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Negative Mach Reflection with Multiple Three-Shock Configurations for Shock Wave Diffraction on a Wedge

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Within the framework of the Euler equations, a numerical study of the structure of a self-similar flow for various types of negative Mach reflection during diffraction of a shock wave by a wedge is performed. Along with the known modes of double and triple Mach reflection, a qualitatively new mode of negative Mach reflection with multiple three-shock configurations is observed. Peculiarities of the transition from multiple Mach reflection to regular reflection when changing the wedge angle are noted.

Keywords: shock wave, three-shock configuration, double Mach reflection, negative Mach reflection

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The problem of shock wave diffraction on a wedge in its classic statement (for an ideal perfect gas with a constant adiabatic exponent) is characterized by an amazing variety of self-similar solutions, which determines the permanent interest in it from classics and modern researchers who are focused on gas dynamics and physics of shock waves. Currently topical and unsolved problems and contemporary classification of Mach reflection modes of shock waves are reviewed in [1–3].

In case of negative double Mach reflection in the first triple configuration, the incident and reflected shock waves are on opposite sides of the straight-line path of triple point motion. The negative double Mach reflection for moderate Mach numbers of the incident shock wave is realized in gases with an adiabatic exponent close to unity (in experiments for heavy gases with complex molecular structure). The negative double Mach reflection in the problem of shock wave diffraction of a wedge was experimentally detected in [4,5]. The conditions for realization of various modes of double Mach reflection (positive and negative) were discussed in [6–8]. In [9], the effects of thin return jets formation near the wedge surface and intense circulation currents behind the Mach stem in case of double Mach reflection were investigated. Domains of existence of stationary three-shock configurations with a negative reflection were analytically studied in [10,11].

In this work, we performed a numerical study of the structure of a self-similar flow for different types of the negative Mach reflection in case of shock wave diffraction on a wedge.

To describe planar unsteady flows of ideal perfect gas in a Cartesian coordinate system (x, y), the Euler equations were used

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ e\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ (e+p)u\end{pmatrix}+\frac{\partial}{\partial y}\begin{pmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ (e+p)v\end{pmatrix}=0,$$

where p is the pressure, ρ is the density, u, v are the components of velocity along x, y axes, respectively, e is the total energy of unit volume, which is $e = p/(\gamma - 1) + \rho(u^2 + v^2)/2$ for the perfect gas with a constant adiabatic exponent γ. The problem is reduced to a dimensionless form, pressure and density of gas in the state of rest in front of the shock wave are used as scales.

The computational domain was chosen in such a way that the disturbances do not reach its limits (Fig. 1). X-axis of a rectangular Cartesian computational grid coincides with the surface of the wedge. The Rankine–Hugoniot relations are used to calculate flow parameters u_s, v_s, ρ_s, p_s behind the shock wave by a given Mach number M_s of the incident shock wave. At the initial moment of time, the shock wave passes through the wedge vertex $x_0 = 1/6$ and is inclined at an angle of 90° — α (α is the angle of the wedge). In the calculation points to the left of it, the flow parameters correspond the flow behind the shock wave, and to the right of it they correspond to the state of rest in front of the wave. The flow was calculated in the domain of $0 \leqslant x \leqslant 4$, $0 \leqslant y \leqslant 1$. Since the point of the incident shock wave intersection with the upper limit of the calculation domain at any time is known, the conditions on the upper, left (inlet) and right (outlet) limits can be defined precisely. At the lower limit within the interval of $0 \leqslant x \leqslant x_0$ the conditions of free outflow were defined, while within the interval of $x_0 \leqslant x \leqslant 4$ (wedge surface) impermeability conditions were defined (implemented as symmetry conditions for p, ρ, u and reflection conditions for v). Dimensionless governing parameters of the problem are the Mach number of the incident shock wave, M_s, the adiabatic exponent of the gas, γ, and the wedge angle, α. Correctly calculated solution to the problem should be stationary in self-similar variables $\xi = (x - x_0)/t$, $\eta = y/t$.

The numerical investigation was carried out using the original program where a TVD-modification [12] of the McCormack explicit finite difference scheme is imple-

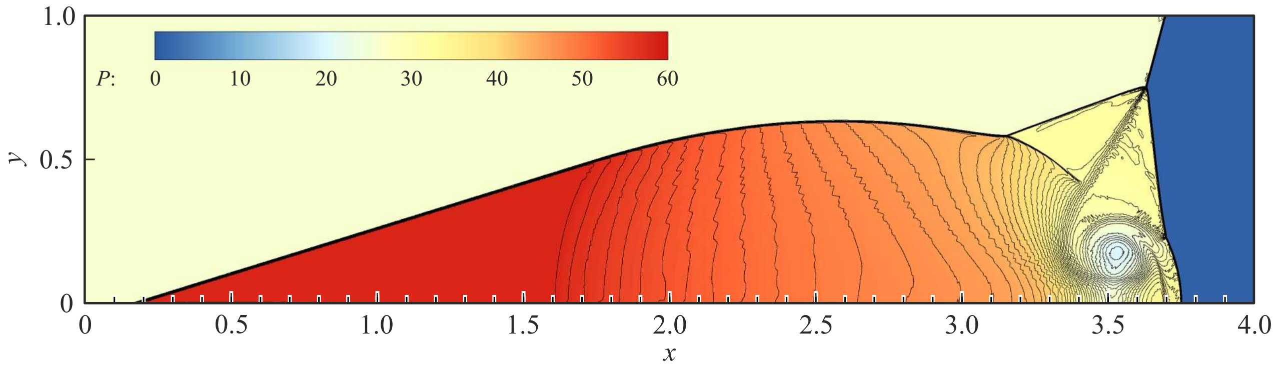

Figure 1. Computational domain and a test calculation for the case negative „triple" Mach–White reflection. Field of pressure is represented with color, densities are represented with contours (colored version of the figure is presented in the electronic version of this paper). $M_s = 4.94$, $\gamma = 1.13$, $\alpha = 15°$.

mented [13], ensuring that monotone behavior of the solution is kept. As shown by the comparative testing using various one-dimensional and two-dimensional problems of modern gas dynamics performed by the authors according to the method of [14], this scheme provides a reliable calculation of non-stationary flows with gas-dynamic discontinuities of various types on fine grids. Standard resolution of the computational grid was 6400×1600, and in specially stipulated cases, a double-resolution grid $12\,800 \times 3200$ was used. In all further calculations some parameters of the problem are fixed: $M_s = 4.94$, $\gamma = 1.13$ (CCl_2F_2 — Freon R12).

For the problem of shock wave diffraction on a wedge verification was carried out in accordance with the method of [15], and also a comparison with experimental and calculated data of [3] was conducted for various Mach reflection modes. Fig. 1 shows the case of negative „triple" Mach–White reflection, which is realized at $\alpha = 15°$, corresponding to the experiment of [3] (see Fig. 1, *h* in the mentioned work). The color represents the pressure distribution, and the contours show the density distribution, which allows visually distinguish between shock waves and tangential discontinuities (color version of the figure is presented in the electronic version of the paper). Main features of the flow are the intense circulating flow under the tangential discontinuity, the powerful return jet along the surface and the presence of additional triple point on the Mach stem (which allows this case to be classified as a „triple" reflection).

In this work, we have identified and investigated the qualitatively new self-similar modes of negative Mach reflection with multiple three-shock configurations, which allows them to be classified as a „multiple" Mach reflection. Fig. 2 shows the flow fields at $\alpha = 35°$ (the resolution of the computational grid is $12\,800 \times 3200$). Vectors represent the „self-similar" field of velocities in accordance with method of [9]. To calculate the „self-similar" velocity components (U, V), the „self-similar" extension vector (which modulus is equal to the distance to wedge vertex divided by time) is subtracted from the physical velocity vector at each calculation point: $U = u - \xi = u - (x - x_0)/t$, $V = v - \eta = v - y/t$. The „self-similar" field of velocities in the vicinity of any typical point of the flow (for example, the triple point) locally coincides with the physical field of velocities in reference system associated with this point. To verify the self-similarity of the solution, a comparison was carried out between the distributions of parameters at different moments of times, which confirmed the scalability of the results. In addition, shadow patterns were built for the field of absolute pressure maxima at each grid point for the entire calculation time (a similar technique is used to visualize the cellular structure of the gaseous detonation), which analysis confirmed the straightness of triple point movement paths.

Fig. 2 shows the distributions of local Mach numbers in the „self-similar" field of velocities and relative temperature (p/ρ), which allow analysis of shock wave configurations and jet flows to be carried out. In the first three-shock configuration *1* the angle of incident shock wave reflection IS is negative, which inevitably leads to the appearance of the second three-shock configuration *2* on the reflected shock $RS1$ with shocks $RS2$, $MS2$ and a tangential discontinuity $T2$. In this case, the flow in an extensive „upper" area over the tangential discontinuity $T1$ is of supersonic behavior. In the „lower " area under the tangential discontinuity $T1$ behind the Mach stem $MS1$ an intensive circulation flow is formed, which leads to the appearance of the third three-shock configuration *3* on the $MS1$. A supersonic return jet Jet can be distinguished that propagates along the surface towards the $MS1$ and is decelerated in the internal shock $IntS$. It has been established that the occurrence of new three-shock configurations over the tangential discontinuity $T1$ is associated with the intensification of the circulation jet flows with an increase in the wedge angle. As a result of the interaction between the internal shock wave $MS2$ and the high-temperature gas jet outflowing from the triangular region limited by the shock $MS1$ and tangential discontinuities $T1$, $T3$, a three-shock configuration is formed *4* that includes oblique shocks $RS4$, $MS4$. This result is in good agreement with the effect of

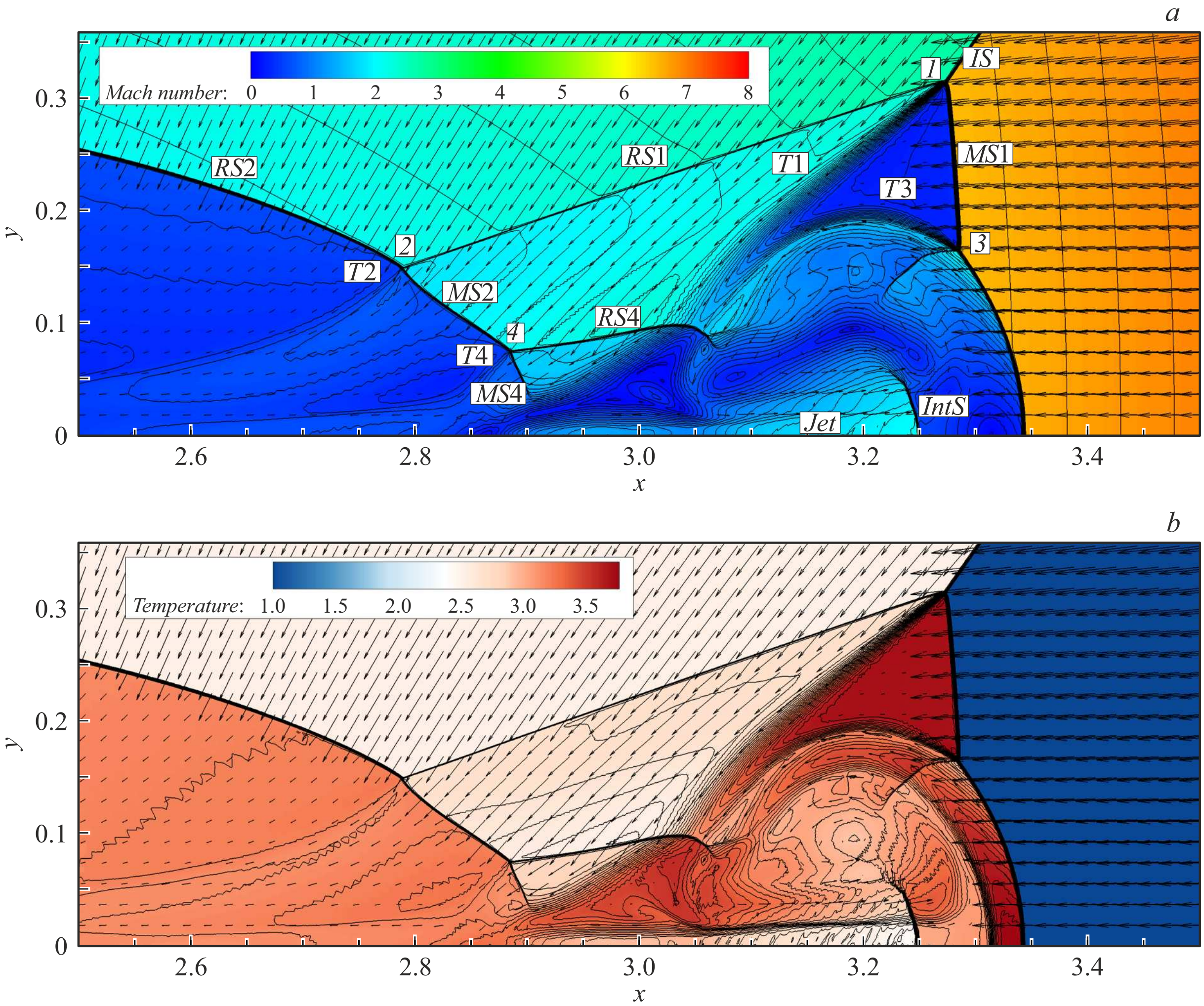

Figure 2. The structure of flow under negative Mach reflection with multiple triple configurations. $M_s = 4.94$, $\gamma = 1.13$, $\alpha = 35^\circ$. *a* is the distribution of Mach numbers in the „self-similar" field of velocities; *b* is the field of temperature. Explanations in the text.

formation of a large-scale gas-dynamic precursor from the interaction of a shock wave with a thin hot-gas layer [16].

Self-similar flow with multiple Mach reflection is arranged in such a way that a significant part of the flow passes into a narrow „throat" near the wedge surface through the shocks $MS2$, $MS4$. In this case, near the wedge surface, an area of jet spreading appears with jets directed along the wedge surface to the wedge vertex and from the vertex towards the main Mach stem $MS1$. Pressure and density in the jet spreading area behind the shock $MS4$ turn out to be very high (in calculations, the pressure is 150 times higher than the initial level). However, the temperature (Fig. 2, *b*) in the jet spreading area remains moderate (3 times higher than the initial temperature), which is less than the temperature immediately behind the Mach stem $MS1$. Thus, the high-temperature effects of a real gas should not have a significant impact on the solution to the problem, and the application of the ideal gas model with constant heat capacities is justified.

Features of the evolution of negative Mach reflection modes are noted at a change in the wedge angle α (Fig. 3). It is shown that with an increase in the wedge angle the modes of double, triple and multiple Mach reflections are realized, and then the transition to regular reflection takes place. It is established that with an increase in the wedge angle the mode of multiple reflection is realized starting from an angle of $\alpha = 25^\circ$ and remains up to the transition to the regular reflection at $\alpha = 41.75^\circ$. In this case, at the final stage the transition occurs abruptly, so that the dependence of inclination angles of the straight-line paths of triple points motion χ on the wedge angle α has vertical tangents at the point of transition (Fig. 3, *a*). In addition, the similarity of the gas-dynamic structures when approaching the critical angle of transition is noted, when at a small change in the wedge angle α the size of the interaction area changes significantly, but the scalability of flow is maintained (Fig. 3, *b*, *c*). At the same time, the behavior of flow at angles close to the critical angle of the transition is qualitatively correspondent to the experimental data (see Fig. 2, *a* in [6] and Fig. 7, *d* in [8]).

The obtained solution with multiple three-shock configurations (the mode of multiple reflection) expands the insight into possible gas-dynamic configurations in the problem

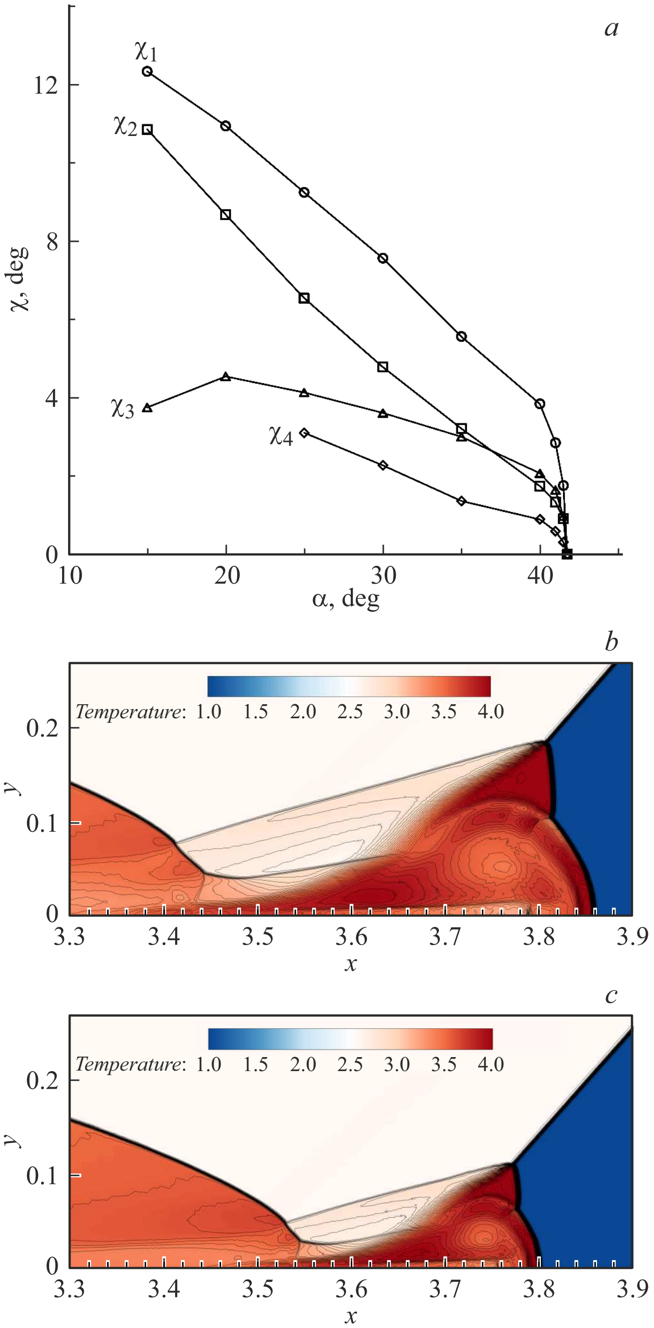

Figure 3. Peculiarities of the transition to regular reflection under a change in the wedge angle α at $M_s = 4.94$, $\gamma = 1.13$. *a* is the inclination angle of the triple points path χ depending on α; *b* and *c* is the field of temperature at $\alpha = 41$ and $41.5°$, respectively.

of shock wave diffraction on a wedge and supplements the modern classification [1–3]. Further investigations are needed to determine the criteria of realization of the identified modes of the negative Mach reflection and to refine the general pattern of transition to the regular reflection.

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Conflict of interest

The authors declare that they have no conflict of interest.

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