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Period of droplet quasi-Bessel beam generated with the round-tip axicon

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We demonstrate the experimental study of the axial intensity distribution of a quasi-Bessel beam with a droplet structure of the central core, formed by an axicon with the round-tip, and the results of the theoretical calculations. We show that the period of droplet quasi-Bessel beam is determined by the shape of the surface rounding and the angle at the top of the axicon lens and depends on the distance to it. The analysis of this dependence makes it possible to restore the shape of the round-tip of the axicon without 3D scanning.

Keywords: bessel beams, axicon, droplet beams.

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In theory, ideal Bessel beams created using axicon can propagate without divergence and carry infinite energy [1]. In practice, intensity distribution of an axicon incident beam is described by Gaussian function [2], which results in formation of a quasi-Bessel beam with a finite length of propagation that depends on the aperture of the generating beam. Quasi-Bessel beams formed in this way can be effectively used to capture and manipulate microscopic objects [3], to implement optical visualization [4], to process materials [5], in light sheet microscopy [6] and THz photonics [7]. The possibilities of their practical use become considerably wider after the quasi-Bessel beam generation using semiconductor lasers and LEDs was demonstrated [8,9].

In recent times, the quasi-Bessel beams are actively studied that have their central core propagating intermittently in the form of „light droplets" [6,10,11]. These beams, known as „droplet beams", are extensively used, in particular, in high-resolution microscopy [11] to obtain contrast images of thick opaque objects without distortions since resolution of the droplet beam is comparable with that of a standard optical system based on Gaussian beam, and „light droplets" maintain their spatial profile and are capable of providing full-featured illuminance inside hollow opaque objects. At the same time, droplet beams can be used in optogenetics as well to study the dynamics of live cell movement [12] and neuronal activity [13].

However, the experimental setups to form droplet quasi-Bessel beam using an expensive spatial light modulator [6,10,11] are complicated in use and require fine setting. Therefor the use of the axicon with round tip (Fig. 1, a) to generate droplet quasi-Bessel beams [14–16] is an attractive alternative since it allows making the experimental setup significantly simpler and cheaper.

The „droplet" structure of the central spot in the quasi-Bessel beam formed by an axicon with round tip is accounted for by the fact that the rounded area is a focusing lens that keeps unchanged the wavevector projection on the symmetry axis when the beam passes through it, while the beam refracted by the conical part of the axicon has smaller wavevector projection depending on the refraction angle [15–17]. The interference results in an oscillation with a half-period of $\Delta z = \lambda/2(1-\cos\gamma) \approx \lambda/\gamma^2$ (at $\gamma \ll 1$) [17], where λ is the wavelength of the beam, γ is the angle of light propagation after refraction on the conical surface of the axicon (Fig. 1, a).

In this work it is shown on the basis of experimental investigation of the axial intensity distribution and results of theoretic calculations that the half-period of intensity pulsing on the axis is not a constant value equal to $\Delta z = \lambda/\gamma^2$, but depends on the distance to the round tip and is determined by the shape of axicon surface. Studying of this dependence and the use of profilometer makes it possible to recover the exact shape of axicon surface without 3D-scanning. Studying of the axicon surface by profilometer allows obtaining the information about the area close to the tip. In turn, the area distant from the tip can be recovered from the results of experimental studying of the oscillation half-period.

To study the period of intensity oscillation, let us consider an overall surface shape that is determined by parabolic and hyperbolic matching [18]:

$$f(r) = \begin{cases} v(r_D + r^2/2r_P), & r \leqslant r_0, \\ v\sqrt{r_H^2 + r^2}, & r > r_0, \end{cases} \qquad (1)$$

where $v = \mathrm{ctg}(\alpha/2)$ is determined by the tip angle of the axicon α; r_P and r_H are radii of curvature of the parabolic and hyperbolic curves multiplied by v; r_D is the distance from the round tip to the tip of ideal axicon,

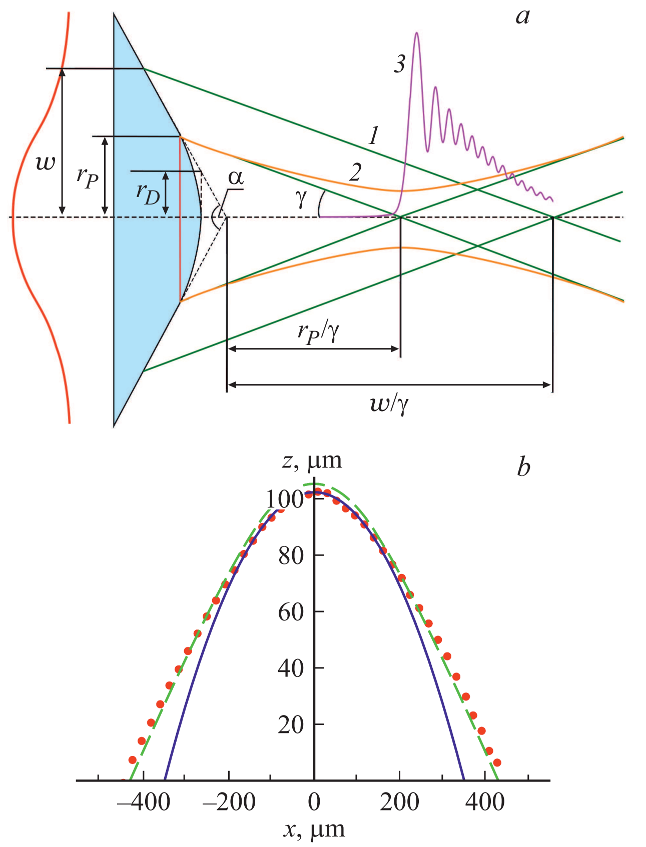

Figure 1. *a* — scheme of the experiment: initial Gaussian beam with a radius of w is incident on an axicon with round tip and an angle of $\alpha = 140^\circ$ (r_D and r_P — parameters of the model that describes the shape of the axicon surface, see the explanation in the text); the beam passed through the conical part of the axicon propagates at an angle of γ relative to the axial coordinate z (line *1*) forming a quasi-Bessel beam near the axis; distances r_P/γ and w/γ determine the beginning and the end of the geometric area of quasi-Bessel beam propagation; the rounded part of the axicon, which is a lens, focuses the beam in the point with coordinate $z = r_P/\gamma$ (curves *2*); the interference of light from the conical and rounded areas forms oscillations of the longitudinal intensity distribution (curve *3*). *b* — profile of the axicon surface measured by the profilometer (points), its approximation by a parabolic curve $\upsilon x^2/2r_P$ (solid line) and profile proposed by the model (1) (dashed line), where $\upsilon = 0.364$, $r_P = 220\,\mu$m and $r_H = 130\,\mu$m.

divided by υ; r_0 is the point of matching. In this case only two parameters of four (r_P, r_H, r_D, r_0) are independent because of the continuity condition imposed on function (1) and on its first derivative in the point of r_0. Thus, the main difference between the model of axicon surface under consideration and well-known models described in literature, i.e. the hyperbolic model [15,17] and the matching of cone with sphere [14] or parabola [16] is the presence of two independent parameters instead of one. In the experiment with measurement of the rounded axicon surface by Sloan DECTAC 3030 profilometer followed by profile processing (Fig. 1, *b*) we obtained the first of the two independent parameters: $r_P = 220\,\mu$m. It should be noted that the profilometer readings possess an inevitable error due to recording of the cross-section out of the plane passing precisely through the center of the axicon. Thus, based on the cross-section shown in Fig. 1, *b* it does not seam possible to determine reliably the second independent parameter. As the second independent parameter explicitly determined from the experiment, we use the distance of r_H. For this purpose we experimentally studied the axial intensity distribution (Fig. 1, *a*) of quasi-Bessel beam formed by an axicon with round tip. The axial distribution was obtained from a set of transversal intensity distributions by taking photos with equal intervals of $10\,\mu$m, which were further processed using ImageJ software [19]. The tip angle of the axicon was $\alpha = 140^\circ$ (Fig. 1, *a*), $w \approx 0.8$ mm — radius of the Gaussian beam at an intensity level of $1/e$ from the intensity at the axis, and refraction index of the axicon is $n \approx 1.5$, which corresponds to $\upsilon = \mathrm{ctg}(\alpha/2) = 0.364$ and a refraction angle of $\gamma = \upsilon(n-1) = 0.182$. The dependence of the number of peak s of the oscillation half-period $\Delta z(s) = z_{s+1} - z_s$, which is determined as a distance between neighboring peaks of the measured intensity distribution, is shown in Fig. 2. In this case the expression to determine half-period for the surface shape under consideration (1) can be written as follows [18]:

$$\Delta z(s) = \frac{1}{2\gamma}\left(\lambda/\gamma + \sqrt{r^{(0)}(s+1)^2 - 4r_H^2} - \sqrt{r^{(0)}(s)^2 - 4r_H^2}\right), \qquad (2)$$

where the following notation is introduced: $r^{(0)}(s) = (s - 1/4)\lambda/\gamma + (r_H^2 + r_P^2)/r_P$. Expression (2) depends on parameters r_P and r_H. In our case parameter r_P is known ($r_P = 220\,\mu$m), and r_H needs to be determined. For the tip angle of axicon $\alpha = 140^\circ$ the paraxial approximation is valid, therefore the formalism under consideration is valid. By approximating the obtained experimental results with formula (2), we determine numerical value of the parameter: $r_H = 130 \pm 14\,\mu$m (Fig. 2). For illustration purposes Fig. 2 also shows the oscillation half-period for two limit cases: hyperbolic matching ($r_H = 220\,\mu$m) and parabolic and straight line matching ($r_H = 0$). Other two parameters of the model (1), as mentioned above, are determined from the continuity condition of the function and the derivative [18]: $r_D = (r_H^2 + r_P^2)/2r_P = 148\,\mu$m and $r_0 = (r_P^2 - r_H^2)^{1/2} = 178\,\mu$m. Physical cause of the arising dependence of the oscillation period on the distance to the axicon tip is in the deviation of the axicon surface from the ideal cone in the areas away from the tip ($r \gg r_P$): $\delta f(r) = [f(r) - \upsilon r] \approx \upsilon r_H^2/2r$. The light refracted on the conical part in the point with radius r is propagated at an angle of γ relative to the axis of symmetry towards the point $z = r/\gamma$ as shown in Fig. 1, *a*. In this case phase factor $\exp[-ik_0(n-1)\delta f(r)]$ arising due to deviation of the axicon surface from the conical shape is also transferred

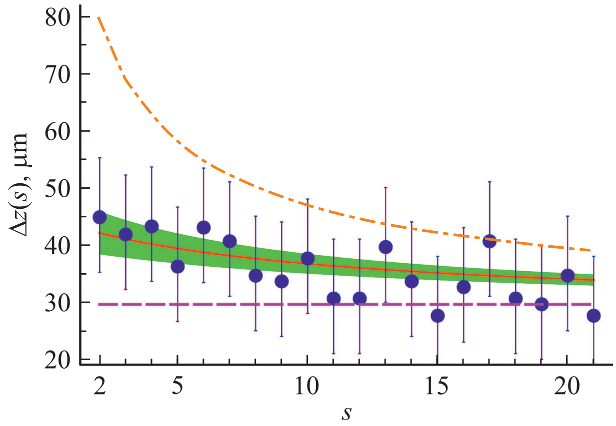

Figure 2. The dependence of oscillation half-period Δz on the number of peak s obtained from the analysis of the experimental data (points). Experimental values were approximated by formula (2), from which the model parameter r_H was determined. The solid curve represents the corresponding half-period for $r_H = 130\,\mu$m, shaded region is the confidence interval for the half-period with a confidence level of 0.95. Also, half-periods for two extreme cases of matching are presented: dash-dotted line — hyperbolic matching ($r_H = 220\,\mu$m), dashed line — parabolic and straight line matching ($r_H = 0$).

from the point with radius r in the axicon plane to the axis of symmetry $z = r/\gamma$. As a result, the amplitude of the field on the axis obtains an additional phase factor, which depends explicitly on the distance r_H:

$$E(z) \propto \exp\left[-ik_0(n-1)\delta f(\gamma z)\right] = \exp\left[-ik_0 r_H^2/2z\right], \quad (3)$$

where k_0 is the wavevector of light in vacuum. It can be seen from formula (3) that the contribution of this term decreases with increase in the axial coordinate z of the number o peak s. As a result, the half-period tends to the constant value λ/γ^2, as can be seen in Fig. 2. In addition, a decrease in the distance r_H leads to suppression of phase factor (3) as well, which makes weaker the dependence of the half-period on the axial coordinate z. It can be seen in Fig. 2 that in accordance with these considerations a hyperbolic approximation of the axicon surface yields the strongest dependence of the „light droplets“ period on the distance. In contrast, the transition to the parabolic surface and cone matching leads to zeroing of phase factor (3) and disappearance of the dependence of oscillation period on the distance. Since the observed effect is determined by the surface properties away from the rounded area of the axicon, these results can be easily extended to the model of cone and sphere matching [14].

Thus, the experimental investigation of the axial intensity distribution of quasi-Bessel beam and results of the theoretical calculation showed that the oscillation period of the axial intensity distribution is not a constant value, but depends on the distance to the round tip. The analysis of this dependence in conjunction with the use of linear scanning data from profilometer makes it possible to recover the shape of conical lens tip rounding without 3D-scanning. In the future the obtained results can be used to optimize the design of bottle beams [20] and to create controlled axial intensity distributions [21]. By varying the oscillation period of axial intensity distribution of the beam the length of „light droplets“will change as well, which can provide new applications in the field of high-resolution microscopy [11], where cavities of various volumes needs to be additionally illuminated inside hollow opaque objects. Also, this feature can be effectively used for laser cutting of glass with various thickness [22].

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] J. Durnin, J. Opt. Soc. Am. A, **4**, 651 (1987). DOI: 10.1364/JOSAA.4.000651
[2] D. McGloin, K. Dholakia, Contemp. Phys., **46**, 15 (2005). DOI: 10.1080/0010751042000275259
[3] B. Shao, S.C. Esener, J.M. Nascimento, E.L. Botvinick, M.W. Berns, Appl. Opt., **45**, 6421 (2006). DOI: 10.1364/AO.45.006421
[4] B. Xiong, X. Han, J. Wu, H. Xie, Q. Dai, Opt. Express, **28**, 9464 (2020). DOI: 10.1364/OE.388808
[5] R. Stoian, M.K. Bhuyan, A. Rudenko, J.-P. Colombier, G. Cheng, Adv. Phys. X, **4**, 1659180 (2019). DOI: 10.1080/23746149.2019.1659180
[6] G. Di Domenico, G. Ruocco, C. Colosi, E. DelRe, G. Antonacci, Sci. Rep., **8**, 17178 (2018). DOI: 10.1038/s41598-018-35006-1
[7] Z. Wu, X. Wang, W. Sun, S. Feng, P. Han, J. Ye, Y. Zhang, Sci. Rep., **7**, 13929 (2017). DOI: 10.1038/s41598-017-12524-y
[8] G.S. Sokolovskii, V.V. Dudelev, S.N. Losev, S.A. Zolotovskaya, A.G. Deryagin, V.I. Kuchnskii, E.U. Rafailov, W. Sibbett, Tech. Phys. Lett., **34** (12), 1075 (2008). DOI: 10.1134/S1063785008120262
[9] G.S. Sokolovskii, V.V. Dudelev, S.N. Losev, A.G. Deryagin, D.A. Vinokurov, A.V. Lyutetskiĭ, N.A. Pikhtin, S.O. Slipchenko, I.S. Tarasov, S.A. Zolotovskaya, E.U. Rafailov, V.I. Kuchinskii, W. Sibbett, Tech. Phys. Lett., **36** (1), 9 (2010). DOI: 10.1134/S1063785010010049.
[10] L. Li, W.M. Lee, X. Xie, W. Krolikowski, A.V. Rode, J. Zhou, Opt. Lett., **39**, 2278 (2014). DOI: 10.1364/OL.39.002278
[11] G. Antonacci, G.Di Domenico, S. Silvestri, E. DelRe, G. Ruocco, Sci. Rep., **7**, 17 (2017). DOI: 10.1038/s41598-017-00042-w
[12] T.A. Planchon, L. Gao, D.E. Milkie, M.W. Davidson, J.A. Galbraith, C.G. Galbraith, E. Betzig, Nature Meth., **8**, 417 (2011). DOI: 10.1038/nmeth.1586
[13] R. Prevedel, Y.-G. Yoon, M. Hoffmann, N. Pak, G. Wetzstein, S. Kato, T. Schrödel, R. Raskar, M. Zimmer, E.S. Boyden, A. Vaziri, Nature Meth., **11**, 727 (2014). DOI: 10.1038/nmeth.2964
[14] G.S. Sokolovskii, V.V. Dyudelev, S.N. Losev, M. Butkus, K.K. Soboleva, A.I. Sobolev, A.G. Deryagin, V.I. Kuchinskii, V. Sibbett, E.U. Rafailov, Quant. Electron., **43** (5), 423 (2013). DOI: 10.1070/QE2013v043n05ABEH015170.

[15] D.V. Chistyakov, S.N. Losev, S.H. Abdulrazak, V.Y. Myl'nikov, E.A. Kognovitskaya, Y.M. Zadiranov, N.G. Deryagin, V.V. Dudelev, V.I. Kuchinskii, G.S. Sokolovskii, Opt. Spectrosc. **127** (5), 848 (2019). DOI: 10.1134/S0030400X19110079.
[16] S.N. Losev, S.K. Abdulrazak, D.V. Chistyakov, V.Y. Myl/nikov, E.A. Kognovitskaya, I.V. Berkutov, Y.M. Zadiranov, N.G. Deryagin, V.V. Dudelev, V.I. Kuchinskii, G.S. Sokolovskii, Tech. Phys. Lett. **44** (10), 887 (2018). DOI: 10.1134/S1063785018100097.
[17] O. Brzobohatý, T. Čižmár, P. Zemánek, Opt. Express, **16**, 12688 (2008). DOI: 10.1364/OE.16.012688
[18] V.Yu. Mylnikov, G.S. Sokolovskii, Optik, **268**, 169797 (2022). DOI: 10.1016/j.ijleo.2022.169797
[19] *ImageJ* [Electronic source]. URL: https://imagej.nih.gov/ij/
[20] T. Du, T. Wang, F. Wu, Opt. Commun., **317**, 24 (2014). DOI: 10.1016/j.optcom.2013.11.059
[21] N. Stsepuro, P. Nosov, M. Galkin, G. Krasin, M. Kovalev, S. Kudryashov, Appl. Sci., **10**, 7911 (2020). DOI: 10.3390/app10217911
[22] J. Dudutis, P. GeČys, G. RaČiukaitis, Opt. Express, **24**, 28433 (2016). DOI: 10.1364/OE.24.028433