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Non-Heisenberg ferrimagnet with single-ion anisotropy

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We have investigated the effect of single-ion anisotropy of the „easy plane" type on the phase states of a ferrimagnet with $S = 1$ and $\sigma = 1/2$ sublattices and non-Heisenberg (bilinear and biquadratic in spins) exchange interaction for the sublattice with $S = 1$. It is shown that taking into account both the non-Heisenberg exchange interaction and the single-ion anisotropy of the sublattice with $S = 1$ leads to the realization of a phase with vector order parameters (ferrimagnetic phase) and a phase characterized by both vector and tensor order parameters (quadrupole-ferrimagnetic). It is shown that taking into account single-ion anisotropy changes the type of phase transition in comparison with an isotropic non-Heisenberg ferrimagnet. A phase diagram is constructed, and the condition for the compensation of the sublattice spins is determined.

Keywords: ferrimagnet, biquadratic exchange interaction, single-ion anisotropy „easy plane", quadrupole-ferrimagnetic phase, phase transition.

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1. Introduction

In recent years, a new and actively developing field of the physics of magnetism has arisen — the spintronics, in which compensated magnets are actively studied and applied (see [1–3]). This interest is due to the fact that for them the dynamic parameters, such as magnetic resonance frequencies, domain wall velocities, and the number of others, are exchange enhanced. In the work [4] it was shown that the spin current actively influences magnetically ordered systems with zero integral magnetic moment, which makes it possible to use antiferromagnetics in spintronics. This circumstance, in principle, makes it possible to increase the speed of the systems for writing and reading information [5–8], and significantly (up to values of the order of THz) to increase the operating frequency of oscillators pumped by spin current [9–11]. However, despite all the attractiveness of antiferromagnetics, they have a high sensitivity of the magnetic order to the presence of defects that violate the sublattice structure of the crystalline sample, which makes it difficult to use them in nanosystems. On the other hand, for ferrimagnetics such as GdFeCo, amorphous alloys of rare-earth elements with elements of the iron group, standard nanotechnologies can be used, the same as for classical materials of nanomagnetism, iron, nickel or permalloy. It is well known that the effects of exchange amplification of dynamic parameters, similar to those known for antiferromagnetics, occur for ferrimagnetics located in the vicinity of the sublattice compensation point [12]. Thus, it becomes possible to use ferrimagnetics located near the compensation point for various ultrafast spintronic devices. In recent papers, ultrafast (with velocities of the order of km/s) dynamics of domain walls [13,14] and high-frequency dynamics of ferrimagnetic vortices [15,16] have been studied experimentally and theoretically. The scheme of a magnetic nano-generator based on ferrimagnetics pumped by spin current, operating in the THz range [17], is proposed. These circumstances make a detailed study of various aspects of the spin dynamics of ferrimagnetics practically important and topical (see the recent review [18]). In addition, for ferrimagnetics (namely, an alloy of rare-earth and transition metals GdFeCo), an ultrafast (over a time of the around few ps) flip of the sublattice magnetizations under the action of a laser pulse with a duration of less than 100 fs [19,20]. It turned out that this effect is directly related to the presence of two sublattices, and an essential role in the formation of the effect is played by the change in the modules of the magnetic moments of the sublattices due to the exchange interaction, such that their sum remains constant [21,22]. Thus, the purely longitudinal evolution of the magnetic moments of sublattices is essential for describing the effect.

It should be noted that a number of problems in the physics of ferrimagnetics have been relatively poorly studied. In particular, the reorientation effect noted above was observed for a ferrimagnetic containing both weakly anisotropic ions and rare-earth ions with a considerable single-ion anisotropy. The presence of considerable single-ion anisotropy leads to essentially quantum effects that are not described by the standard phenomenological theory [23]. Complete description of such effects requires taking into account the dynamics of tensor variables, which are quantum averages of operators bilinear in spin components, which goes beyond the Landau–Lifshitz equation [24,25]. The effect of quantum spin reduction is characteristic of magnetics with single-ion anisotropy of the „easy plane"type [26–30].

It can be assumed that the spin reduction effect can be proposed to describe the ultrafast longitudinal „switching" of spins [29–30].

It should be emphasized that the effect of quantum spin reduction is realized not only in strongly anisotropic magnetics, but also in the so-called non-Heisenberg magnetics. By the term „non-Heisenberg magnetics" we mean magnetically ordered systems in which higher spin invariants of the form $(\mathbf{S}_1, \mathbf{S}_2)^n$ with values n up to $2S$, where S is the value of the magnetic ion spin, play an important role both in the formation of static and dynamic properties [31–34]. Thus, the Hamiltonian for the isotropic exchange interaction of a magnetic with $S = 1$ contains both the bilinear $(\mathbf{S}_1, \mathbf{S}_2)$ and the biquadratic term $(\mathbf{S}_1, \mathbf{S}_2)^2$ [25,31,32–37]. In recent works on the dynamic and static properties of isotropic and exchange-anisotropic non-Heisenberg ferrimagnetics with sublattices $S = 1$ and $\sigma = 1/2$ and taking into account the biquadratic exchange interaction in the sublattice with spin 1 [38,39], it was shown that in such systems, depending on the ratio of material parameters, it is possible to realize both a ferrimagnetic phase characterized by dipole order parameters and a phase whose state is described by both dipole and tensor order parameters (quadrupole-ferrimagnetic). Moreover, in this „mixed" state, compensation of the magnetic moments of the sublattices is possible, i.e. compensation line exists.

Thus, the question of the properties of non-Heisenberg ferrimagnetic currents, taking into account the influence of single-ion anisotropy of the type „easy plane" is not only of academic interest, but also of great practical importance.

2. Model

As a system under study, we consider the two-sublattice anisotropic magnetic with the spin of the magnetic ion of the first sublattice $S = 1$ and the second — $\sigma = 1/2$, and non-Heisenberg exchange interaction for the sublattice with $S = 1$. In this case, in the first sublattice, both bilinear exchange and biquadratic exchange interactions are taken into account, as well as single-ion anisotropy of the „easy plane"type. The Hamiltonian of such system can be represented in the form:

$$H = -\frac{1}{2}\sum_{n,n'}\left[J^{(2)}(n-n')(\mathbf{S}_n, \mathbf{S}_{n'}) + K(n-n')(\mathbf{S}_n, \mathbf{S}_{n'})^2\right]$$
$$-\frac{1}{2}\sum_{m,m'} J^{(1)}(m-m')(\boldsymbol{\sigma}_m \boldsymbol{\sigma}_{m'})$$
$$-\frac{1}{2}\sum_{n,m} A(n-m)(\boldsymbol{\sigma}_m \mathbf{S}_n) + \frac{\beta}{2}\sum_n (S_n^x)^2, \qquad (1)$$

where $J^{(1)} > 0$ is the exchange interaction constant for a sublattice with spin $\sigma = 1/2$; $J^{(2)} > 0$, $K > 0$ are bilinear and biquadratic exchange interaction constants for $S = 1$; $A < 0$ is the inter-sublattice interaction constant, $\beta > 0$ is the single-ion anisotropy constant of „easy plane" type (basic plane ZOY). Further consideration will be carried out for the case of low temperatures ($T \ll T_{\mathrm{N}}$, T_{N} — Neel temperature).

The change in phase states is associated with a change in the value of material parameters (and their relationship between them) [24,32,36,38–41]. Variation of system parameters can occur, for example, by changing the concentration of magnetic ions, or by applying external mechanical stresses, leading to deformation of the crystal lattice. In the context of this work, it is not important how the material constants change in the model under consideration.

Let us choose the OZ axis as the quantization axis. Then, the average value of the spin for the first sublattice will be parallel to this axis, and the second sublattice — antiparallel to this axis. This orientation for the magnetic moments of the sublattices is due to the fact that the constant of the inter-sublattice exchange interaction is $A < 0$, which determines the antiparallel orientation of the magnetic moments of the sublattices. For the convenience of calculations, we turn the second sublattice so that the directions of the quantization axes of both sublattices coincide. The unitary rotation of $U(\varphi) = \prod_l \exp(i\varphi\sigma_l^x)$ by the angle $\varphi = \pi$ leads to the following transformations of the components of the second spin operator sub-lattice

$$\sigma_m^x \to \sigma_m^x, \quad \sigma_m^y \to -\sigma_m^y, \quad \sigma_m^z \to -\sigma_m^z.$$

It should be noted that such transformations preserve the standard commutation relations for the components of the spin operators.

Further calculations will be carried out using the Stevens operators [42], since the average values of these operators implement the full set of dynamic system variables.

Then, the Hamiltonian of the system under study takes the form

$$H = -\frac{1}{2}\sum_{m,m'} J^{(1)}(m-m')(\sigma_m^x\sigma_{m'}^x + \sigma_m^y\sigma_{m'}^y + \sigma_m^z\sigma_{m'}^z)$$
$$-\frac{1}{2}\sum_{n,n'}\left[J^{(2)}(n-n') - \frac{K(n-n')}{2}\right](S_n^xS_{n'}^x + S_n^yS_{n'}^y + S_n^zS_{n'}^z)$$
$$-\frac{1}{4}\sum_{n,n'} K(n-n')\bigg(\frac{1}{3}O_{2n}^0O_{2n'}^0 + O_{2n}^1O_{2n'}^1 + \tilde{O}_{2n}^1\tilde{O}_{2n'}^1$$
$$+ O_{2n}^2O_{2n'}^2 + \tilde{O}_{2n}^2\tilde{O}_{2n'}^2\bigg) - \frac{1}{2}\sum_{m,n} A(m-n)(\sigma_m^xS_n^x - \sigma_m^yS_n^y$$
$$-\sigma_m^zS_n^z) + \frac{\beta}{12}\sum_n O_{2n}^2 + \frac{\beta}{8}\sum_n (S_n^+S_n^- + S_n^-S_n^+), \qquad (2)$$

where $S^{\pm} = S^x \pm iS^y$; $O_2^0 = 3(S^z)^2 - S(S+1)$; $O_2^1 = \frac{1}{2}\left[S^z, (S^+ + S^-)\right]_+$; $\tilde{O}_2^1 = \frac{1}{2i}\left[S^z, (S^+ - S^-)\right]_+$ $\tilde{O}_2^2 = \frac{1}{2i} \times \left[(S^+)^2 - (S^-)^2\right]$ — Stevens operators.

Separating out in Hamiltonian (2) the average fields associated with both the dipole order parameters $\langle S^z \rangle$ and the quadrupole ones ($q_2^t = \langle O_2^t \rangle$), we obtain the one-node Hamiltonian

$$H_0 = \overline{H}_\sigma \sigma_n^z + \overline{H}_S S_n^z - B_2^0 O_{2n}^0 - B_2^2 O_{2n}^2 + \frac{\beta}{8}(S^+S^- + S^-S^+) + \Delta, \quad (3)$$

where

$$\overline{H}_S = \left(J_0^{(2)} - K_0/2\right)\langle S^z \rangle - \frac{1}{2}A_0\langle \sigma^z \rangle,$$

$$\overline{H}_\sigma = J_0^{(1)}\langle \sigma^z \rangle - \frac{1}{2}A_0\langle S^z \rangle, \quad (4)$$

$$B_2^0 = \frac{K_0}{6}q_2^0, \quad B_2^2 = \frac{K_0}{2}q_2^2 - \frac{\beta}{12},$$

$$\Delta = \frac{1}{2}J_0^{(1)}\langle \sigma^z \rangle^2 + \frac{1}{2}\left(J_0^{(2)} - \frac{K_0}{2}\right)\langle S^z \rangle^2 + \frac{K_0}{4}\left(\frac{(q_2^0)^2}{3} + (q_2^2)^2\right) - \frac{1}{2}A_0\langle S^z \rangle\langle \sigma^z \rangle.$$

Here, $J_0^{(1)}$, $J_0^{(2)}$, K_0 and A_0 are zero Fourier components of the corresponding exchange integrals.

Both single-ion anisotropy and biquadratic exchange interactions can be correctly taken into account using the diagram technique for Hubbard operators [43–46]. These operators are built on the basis of the eigenfunctions of the operators $S^z(|M\rangle)$, $M = -1, 0, 1$ and $\sigma^z(|m\rangle)$, $m = -1/2, 1/2$ for the first $X^{M'M} = |M'\rangle\langle M|$ and the second $Y^{m'm} = |m'\rangle\langle m|$ sublattices, respectively, and describe the transition of a magnetic ion from the M' state to the M state and from the m' state to the m state. The relation between the spin and Stevens operators and the Hubbard operators has the form

$$S^z = X^{11} - X^{-1-1}, \quad O_2^2 = X^{1-1} + X^{-11},$$

$$O_2^0 = X^{11} - 2X^{00} + X^{-1-1},$$

$$\sigma^z = \frac{1}{2}\left(Y^{\frac{1}{2}\frac{1}{2}} - Y^{-\frac{1}{2}-\frac{1}{2}}\right), \quad \sigma^+ = Y^{\frac{1}{2}-\frac{1}{2}}, \quad \sigma^- = (\sigma^+)^+.$$

Then, in terms of the Hubbard operators, the one-node Hamiltonian (3) can be represented in the form

$$H_0 = -\frac{1}{2}\overline{H}_\sigma\left(Y^{\frac{1}{2}\frac{1}{2}} - Y^{-\frac{1}{2}-\frac{1}{2}}\right) - \overline{H}_S(X^{11} - X^{-1-1}) - B_2^2(X^{1-1} + X^{-11}) - B_2^0(X^{11} - 2X^{00} + X^{-1-1}) + \frac{\beta}{4}(X^{11} + 2X^{00} + X^{-1-1}) + \Delta. \quad (5)$$

As you can see, the Hamiltonian (5) is off-diagonal, and to diagonalize it we use the unitary transformation [44]:

$$\tilde{H}_0 = U(\alpha)H_0U^+(\alpha),$$

whose explicit form is: $U(\alpha) = 1 + (\cos\alpha - 1) \times (X^{11} + X^{-1-1}) + \sin\alpha(X^{1-1} - X^{-11})$.

As a result, we obtain the Hamiltonian (5) in the diagonal form

$$\tilde{H}_0 = E_1X^{11} + E_0X^{00} + E_{-1}X^{-1-1} + \varepsilon_{\frac{1}{2}}Y^{\frac{1}{2}\frac{1}{2}} + \varepsilon_{-\frac{1}{2}}Y^{-\frac{1}{2}-\frac{1}{2}}, \quad (6)$$

where

$$E_1 = -B_2^0 - \frac{\beta}{4} - \overline{H}_S\cos 2\alpha - B_2^2\sin 2\alpha + \Delta,$$

$$E_0 = -2B_2^0 + \frac{\beta}{2} + \Delta,$$

$$E_{-1} = -B_2^0 - \frac{\beta}{4} + \overline{H}_S\cos 2\alpha + B_2^2\sin 2\alpha + \Delta,$$

$$\varepsilon_{\frac{1}{2},-\frac{1}{2}} = \mp\overline{H}_\sigma\langle \sigma^z \rangle, \quad (7)$$

energy levels of magnetic ions of the first and second sublattices, respectively.

The wave functions of the sublattices have the form

$$\Psi(1) = \cos\alpha|1\rangle + \sin\alpha|-1\rangle; \quad \Psi(0) = |0\rangle$$

and

$$\Psi(-1) = -\sin\alpha|1\rangle + \cos\alpha|-1\rangle,$$

$$\Phi\left(\frac{1}{2}\right) = \left|\frac{1}{2}\right\rangle \quad \text{and} \quad \left|-\frac{1}{2}\right\rangle. \quad (8)$$

The relationship of spin operators with Hubbard operators constructed on the basis of the eigenfunctions of the Hamiltonian (8) now has the form

$$S_n^z = \cos 2\alpha(X_n^{11} - X_n^{-1-1}) - \sin 2\alpha(X_n^{1-1} + X_n^{-11});$$

$$S_n^+ = \sqrt{2}\left[\sin\alpha(X_n^{01} - X_n^{-10}) + \cos\alpha(X_n^{0-1} + X_n^{10})\right],$$

$$S_n^- = (S_n^+)^+,$$

where, α is the unitary u–v transformation parameter defined by the relation

$$\overline{H}_S\sin 2\alpha = -B_2^2\cos 2\alpha.$$

From the relationship between spin operators and Hubbard operators, we can determine the order parameters of the first sublattice as a function α:

$$\langle S^z \rangle = \cos 2\alpha, \quad q_2^2 = \langle O_2^2 \rangle = \sin 2\alpha, \quad q_2^0 = \langle O_0^2 \rangle = 1.$$

The second sub lattice is described only by the dipole parameter in the order $\langle \sigma^z \rangle$ and plays the role of the „bias“ field.

3. Free energy analysis

Since we consider the system at low temperatures, the free energy density practically coincides with the energy levels of the magnetic ion of the ground state. As follows from relations (7), the lowest levels of the first and second sublattices are the levels E_1 and $\varepsilon_{\frac{1}{2}}$, respectively. Consequently, the free energy density of the considered ferrimagnetic can be represented in the form

$$F = E_1 + \varepsilon_{\frac{1}{2}},$$

Taking into account relations (4) and (7), for the free energy density we obtain

$$F = -\frac{1}{12}K_0 - \frac{1}{4}\beta - \frac{1}{2}J_0^{(1)}\langle\sigma^z\rangle^2 - \frac{1}{2}\left[J_0^{(2)} - K_0\right]\langle S^z\rangle^2 + \frac{1}{2}A_0\langle\sigma^z\rangle\langle S^z\rangle + \frac{1}{12}\beta\sin 2\alpha.$$

Considering that $\langle\sigma^z\rangle = 1/2$, $\langle S^z\rangle = \cos 2\alpha$, and also that the inter-sublattice interaction constant $A < 0$, we get the following expression:

$$F = -\frac{1}{4}\left[\beta + \frac{4}{3}K_0 + \frac{1}{2}J_0^{(1)}\right] - \frac{1}{4}|A_0|\cos 2\alpha + \frac{1}{12}\beta\sin 2\alpha - \frac{1}{2}\left[J_0^{(2)} - K(0)\right]\cos^2 2\alpha. \quad (9)$$

An analysis of the free energy density (9) makes it possible to determine the transformation parameter $\alpha u{-}v$ for various ratios of the material parameters of the system.

In the general case, the equation for the parameter α has the form

$$\frac{|A_0|}{2}\sin 2\alpha + \frac{\beta}{6}\cos 2\alpha + 2(J_0^{(2)} - K_0)\cos 2\alpha\sin 2\alpha = 0. \quad (10)$$

As follows from equation (10), the magnetization of the sublattice with $S = 1$ essentially depends on the ratio of material parameters, and the magnetization of the sublattice with spin $1/2$ remains constant and plays the role of the „bias" field. It should be noted that the condition $\langle\sigma^z\rangle = 1/2$ arises naturally from the connection of the z-th component of the operator σ with the Hubbard operators $Y^{m'm}$ and is exact in our case $T = 0$.

Let us consider in more detail the solutions of equation (10) at various ratios of material parameters and low temperatures.

Thus, if the single-ion anisotropy constant is much smaller than the bilinear and biquadratic exchange interactions, and the bilinear exchange, in turn, exceeds the biquadratic one ($J_0 > K_0 \gg \beta$), then with such a ratio of material parameters, the solution of the equation (10) can be represented in the form

$$\sin 2\alpha = -\frac{\beta/3}{4(J_0^{(2)} - K_0) + |A_0|}.$$

Since we assume that the single-ion anisotropy constant is the smallest parameter of the system, and $J_0 > K_0$, then $\sin 2\alpha \sim 0$, and hence $\cos 2\alpha \sim 1$, i.e., the magnetization of the first sublattice practically reaches its maximum possible value $\langle S^z\rangle \approx 1$, thus the state of the system is close to ferrimagnetic. This means that ferrimagnetic ordering (FiM) occurs in the system with sublattice state vectors

$$|\Psi(1)\rangle = |1\rangle \quad \text{and} \quad \left|\Phi\left(\frac{1}{2}\right)\right\rangle = \left|\frac{1}{2}\right\rangle,$$

and order parameters

$$|\langle\sigma^z\rangle| = \frac{1}{2}, \quad \langle S^z\rangle = \cos 2\alpha \approx 1, \quad q_2^0 = 1, \quad q_2^2 \approx 0.$$

As can be seen, in this state the first and second sublattices are close to saturation, but the magnetization vectors of the sublattices are anti-collinear. It should be noted that the sublattice $S = 1$ reaches saturation asymptotically, i.e., at sufficiently large values of the bilinear exchange interaction constant.

Let us now consider the opposite case, when the predominant parameter of the first sublattice is the biquadratic exchange interaction. In this case, the solution of equation (10) has the form

$$\cos 2\alpha = \frac{|A_0|}{4\left(K_0 - J_0^{(2)}\right) + \beta/3}. \quad (11)$$

Since $\cos 2\alpha$ determines the average magnetic moment (at the node) of the first sublattice, this value must be positive, i.e.,

$$\frac{|A_0|}{4\left(K_0 - J_0^{(2)}\right) + \beta/3} > 0.$$

Also, the $\cos 2\alpha$ function is limited. Thus, at $K(0) > \beta > J(0)$, the state is realized in the system with the magnetization of the first sublattice significantly less than the maximum possible, and the second sublattice retains the saturated value of the magnetization ($|\langle\sigma^z\rangle| = 1/2$). The quadrupole order parameters of the first sublattice in this case have the form

$$q_2^2 = \langle O_2^2\rangle = \sin 2\alpha < 1, \quad q_2^0 = \langle O_2^0\rangle = 1.$$

Thus, the system realizes the phase in which both the vector order parameter of the first sublattice ($\langle S^z\rangle$) and the components of the quadrupole moment tensor (q_2^2) of the first sublattice take intermediate values between zero and one, and the second sublattice plays the role of a constant „bias field". Thus, at large values of the biquadratic exchange interaction constant and a significant single-ion anisotropy, the effect of quantum spin reduction appears in the first sublattice [23,29,34]. We will call such the state as the quadrupole-ferrimagnetic state (QFiM).

The ground state vectors of the sublattices in the QFiM phase have the form

$$|\Psi(1)\rangle = \cos\alpha|1\rangle + \sin\alpha|-1\rangle, \quad \left|\Phi\left(\frac{1}{2}\right)\right\rangle = \left|\frac{1}{2}\right\rangle.$$

The magnetization vectors of the first and second sublattices are anti-collinear, and, consequently, in this phase, taking into account the quantum spin reduction of the first sublattice [23,29,34], sublattice spins can be compensated. From the condition $\langle S^z \rangle = -\langle \sigma^z \rangle$, and taking into account that $\langle \sigma^z \rangle = 1/2$, we obtain

$$\frac{|A_0|}{4\big(K_0 - J_0^{(2)}\big) + \beta/3} = -1/2.$$

The solution to this equation has the form

$$|A(0)| = -2\big(K_0 - J_0^{(2)}\big) - \beta/6. \tag{12}$$

Thus, equation (12) describes a surface in variables (J, K, A, β) on which the total average spin of sublattices is equal to zero ($\langle S^z + \sigma^z \rangle = 0$). It should be emphasized that in this case we are talking about the compensation of the sublattice spins, and not about the compensation of the magnetic moments of the sublattices. The point is that the magnetic moment is related to the spin moment of the sublattices as $\mathbf{M} = -g\mu_B \mathbf{S}$, where Land' g-factor (g-factor). Since in our model the sublattices are not equivalent, it is logical to assume that the g-factors of the sublattices are not equal, and, consequently, the magnetic moments of the sublattices on the compensation plane are also not equal [18]. Thus, although the spin moments compensate each other for the ratios of material parameters that we have determined, the integral magnetic moment in this case may not be equal to zero and reach a sufficiently large value, greater, for example, than for weak ferromagnets (AFM with the Dzyaloshinskii–Moriya interaction). Moreover, this resulting magnetic moment is parallel to the anti-ferromagnetism vector, and the dynamics of a ferrimagnetic at the compensation point can be considered as „antiferromagnetic" [18].

Equation (12) is more convenient to rewrite in the reduced variables $y = |A|/K$, $x = J/K$, $y = \beta/K$. Then

$$y = 2x - 2 - z/6. \tag{13}$$

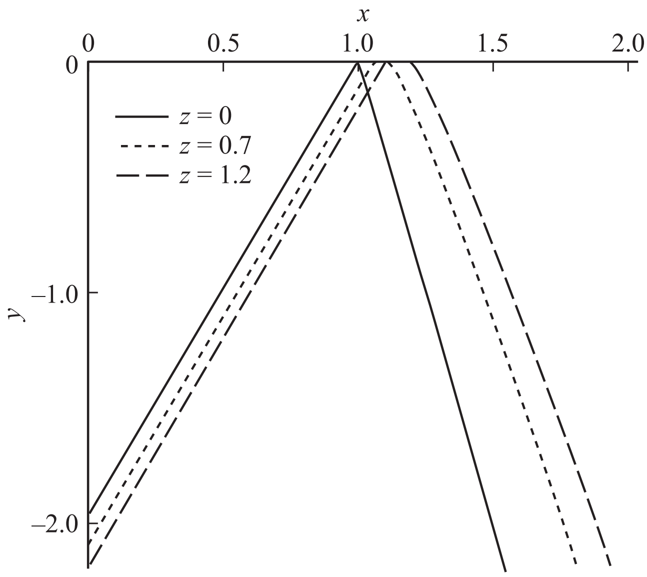
It should be noted that in the absence of inter-sublattice exchange interaction ($A = 0$) $\langle S^z \rangle = \cos 2\alpha = 0$, i.e., parameter $\alpha = \pi/4$. This means that for $A = 0$ the nematic state [49–54] is realized in the first sublattice, whose parameters have the form around

$$\langle S^z \rangle = 0, \quad q_2^2 = \langle O_2^2 \rangle = 1, \quad q_2^0 = \langle O_2^0 \rangle = 1.$$

In this case, the „bias field", i.e., the second sublattice has no effect on the first one.

From the equality of the free energy density in the FiM and QFiM phases, we obtain the phase transition surface between these phases

$$\big[|A_0| - 4\big(K_0 - J_0^{(2)}\big) - \beta/3\big]^2 + \frac{\beta}{3}\big[4\big(K_0 - J_0^{(2)}\big) + \beta/3\big] = 0,$$



Cross section of the phase diagram of an easy-plane non-Heisenberg ferrimagnetic at various values of the single-ion anisotropy constant.

or in reduced variables (x, y, z)

$$[y - 4(1 - x) - z/3]^2 + \frac{z}{3}\big[4(1 - x) + z/3\big] = 0. \tag{14}$$

The results obtained make it possible to construct a phase diagram of the system under study, moreover, it is more convenient to depict it in the reduced variables on the (x, y) plane, for different values z, i.e., for different values of the single-ion anisotropy constant β. Schematically, this diagram is shown in the figure.

It can be seen from this phase diagram and relations (13) and (14) that for $z = 0$ ($\beta = 0$) our results go over exactly to the results of the work [38,39], in which the phase states of isotropic and exchange-anisotropic non-Heisenberg ferrimagnetics are studied. An analysis of the results obtained in this work indicates that single-ion anisotropy significantly increases the region of existence of the QFiM phase and shifts both the phase transition lines and the compensation lines to the region of large values of the bilinear exchange interaction of the sublattice $S = 1$. This result is easy to understand if we pay attention to expression (11), from which it follows that even for $J_0^{(2)} \sim K_0$ the average value of the magnetic moment of the sublattice with $S = 1$ will be less than the nominal value for large values of the single-ion anisotropy constant ($\beta > |A_0|$).

It is also of interest to determine the type of phase transition QFiM-FiM phase. For this, we consider the free energy density (9) in the vicinity of the phase transition QFiM-FiM, i.e., in the vicinity of the line defined by relation (14). Since the second sublattice plays the role of the „bias field", and its magnetization in both phases is the same and constant ($|\langle \sigma^z \rangle| = 1/2$), we will focus our attention on the first sublattice. Since the average magnetic moment of the first sublattice is equal to $\cos 2\alpha$, the parameter α actually determines the order parameter of the system. This statement requires some comment. As noted earlier, when analyzing the FiM phase, the magnetization of the sublattice with $S = 1$ reaches saturation asymptotically. This means that the parameter α is not exactly equal to zero in the FiM phase, but also tends to zero asymptotically at large values of the bilinear exchange interaction, and reaches the exact value $\alpha = 0$ in the isotropic case for $\beta = 0$. Therefore, the minimum free energy density in the anisotropic case is not reached at the point $\alpha = 0$, but is somewhat shifted (due to the smallness α) near the phase transition line. Expanding the free energy density (9) in a series with respect to this parameter in the QFiM phase in the vicinity of the phase transition line ($\alpha \to 0$) we get

$$F = F_0 + A\alpha + \Lambda\alpha^2 + B\alpha^3 + \Theta\alpha^4 \ldots, \qquad (15)$$

where

$$A = \frac{\beta}{6}, \quad \Lambda = 2J_0^{(2)} - 2K_0 + \frac{1}{2}|A_0|, \quad B = -\frac{\beta}{9},$$

$$\Theta = -\frac{1}{6}|A(0)| - \frac{8}{3}J_2(0) + \frac{8}{3}K(0),$$

or in variables (x, y, z)

$$A = \frac{z}{6}K_0, \quad B = -\frac{z}{9}K_0, \quad \Lambda = \frac{K(0)}{2}(4x - 4 + y),$$

$$\Theta = \frac{K(0)}{6}(-y - 16x + 16).$$

The presence of the term linear in α in expression (15) indicates that the $S = 1$ sublattice is unsaturated in the FiM phase. This behavior of the α parameter is associated with the presence of quadrupole means $\langle S^iS^j + S^jS^i \rangle$, which in the case under consideration exists

$$q_2^2 = \left\langle (S^x)^2 - (S^y)^2 \right\rangle = \frac{1}{2}\left\langle (S^+)^2 + (S^-)^2 \right\rangle = \sin 2\alpha.$$

An analysis of the free energy density (15) allows one to interpret the QFiM-FiM phase transition as a first-order phase transition. Since the coefficient $\Theta > 0$ in the QFiM phase, the cubic parabola defined by the equation

$$A + 2\Lambda\alpha + 3B\alpha^2 + 4\Theta\alpha^3 = 0,$$

has two minima, neither of which coincides with $\alpha = 0$.

In the case of an isotropic ferrimagnetic, the free energy density in the vicinity of the phase transition line QFiM-FiM has

$$F = F_0 + \Lambda\alpha^2 + \Theta\alpha^4 + \ldots,$$

and the quantities Λ and Θ have the form given above. Therefore, in the case of an isotropic non-Heisenberg ferrimagnetic, the considered phase transition is a second-order transition.

4. Discussion of results

The studies performed have shown that in a non-Heisenberg ferrimagnetic with a single-ion anisotropy of the „easy plane“ type and $S = 1$ and $\sigma = 1/2$ sublattices, it is possible to realize as a ferrimagnetic state with an integral magnetic moment $\left\langle S^z + \sigma^z \right\rangle = 1/2$, as well as a phase in which both vector order parameters of the first and second sublattices are present ($\langle S^z \rangle$, $\langle \sigma^z \rangle$), as well as the tensor order parameter for the first sublattice, the presence of which is due both to the influence of the biquadratic exchange interaction of the first sublattice and to allowance for easy-plane anisotropy in this sublattice. Accounting for these interactions leads to a quantum reduction in the spin of the first sublattice, but does not affect the value of the magnetic moment of the second sublattice. In this case, the second sublattice plays the role of the „bias“ field, and does not allow for any values of the biquadratic exchange interaction and single-ion anisotropy to transfer the first sublattice to the spin nematic state [31]. We called this phase the quadrupole-ferrimagnetic phase. Since in this phase the average value of the spin of the first sublattice varies depending on the ratio of the material parameters of the sublattice, compensation of the sublattice spins is possible in this state. We have obtained the equation of the compensation line in the space of material parameters, as well as the phase transition line „ferrimagnetic — quadrupole-ferrimagnetic phase“. It should be noted that our results agree with the results [38,39], in which the properties of isotropic and exchange-anisotropic non-Heisenberg ferrimagnetics with sublattices $S = 1$, $\sigma = 1/2$ were investigated. As noted earlier, taking into account the easy-plane single-ion anisotropy in the sublattice with $S = 1$ significantly expands the region of stability of the QFiM- phase compared to isotropic ferrimagnetic, and, most interestingly, makes the transition QFiM-FiM phase transition of the first order. This behavior of the system under consideration requires a detailed study of the system dynamics. Also, the behavior of the spectra of elementary excitations in the vicinity of the compensation line is of great interest. The authors hope to carry out these studies in the near future.

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Conflict of interest

The authors declare that they have no conflict of interest.

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