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Electrokinetic repeater of acoustic vibrations

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It is shown that in an electrokinetic acoustoelectric transducer, when a constant DC electric field (pump voltage) is applied during reception of an acoustic signal, it becomes possible to retransmit the initial external acoustic field. This is due to the presence of simultaneously reverse electrokinetic phenomena in the transducer: electroosmosis and flow potential. The necessary theoretical substantiation of this phenomenon is presented. The data of a full−scale experiment confirming the theory are presented.

Keywords: acoustoelectric conversion, electrokinetic phenomena, current potential, energy pumping, electrokinetic repeater.

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The authors have earlier considered in [1,2] and other papers the acoustoelectric conversion based on such an electrokinetic phenomenon as electroosmosis. In [3] there was suggested an acoustoelectric conversion based on the electrokinetic phenomenon reverse to electroosmosis, i. e. flow potential that is sometimes referred to as current potential [4]. Analysis of studies [1–3] showed that in both the cases of direct and reverse conversion almost identical mathematical and physical models are applicable, which is caused by the conversion reversibility and, hence, by the reversibility of electrokinetic transducers.

This paper considers one more effect associated with electrokinetic transducers, namely, the possibility of realizing in them the retransmission mode consisting in that the acoustoelectric transducer can, in its turn, retransmit the acoustic field with amplification. Below are given the necessary theoretical and experimental results.

Consider the repeater in the case when a porous structure is simulated by a capillary filled with liquid fluid. The possibility of the retransmission mode was revealed in analyzing the motion equation as applied to the acoustoelectric conversion when constant DC electric field $\mathbf{E}_0$ and external acoustic pressure field p_a were simultaneously applied to the ends of the capillary filled with fluid. Here the process is analyzed by using the Navier-Stokes equation in the following form [1,2]:

$$\rho_\Sigma(\partial \mathbf{v}_\Sigma/\partial t + (\mathbf{v}_\Sigma \cdot \nabla)\mathbf{v}_\Sigma) = -\nabla p_\Sigma + \eta\Delta\mathbf{v}_\Sigma + (\xi + \eta/3)\nabla\nabla\cdot\mathbf{v}_\Sigma + \rho_{el}\mathbf{E}_0 + \mathbf{F}. \tag{1}$$

Here $\rho_\Sigma = \rho_0 + \rho$, $\mathbf{v}_\Sigma = \mathbf{v}_0 + \mathbf{v}$, $p_\Sigma = p_0 + p$ are the fields of the fluid density, velocity and pressure, respectively, $\mathbf{E}_0 = \mathrm{const}$ is the DC electric field vector directed along the capillary axis,

$$\mathbf{F} = (\rho_e \varepsilon\varepsilon_0 \tilde{\xi}/\eta\sigma)\nabla p(\mathbf{x}, t) \tag{2}$$

is the external bulk force that is the source of the process of flow potential [3], η and ξ are the dynamic and bulk viscosities, respectively, ρ_e is the electric charge density induced by the presence of a double layer in electrokinetic processes, ε is the dielectric permeability, ε_0 is the electric constant, $\tilde{\xi}$ is the electrokinetic potential (zeta-potential), σ is the fluid conductivity. Index 0 corresponds to the electroosmosis process generated by the DC electric field $\mathbf{E}_0$, parameters without indices correspond to the remaining processes whose sources are the external acoustic field, flow potential, and also pump-induced fields.

The acoustic process in the capillary is described in terms of compressible liquid in the linearized form

$$\rho_0\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}_0 \cdot \nabla)\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v}_0\right) = -\nabla p + \eta\Delta\mathbf{v} + \left(\xi + \frac{\eta}{3}\right)\nabla\nabla\cdot\mathbf{v} + \mathbf{F} \tag{3}$$

jointly with the continuity equation for compressible liquid $\frac{\partial \rho}{\partial t} + \rho_0\nabla\cdot\mathbf{v} = 0$.

Equation (3) is linear with respect to acoustic fields $\mathbf{v}$ and p which, after expressing them as the following sums

$$\mathbf{v} = \mathbf{v}' + \mathbf{v}_a, \quad p = p' + p_a \tag{4}$$

obey the following equations [3]:

$$\rho_0\partial\mathbf{v}_a/\partial t = -\nabla p_a + \eta\Delta\mathbf{v}_a + (\xi + \eta/3)\nabla\nabla\cdot\mathbf{v}_a + \mathbf{F}_a, \tag{5}$$

$$\rho_0\big(\partial\mathbf{v}'/\partial t + (\mathbf{v}_0 \cdot \nabla)\mathbf{v}' + (\mathbf{v}_0 \cdot \nabla)\mathbf{v}_a + (\mathbf{v}' \cdot \nabla)\mathbf{v}_0 + (\mathbf{v}_a \cdot \nabla)\mathbf{v}_0\big) = -\nabla p' + \eta\Delta\mathbf{v}' + (\xi + \eta/3)\nabla\nabla\cdot\mathbf{v}' + \mathbf{F}'. \tag{6}$$

Here, according to (2) and the second equation of (4),

$$\mathbf{F} = (\rho_e\varepsilon\varepsilon_0\tilde{\xi}/\eta\sigma)\nabla\big(p_a(\mathbf{x}, t) + p'(\mathbf{x}, t)\big) = \mathbf{F}_a + \mathbf{F}'. \tag{7}$$

We interpret $\mathbf{F}$ included in expression (7) as Coulomb force $\mathbf{F} = \rho_e \mathbf{E}$, where $\mathbf{E} = \mathbf{E}_a + \mathbf{E}'$ are the vectors of certain AC electric fields (see [3]),

$$\mathbf{E}_a = (\varepsilon\varepsilon_0\tilde{\xi}/\eta\sigma)\nabla p_a, \quad \mathbf{E}' = (\varepsilon\varepsilon_0\tilde{\xi}/\eta\sigma)\nabla p'. \qquad (8)$$

Taking into account (7) and (8), it is possible to rewrite equations (5) and (6) in the equivalent form:

$$\rho_0 \frac{\partial \mathbf{v}_a}{\partial t} = -\nabla p_a + \eta\nabla\mathbf{v}_a + (\xi + \eta/3)\nabla\nabla\cdot\mathbf{v}_a + \rho_e\mathbf{E}_a, \qquad (9)$$

$$\rho_0\big(\partial\mathbf{v}'/\partial t + (\mathbf{v}_0\cdot\nabla)\mathbf{v}' + (\mathbf{v}'\cdot\nabla)\mathbf{v}_0\big) = -\nabla p' + \eta\Delta\mathbf{v}'$$

$$+ (\xi + \eta/3)\nabla\nabla\cdot\mathbf{v}' - \rho_0\big[(\mathbf{v}_0\cdot\nabla)\mathbf{v}_a + (\mathbf{v}_a\cdot\nabla)\mathbf{v}_0\big] + \rho_e\mathbf{E}'. \qquad (10)$$

Field $(\mathbf{v}_a, p_a)$ is the acoustic field induced in the capillary by the external pump−free acoustic field, while the bulk source in the right−hand part of (9) is created by pressure p_a of the external acoustic field. Field $(\mathbf{v}', p')$ in (10) is the acoustic field generated inside the capillary by applying stationary field $\mathbf{E}_0$ (the presence in the (10) right−hand part of the bulk source $-\rho_0\big[(\mathbf{v}_0\cdot\nabla)\mathbf{v}_a + (\mathbf{v}_a\cdot\nabla)\mathbf{v}_0\big]$ caused by the nonlinearity of the problem with the elecroosmotic flow vector $\mathbf{v}_0$), i.e. by pumping. To consider the electrokinetic repeater, let us analyze equation (10).

Assume that the capillary axis is oriented along the Oz axis. Paper [3] has shown that under the assumption of a thin double layer $\kappa a \gg 1$ (a is the capillary radius, $\kappa = 1/\lambda_D$, λ_D is the double layer Debye length or thickness) the electroosmotic velocity is defined as $\mathbf{v}_0 = (0, 0, U_{eo})$, $U_{eo} = E_0\frac{\varepsilon\varepsilon_0}{\eta}\tilde{\xi} = \text{const}$. Thus, $\mathbf{v}_0 = (0, 0, U_{eo})$. As per [5], the following equations are valid in the Cartesian and cylindrical frames of reference at $\mathbf{v}_0 = \text{const}$:

$$(\mathbf{v}_0\cdot\nabla)\mathbf{v}' = U_{eo}\partial\mathbf{v}'/\partial z = E_0(\varepsilon\varepsilon_0/\eta)\tilde{\xi}\partial\mathbf{v}'/\partial z,$$

$$(\mathbf{v}_0\cdot\nabla)\mathbf{v}_a = U_{eo}\partial\mathbf{v}_a/\partial z = E_0(\varepsilon\varepsilon_0\tilde{\xi}/\eta)\partial\mathbf{v}_a/\partial z. \qquad (11)$$

Rewrite (10) taking into account equations (8), (11) and evident equation $\nabla\mathbf{v}_0 \equiv 0$

$$\rho_0(\partial\mathbf{v}'/\partial t) = -\nabla p' + \eta\Delta\mathbf{v}' + (\xi + \eta/3)\nabla\nabla\cdot\mathbf{v}'$$

$$-\rho_0 U_{eo}\partial(\mathbf{v}_a + \mathbf{v}')/\partial z + \rho_e(\varepsilon\varepsilon_0\tilde{\xi}/\eta\sigma)\nabla p'.$$

Assuming that the process is potential, $\mathbf{v}' = \nabla\Phi'$, $\mathbf{v}_a = \nabla\Phi_a$ as in [2], let us reduce the last equation to the scalar form:

$$\rho_0\partial\Phi'/\partial t = -p' + (\xi + 4\eta/3)\Delta\Phi' - \rho_0 U_{eo}\partial(\Phi_a + \Phi')/\partial z$$

$$+\rho_e(\varepsilon\varepsilon_0\tilde{\xi}/\eta\sigma)p'. \qquad (12)$$

From the continuity equation and condition of the fluid barotropy, obtain via the scalar potential $\partial p'/\partial t = -\rho_0 c^2\Delta\Phi'$. For the harmonic case with temporal factor $e^{-i\omega t}$, let us express the pressure amplitude p' through the potential amplitude Φ' as $p' = (\rho_0 c^2/i\omega)\Delta\Phi'$ and transform (12) to

$$-\rho_0 i\omega\Phi' = -(\rho_0 c^2/i\omega)\Delta\Phi' + (\xi + 4\eta/3)\Delta\Phi'$$

$$-\rho_0 U_{eo}\partial(\Phi_a + \Phi')/\partial z + (\rho_e\varepsilon\varepsilon_0\tilde{\xi}\rho_0 c^2)/(\eta\sigma i\omega)\Delta\Phi'.$$

After the transformations, this equation takes the form of the nonuniform Helmholtz's equation:

$$\Delta\Phi' + k^2\Phi' = U_{eo}\big(k^2/i\omega\big)\partial(\Phi_a + \Phi')/\partial z, \qquad (13)$$

where k is the relevant wave number defined by the following relation:

$$k = k_0/\Big(1 - \rho_e(\varepsilon\varepsilon_0\tilde{\xi})/\eta\sigma - i\omega/(\rho_0 c^2)(\xi + 4\eta/3)\Big)^{1/2},$$

where $k_0 = \omega/c$ is the wave number for a frictionless homogeneous fluid. Rewrite equation (13) as

$$\Delta\Phi' + k^2\Phi' = U_{eo}(k^2/i\omega)(\upsilon_{za} + \upsilon'_z),$$

where υ_{za} and υ'_z are the z-components of velocities $\mathbf{v}_a$ and $\mathbf{v}'$, respectively. Rewrite the last relation taking into account electroosmotic velocity U_{eo}:

$$\Delta\Phi' + k^2\Phi' = E_0\big(\varepsilon\varepsilon_0\tilde{\xi}k^2/(\eta i\omega)\big)(\upsilon_{za} + \upsilon'_z). \qquad (14)$$

Relation (14) shows that potential Φ' of the repeated flow velocity $\mathbf{v}'$ is directly proportional to amplitude E_0 of the pump electric field $\mathbf{E}_0$ and amplitude z- of the velocity component $\mathbf{v}_a$ induced by the external acoustic field $(p_a, \mathbf{v}_a)$. In the absence of pump field $E_0 = 0$, the retransmission mode is absent: $p' = 0$, $\mathbf{v}' = 0$. In addition, relation (14) demonstrates the influence of other parameters of the process, for instance, inverse proportionality of the repeating effect to angular frequency ω. Other peculiar features of the equation (14) solution behavior are briefly described in [2].

To verify the presented theory, a number of experiments were conducted. As a source of sound, acoustic columns emitting harmonic acoustic oscillations 1 kHz in frequency were used. The following experiments were performed sequentially:

(i) In the first experiment, a laboratory noise−level meter was installed in the wave zone at a certain distance from the columns. The columns and noise−level meter were separated by free space. After switching−on the acoustic field, the noise meter fixed the sound level of 61 dB.

(ii) At the same „noise meter−loud speaker“ geometry, a receiving matrix (electrokinetic acoustoelectric transducer [3]) was installed between the noise meter and acoustic columns in line with them. In this case, no pumping was supplied to the matrix. The noise meter fixed the acoustic pressure of 56.5 dB.

(iii) At this stage, the retransmission mode was tested. The experimental conditions were almost the same as in (ii); the only difference was that the receiving matrix was supplied with pump voltage of 1320 V. When the receiving matrix (i.e., the repeater) was pumped, the noise meter fixed the acoustic pressure of 59.5 dB.

Hence, the noise meter signal increased by 3 dB in the retransmission mode and in the presence of pumping.

This paper shows that the existence in the electrokinetic transducer of two reverse electrokinetic phenomena (electroosmosis and flow potential) provides the opportunity for obtaining the effect of the initial acoustic signal repetition. This becomes possible due to the fact that the pumping–induced nonlinearity of the process hydrodynamic model leads to arising in the initial acoustoelectric receiver of the electroacoustic conversion in addition to the acoustoelectric process; this is just what gives rise to the repeating effect. To find out how to better utilize the repeating effect, additional experimental investigations are needed.

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Conflict of interests

The authors declare that they have no conflict of interests.

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