between a fermion ensemble and the gauge field is identical to the coupling between electrons and an electromagnetic field. Thus, a phenomenon similar to the Hall effect is likely to manifest itself in a system of fermions controlled by the gauge field. The „Hall effect" in this case consists in that the „electric" component of gauge field a_μ induces a fermion current in the direction perpendicular to the direction of „magnetic" component b of gauge field a_μ. There is an opinion that the Hall conductivity of a filled fermion band is quantized by field b as an integer multiplied by $1/2\pi$. The density of controlled fermions in a chiral spin liquid may be adjusted without creating fermions in the conduction band or holes in the valence band. A slow variation of the direction (rotation) of „magnetic" flux $\Phi = \int d^2xb$ of gauge field a_μ induced by external factors is sufficient for this. The flux rotation induces a circular „electric" field e_θ that, in turn, generates a current of controlled fermions in the radial direction. As a result, charge accumulates around „magnetic" flux Φ. It was demonstrated that the total number of fermions induced by the variation of flux Φ is $N = -\sigma_{xy}\Phi = -\Phi/\pi$. According to the experimental data obtained in this study, the variation of the external magnetic field may act as such a factor that alters flux Φ of gauge field a_μ. The striking difference in quantization of the spinon spectrum in $\text{SmMnO}_{3+\delta}$ and $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$ is apparently attributable to the difference in energies of the interchain exchange interaction.

4.2. Formation of bosons in the form of 1D charge/spin density waves in systems of AFM spin chains induced by confinement of spinon pairs

The mechanism of formation of a gapless mode of fundamental longitudinal oscillations of a system of AFM spin chains in $\text{Yb}_2\text{Pt}_2\text{Pb}$ was studied by high-resolution neutron scattering in strong magnetic fields in [8]. Earlier experiments on neutron scattering have demonstrated that $4f$-orbital overlapping imparts unusual magnetic properties to $\text{Yb}_2\text{Pt}_2\text{Pb}$ metal, where orthogonal pairs of Yb ions with pseudospin $S = 1/2$ are located in tetragonal ab planes. It was found that low-energy spinons are the excitations of the magnetic system of spin chains in zero magnetic field. These spinons form a strongly disperse excitation continuum with its width exceeding the energy gap considerably with wavevector $\mathbf{q}_{\text{L}}$ directed along the chains. The boundaries of the two-spinon excitation continuum are set by the extreme values of energy E and linear momentum q of a spinon pair, which are conserved by one particle and one hole with spin $S^z = \pm 1/2$, respectively. In zero magnetic field, chemical potential $\nu = 0$ is in the middle of spinon energy gap Δ_{S} separating the bands of particles and holes with strong dispersion. An entirely flat continuum of spinon excitations is established for spinons with a perpendicular direction of wavevector $\mathbf{q}_{\text{HH}}$. This is indicative of complete uncoupling of spinons from different chains. The flat dispersion of interchain excitations in zero magnetic field suggests that the interchain interaction at low excitation energies is frozen when spinon gap $\Delta_{\text{S}} \neq 0$. It was found for spinons with wavevector $\mathbf{q}_{\text{L}}$ that an increase in magnetic field intensity within $0 \leq H \leq 30\,\text{kOe}$ is accompanied by a significant alteration of the shape of the spinon excitation spectrum in $\text{Yb}_2\text{Pt}_2\text{Pb}$. This is attributable to the fact that an increasing magnetic field alters the occupancy of disperse spinon bands with $S^z = \pm 1/2$ due to the shift of chemical potential μ in energy gap Δ_{S}. The Zeeman interaction with a magnetic field reduces the system potential to $\mu = -g\mu_{\text{B}}HS^z$, which may result in gap closure in sufficiently strong fields. It was established that the potential needed for gap closure and the emergence of holes at temperatures $k_{\text{B}}T < \Delta_{\text{S}}$ is $|\mu| = \Delta_{\text{S}} = 0.095\,\text{meV}$. This value is attained when the field increases to critical $H_{c1} = 5\,\text{kOe}$. In the limit of relatively weak fields, a jump in magnetization was observed instead of a linear increase in a magnetic field increasing from zero to H_{c1}. It was demonstrated that gap closure at $H \geq H_{c1}$ results in confinement of spinon pairs, and a gapless mode of fundamental longitudinal oscillations of the system of spin chains arises from this confinement. The confinement of low-energy excitations (spinons) with fractional quantum numbers in a system of AFM chains of spins $S = 1/2$ coupled by weak exchange interactions in Heisenberg CaCu_2O_3 AFM with critical temperature $T_{\text{N}} = 25\,\text{K}$ of AFM ordering was examined in [9]. It is typical of states with a spatially restricted spinon motion for spinons in a system of chains to be coupled by an interaction that becomes stronger with distance. The infeasibility of individual observation of such excitations is one of the corollaries of this. The authors of [9] draw an analogy between spinon confinement in a system of AFM chains of spins $S = 1/2$ and similar phenomena known from particle physics, where heavy particles (baryons and mesons) are created due to quark confinement. A simplified pattern of spinon confinement is observed in chains with a significant Ising-like anisotropy of the exchange interaction, where the ground states have a finite Néel order. Spinons then act as domain walls separating two degenerate states with opposite magnetizations. The pattern is different in the case of chains with Heisenberg exchange between spins. Although domain walls are always created in pairs in an individual chain, no energy is lost when they are moving relative to each other. Therefore, spinons in an individual chain are not coupled into pairs of spins $S = 1/2$ by strong attraction. In contrast to the Ising limit, only two types of chain excitation (triplet and singlet) exist at the point with zero anisotropy of exchange interaction between spins. Excitations in an individual chain (spinons) with spin $S = 1/2$ are confined even by an infinitesimal interchain coupling. The majority of spin ladders studied earlier do not exhibit this effect, since a strong interchain interaction suppresses the excitation of spinons at all excitation energies. The results of neutron-scattering studies reported in [9] correspond to the system of weakly coupled chains in spin ladders in $CaCu_2O_3$ cuprate with orthorhombic symmetry $Pmmn$. The crystal structure of $CaCu_2O_3$ features CuO layers stacked in the c direction; spin chains lie in planes ab along the b direction and are shifted by half a unit cell along axis a. Ladders are coupled by several weak interactions. Within planes ab, Cu^{2+} ions on neighboring ladders are coupled via Cu−O−Cu bonds. Just as the other planar cuprates, $CaCu_2O_3$ supports four-spin exchange interaction J_{cyclic} that couples four copper ions, which form rectangular plaquettes in planes ab. According to the results of calculations performed by other research groups, the primary exchange constants are $J_{\text{leg}} = -147\,\text{meV}$; $J_{\text{rung}} = -15\,\text{meV}$; and $J_{\text{cyclic}} = 4\,\text{meV}$, where $J_{\text{leg}}, J_{\text{rung}}$ are the leg and rung exchange constants, respectively. The region between two spinons (domain walls) in a chain consists of inverted spins. If such a chain is coupled antiferromagnetically with another chain in a spin ladder, these inverted spins lose energy (compared to the case when they are parallel to spins of the neighboring chain). This energy loss, which is proportional to the distance between spinons, incudes their confinement. It was found in [9] that the magnetic properties of the system of chains in ladders at high excitation energies are similar to those of uncoupled individual chains. The neutron-scattering signal corresponds in this case to scattering typical of an individual chain of spins $S = 1/2$ with an interchain leg exchange constant $J_{\text{leg}} = -162\,\text{meV}$. At the same time, integral spin excitations typical of strongly coupled chains are dominant at low energies of excitation of chains by neutrons. Thus, weakly coupled ladders in $CaCu_2O_3$ differ significantly from the well-studied strongly coupled spin ladders that always remain in the strong confinement regime and support only magnon excitations ($S = 1$). A system of Ising-like weakly coupled antiferromagnetic XXZ chains in $SrCo_2V_2O_8$ in an external magnetic field was studied in [10] by high-resolution terahertz spectroscopy. A series of excitations with characteristic Zeeman splitting in the external magnetic field was observed at low temperatures. The authors identified these magnetic excitations as confined spinon-pair excitations. The absorption spectrum measured in light transmission through the sample at a temperature of 6.5 K (slightly higher than $T_N = 5\,\text{K}$) within an energy interval of 1.3−6 meV contained intense absorption line E_1 of light with an oscillation frequency around $\omega = 0.35\,\text{THz}$ and a subsequent series of absorption lines with energies $E_2, E_3, \ldots, E_9$, which were abruptly reduced to zero intensity when the frequency increased to $\omega = 1.2\,\text{THz}$. These absorption lines were not observed when the AC magnetic field of light wave $h(\omega)$ was directed parallel to the spins in chains along axis c of the crystal lattice. This is indicative of the magnetic nature of lines of the discovered discrete light absorption spectrum, which is governed by the excitation of spinon pairs. The measured dependence of intensity of these lines on the external DC magnetic field verified this conclusion. In a single Ising chain, two spinons are created by a spin flip. Spinons, each carrying spin $-1/2$, may move freely along the chain by subsequent spin flips without any loss of energy. This results in the formation of a highly degenerate first excited state with energy J, which is equal to the intrachain nearest-neighbor exchange interaction. A finite transverse component of the intrachain exchange interaction lifts the degeneracy, leading to an excitation continuum of two spinons that propagate independently along the chain. In the presence of weak interchain interactions, two spinons are coupled, since their separation frustrates the interchain exchange interaction. The interchain interaction acts as an attractive potential $V(z)$ for spinons that is proportional to distance z between them. Thus, the weak interchain interaction in the reason behind linear confinement of spinons into coupled pairs. Potential $V(z)$ induces quantization of the excitation continuum of the system of chains in $SrCo_2V_2O_8$ into discrete levels E_i of coupled spinon pairs. The dynamics of excitation of spinon pairs in an Ising-like antiferromagnet ($BaCo_2V_2O_8$) at temperatures below $T_N \approx 5.5\,\text{K}$ was studied by inelastic neutron scattering in [11]. The authors note that a one-dimensional antiferromagnet is of special interest, since quantum fluctuations in a one-dimensional system of spins suppress the classical long-range Néel order. The ground state becomes disordered in this case. The excitation spectrum of this disordered ground state of a 1D system is a continuum of many excitation pairs with spin $S = 1/2$ (spinons) similar to domain walls in an Ising magnet. However, a one-dimensional system of spins may become AFM-ordered at very low temperatures in the presence of even a very weak coupling between chains. Three possible spin states $S = \pm 1, 0$ for spinon pairs then transform into two transverse oscillation modes and a third (collective) excitation that corresponds to fluctuations parallel to the ordered momentum (i.e., longitudinal oscillations). An unusual spectrum of spin excitations, which was explained in terms of spinon confinement induced by the interchain interactions, was discovered in [11] at measurement temperature $T = 1.6\,\text{K}$ below T_N. These excitations consist of two types of alternating series with transverse and longitudinal polarizations. Similar series of excitations of Ising spin chains have been observed earlier in $CsCoCl_3$, $CsCoBr_3$, and $CoNb_2O_6$. Longitudinal oscillations in chains correspond to longitudinal fluctuations of the ordered momentum. The obtained experimental data revealed that $BaCo_2V_2O_8$ with moderate Ising anisotropy and significant interchain interactions satisfies well the requirements for observation of long-lived longitudinal oscillations of a 1D-system of spins. Summing up the results of analysis of experimental data, the authors of [11] conclude that excitations in $BaCo_2V_2O_8$ at temperatures below T_N are quantized due to the weak interchain interaction and consist of two alternating series of transverse oscillations and a remarkably strong longitudinal Zeeman ladder. It is assumed that longitudinal modes are stabilized owing to moderate Ising anisotropy, which prevents their decomposition into discrete gap transverse modes. Magnetic excitations in $BaCo_2V_2O_8$ were studied in [12] by elastic and inelastic neutron scattering at temperature $T = 3.5\,K$ as functions of intensity H of a transverse DC magnetic field within the $0-10\,T$ interval. The field was oriented perpendicular to the system of AFM chains with Ising-like exchange along axis c of the crystal lattice. Earlier studies into the influence of a longitudinal magnetic field on the magnetic properties of $S = 1/2$ 1D antiferromagnet with Ising-like anisotropy have revealed that a quantum phase transition from an ordered state with long-range AFM ordering to a disordered phase similar to a spin liquid is likely to occur in the process of H growth. This transition is attributable to the development of a longitudinal spin correlation incommensurate with the crystal lattice in an increasing field. A finite magnitude of interchain exchange interactions induces the emergence and growth of a phase with a long-range longitudinal spin density wave and initiates variations of its incommensurability with the lattice. The influence of transverse magnetic fields on phase transitions in similar systems of spin chains has also been examined. It was demonstrated that a transverse external magnetic field also affects the magnetic order in $BaCo_2V_2O_8$ (e.g., suppresses it at $H \approx 1\,T$). We are mostly interested in the results of examination of the influence of transverse fields on the spinon spectrum in [12]. It was found that the peak features of series of spinon excitations split gradually as the external magnetic field increases. It was demonstrated clearly that the behavior of the magnetic excitation spectrum in $BaCo_2V_2O_8$ in a magnetic field is characterized well by a model based on the theory of an antiferromagnetic XXZ chain of spins with $S = 1/2$. This model predicts that the dynamic magnetic structure factor of the spin component along the chain increases with H, and well-marked incommensurate spin correlations emerge. A radical modification of quantum excitations in $BaCo_2V_2O_8$ in a transverse field at H exceeding the critical field value was discovered in [13] in neutron-scattering experiments. It was demonstrated that this modification is induced by the quantum phase transition between two different types of soliton-like topological excitations. The process of spinon confinement in a quasi-one-dimensional anisotropic Heisenberg antiferromagnet ($SrCo_2V_2O_8$) was studied later in [14] by inelastic neutron scattering within a wide temperature interval above and below critical temperature $T_N = 5.2\,K$ of the phase transition to the AFM state. The spinon continuum observed at $T = 6\,K$ in the form of an intense peak feature smeared within a wide interval of spinon excitation energies is in good agreement with the theoretical prediction for the transverse dynamical structure factor of a Heisenberg XXZ chain at zero temperature. At temperatures below T_N, pairs of spinons are confined, and two series of meson-like bound states with longitudinal and transverse polarizations, which correspond to the propagation of excitation of an XXZ chain by neutrons in longitudinal and transverse directions, are observed. It was found that the broad peak feature transforms into two series of narrow sharp peaks when temperature drops to $T = 1.5\,K$. The width and the intensity of these narrow peaks depend strongly on the excitation conditions. The results of our study of $SmMnO_{3+\delta}$ follow closely the pattern of boson formation in the regime of spinon confinement.

5. Conclusion

Quantum oscillations of the temperature dependence of magnetization of $SmMnO_{3+\delta}$ in magnetic field $H = 100$ and 350 Oe, 1 and 3.5 kOe within a temperature interval of $4.2-12\,K$, which are induced by Landau quantization of the spectrum of spinon pairs with collinear spins $S = 1/2$ in a weak Mott insulator, were studied. Oscillations of magnetization $M(T)$ correspond to the spectrum (quantized by gauge field b) of low-energy excitations (spinons) of the ground RVB state of a gap Z_2 quantum spin liquid in the form of periodic paired peak features of „supermagnetization", which are superimposed on the exponential decay of $M(T)$ at $T \to 0$. The continuum of thermal excitations of „supermagnetization"of spinon pairs in magnetic field $H = 100\,Oe$ is divided into three overlapping Landau bands with energies E_n $(n = 1, 2, 3)$ and fractional band filling factors ν. The excitation of spinon pairs in the band with $n = 3$ around mean temperature $T_{spinon} \cong 8\,K$ in the form of a symmetric intense „supermagnetization"doublet consisting of two broad overlapping peaks produces the primary contribution to magnetization. This suggest that pseudogap $\Delta s \approx 0.4\,meV$ is present in the excitation continuum of spinon pairs in $SmMnO_{3+\delta}$. The doublet excitation of spinon pairs proceeds in the immediate vicinity of temperature $T_{KT} = T_c \cong 12\,K$ of the topological Kosterlitz–Thouless phase transition of depairing of 2D vortex–antivortex pairs in a coherent superconducting state. This is indicative of a crossover of excitation energies of spinon pairs and free 2D vortices in $SmMnO_{3+\delta}$. As the magnetic field intensity increases to $H = 350\,Oe$, the symmetric double-humped peak feature of $M(T)$ centered around temperature $T_{spinon} \cong 8\,K$ of gap excitation of spinons with $S = 1/2$ transforms into a broad asymmetric peak. This is accompanied by significant broadening of the spectrum of excitation of spinon pairs in overlapping Landau bands with $n = 1, 2$, and 3. The obtained result is indicative of the emergence of „giant" fluctuations that form in the ground

QSL state in a magnetic field with increasing H and induce smearing of the spectrum of low-energy spinon excitations in the Landau band with number $n = 3$ over temperature. As the field increases further to $H = 1\,\mathrm{kOe}$, the broad asymmetric „supermagnetization"peak transforms into a symmetric sinusoid peak typical of the excitation continuum of spinon pairs with strong dispersion. This transformation of the spinon pair spectrum in an increasing field is attributable to the growth of the transverse component of the external magnetic field that intensifies the interchain interaction, reduces pseudogap Δ_s, and, consequently, leads to an increase in the density of spinon excitations in AFM spin nanochains in $\mathrm{SmMnO}_{3+\delta}$. The thermal excitation spectrum of spinons changes dramatically in magnetic field $H = 3.5\,\mathrm{kOe}$: a new type of quantization of the spinon spectrum in the form of threshold magnetization features shaped like narrow steps (plateaus), which correspond to integer filling with spinons of three Landau bands with a finite gap, emerges in Landau bands $n = 1, 2$, and 3. The height of thresholds and the width of steps increase with temperature. The emergence of new quantum oscillations of the temperature dependences of magnetization in the strong-field measurement regime is attributable to the gauge-induced Landau quantization of a dense (incompressible) gas of spinons with fractional spin $S = 1/2$ that move in circular orbits in the direction opposite to that of „magnetic" component b of the gauge field.

The dependence of thermal excitations of a Luttinger liquid in the form of charge and spin density waves at $T \leq 60\,\mathrm{K}$ on the external magnetic field was also examined in the present study. A symmetrical double-peak feature of the sample „supermagnetization" in the form of two well-marked overlapping magnetization $M(T)$ peaks of the same intensity around the temperatures of 48 and 56 K is formed within the $T_1 < T < T_2$ temperature interval with a width around 15 K in magnetic field $H = 100\,\mathrm{Oe}$. These peaks are induced by the thermal excitation of fluctuating quasi-one-dimensional fragments of uncoupled charge and spin density waves. The pattern changes dramatically when the magnetic field increases to $H = 1\,\mathrm{kOe}$: instead of a symmetric excitation doublet of uncoupled charge and spin density waves within a temperature interval of $44-58\,\mathrm{K}$, an asymmetric „supermagnetization" peak, which corresponds to the excitation of a coupled state of fragments of charge/spin density waves, forms within a temperature interval of $40-56\,\mathrm{K}$. When the external magnetic field increases further to $H = 3.5\,\mathrm{kOe}$, a broad sinusoid peak of the temperature dependence of magnetization of $\mathrm{SmMnO}_{3+\delta}$ emerges within a temperature interval of $32-60\,\mathrm{K}$. This peak is similar in nature to the excitation of longitudinal spin waves (magnons) in a system of AFM Heisenberg spin chains. Thus, the excitation of a mixed state of charge and spin density waves transforms into the excitation of exclusively longitudinal 1D-waves of spin density (magnons) as the intensity of the external magnetic field increases in the high-temperature regime of measurement of magnetization $M(T)$. According to the field dependences of magnetization $M(H)$ measured in different regimes at temperatures around $T = 0$, an increasing intensity of the external magnetic field eventually induces a quantum phase transition to the LL state in QSL. Peak features of „supermagnetization" with different degrees of overlapping, which are typical of the excitation of gapless uncoupled charge and spin density weaves in an LL, form as a result of this transition around $H = 0$. We believe that this behavior of magnetization $M(T)$ in a filed with increasing H is attributable to the fact that the transverse component of the external magnetic field closes spinon gap Δ_s in $\mathrm{SmMnO}_{3+\delta}$ and facilitates the transition of a system of zigzag AFM spin chains weakly coupled by the exchange interaction to a state similar to a spin LL. The spatial separation of charge and spin excitations is a specific feature of this state. Many-particle excitations in the form of 1D charge and spin density waves, which follow the Bose–Einstein statistics, generally substitute for elementary excitations in the form of spinon pairs in this quasi-one-dimensional case.

Conflict of interest

The authors declare that they have no conflict of interest.

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