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Double negative media based on antiferromagnetic metamaterials

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The paper presents the theoretical study results of the dispersion characteristics of electromagnetic waves existing in antiferromagnetic (AFM) metamaterial. The AFM metamaterial consists of a transversely magnetized antiferromagnet, inside of which a two-dimensional periodic structure of thin conducting wires surrounded by insulators is placed. It has been established that the AFM metamaterial has two frequency ranges, in which there are backward waves, and the material pa-rameters of the medium are twice negative. The indicated areas are located in the terahertz range.

Keywords: metamaterials, left-handed medium, antiferromagnetics, spin waves.

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It is known that metamaterials are man-made media with properties not seen in conventional natural media [1]. A special class among metamaterials includes twice negative media (or „left-handed" media), where dielectric ε and magnetic μ permeabilities are negative values simultaneously [2]. This causes a negative refraction index of the medium and propagation of a backward electromagnetic wave (EMW) in it.

Beginning from the 2000s, both in this country [3,4] and abroad [5–9] the concept of twice negative media, tunable by the magnetic field, began developing. Ferromagnetic materials were used for their creation. These were either films [4,6] and plates [7] of yttrium-iron garnet, or ferrite rods [8] or films of Ba*M* ferrites and hexaferrites [9] operating in the microwave range. It is known that [10] ferromagnetics (FM) are μ-negative media where magnetic permeability assumes negative values in a certain frequency interval located in the microwave range. Combination of their properties with properties of ε-negative media, represented by periodic lattices of thin wires, made it possible to implement twice negative media without using additional subwave elements in the form of ring open resonators.

In addition to the creation of twice negative media based on FMs operating in the microwave range, attempts were made at creating similar media and functional devices for the terahertz range [1,11–13]. Thus, [14] describes numerical simulation of the refraction index for a metamaterial based on LuBiIG ferrite films and silver strips; the material had negative values at frequencies above 100 GHz. Retuning of the frequency band of the given metamaterial by an external constant magnetic field has been demonstrated. Another magnetic material operating in the terahertz region is the antiferromagnetic (AFM) [15–17]. As distinct from FM, the AFM crystalline lattice consists of two magnetic sublattices. Due to this, AFM has four characteristic frequencies at which magnetic permeability changes its sign. Thus, this paper is aimed at creating an AFM-based twice negative metamaterial with two frequency bands of backward EMW, located in the terahertz range.

The analyzed structure is a limitless AFM where a two-dimensional (2D) periodic structure of thin wires with period a is located. It is assumed that the structure period is considerably less than the EMW length λ, i.e. $a \ll \lambda$. The case of transverse magnetization is studied, when an external constant magnetic field $\mathbf{H}_0$ is directed along the wires, while the wave vector $\mathbf{k}$ is perpendicular to the magnetic field $\mathbf{H}_0$ ($\mathbf{k} \perp \mathbf{H}_0$). With such magnetization, linearly polarized EMWs exist in the magnetic material [10]. In this case the EMW electric field is directed along the wires, while the EMW magnetic field is orthogonal to the electric field and the wave vector $\mathbf{k}$. This structure is schematically shown in Fig. 1, *a*.

[5] has shown that in order to make a twice negative medium of a magnetic material requires, the conducting

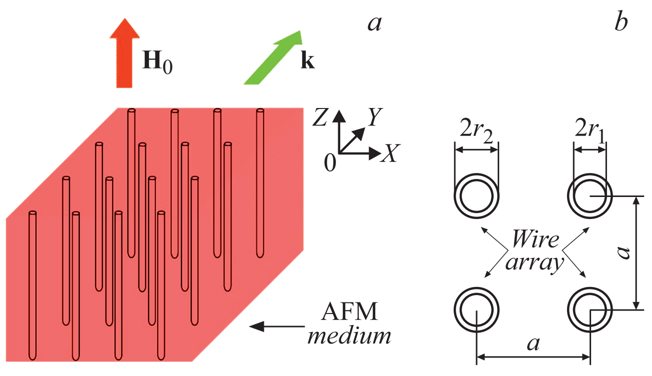

Figure 1. Schematic images of a transversely magnetized limitless AFM-metamaterial (*a*) and 2D-periodic structure (top view) of thin wires with radius r_1, surrounded by a non-magnetic insulator with radius r_2 (*b*).

wires located inside the magnetic matrix must be surrounded with non-magnetic dielectric. The chosen wire radius r_1 was slightly less than the structure period a, while the outer radius of insulating shell r_2 was chosen on the conditions that $r_2 \cong (r_1 a)^{1/2}$ and $r_1 \ll r_2 \ll a$ (Fig. 1, *b*).

It is known [10] that the tensor of high-frequency magnetic permeability $\overleftrightarrow{\mu}$ for a magnetic material, magnetized along the 0Z axis ($\mathbf{H}_0 \parallel 0Z$), is set as

$$\overleftrightarrow{\mu} = \begin{pmatrix} \mu & j\mu_a & 0 \\ -j\mu_a & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where the diagonal μ and off-diagonal μ_a components of the tensor for AFM with an „easy" axis of anisotropy that coincides with the 0Z axis, will be written as follows [17]:

$$\mu = 1 + 8\pi\gamma_s^2 M_s H_A(\omega_+\omega_- - \omega^2)/\left[(\omega_+^2 - \omega^2)(\omega_-^2 - \omega^2)\right],$$

$$\mu_a = 8\pi\gamma_s^2 M_s H_A \omega(\omega_- - \omega_+)/\left[(\omega_+^2 - \omega^2)(\omega_-^2 - \omega^2)\right], \tag{2}$$

where γ_s is the averaged g-factor, M_s is averaged static magnetization of sublattices, H_A is anisotropy field, $\omega_+ = \gamma_s(H_C + H_0)$, $\omega_- = \gamma_s(H_C - H_0)$ are frequencies of antiferromagnetic resonance, $H_C = [H_A(2H_E + H_A)]^{1/2}$ is sublattice „turnover" field, H_E is field of homogeneous exchange interaction between sublattices.

When solving the electrodynamic problem in the approximation of uniform plane waves for a transversely magnetized limitless AFM-metamaterial, we obtain the following dispersion equation

$$k = k_0(\mu_{eff\perp}\varepsilon_{eff\perp})^{1/2}, \tag{3}$$

where k is the wave number of EMW in the medium, $k_0 = \omega/c$ is wave number of EMW in vacuum, $\omega = 2\pi f$ is circular frequency, f is linear frequency, $\mu_{eff\perp}$ is effective magnetic permeability of a transversely magnetized magnetic material that is determined on the basis of the following expression [10]:

$$\mu_{eff\perp} = (\mu^2 - \mu_a^2)/\mu, \tag{4}$$

$\varepsilon_{eff\perp}$ is effective permittivity of a transversely magnetized magnetic material that had the following form in [5]:

$$\varepsilon_{eff\perp} = \varepsilon_r\left[1 - \omega_{p\perp}^2/(\omega^2 + i\alpha_\perp)\right], \tag{5}$$

$$\omega_{p\perp}^2 \cong 2\pi/\langle\varepsilon_f a^2\mu_0\{\ln(r_2/r_1) + \mu_{eff\perp}[\ln(a/r_2) - (3 + \ln 2 - \pi/2)/2]\}\rangle, \tag{6}$$

$\alpha_\perp = \varepsilon_f\omega\omega_{p\perp}^2/\sigma_{eff}$, $\varepsilon_f = \varepsilon_0\varepsilon_r$ is absolute dielectric constant of the magnetic material, $\varepsilon_0 = 1/(\mu_0 c^2)$ is electric constant, μ_0 is magnetic constant, ε_r is relative dielectric constant of the magnetic material, $\sigma_{eff} = \pi r_1^2\sigma/a^2$ is effective conductivity of the wire structure, σ is electrical conductivity of the wire. The expression (5) was obtained on the assumption of uniform density of current passing through the wire. This assumption is fulfilled when wire radius is considerably less than skin-layer depth δ, i.e. $r_1 \ll \delta = (2/\mu_0\sigma\omega)^{1/2}$ [5].

The conditions, at which $\mu_{eff\perp} < 0$, are written as follows for the AFM

$$\omega_{\perp 1} < \omega < \omega_{ar1}, \tag{7a}$$

$$\omega_{\perp 2} < \omega < \omega_{ar2}, \tag{7b}$$

where $\omega_{\perp 1,2} = [\pm(\omega_+ - \omega_-) + D_1^{1/2}]/2$ are two frequencies of AFM-resonance at transverse magnetization, $\omega_{ar1,2} = [(\omega_+^2 + \omega_-^2 + 8\pi\gamma_s^2 M_s H_A \pm D_2^{1/2})/2]^{1/2}$ are two frequencies of AFM-antiresonance,

$$D_1 = (\omega_+ + \omega_-)^2 + 32\pi\gamma_s^2 M_s H_A,$$

$$D_2 = (\omega_+^2 + \omega_-^2 + 8\pi\gamma_s^2 M_s H_A)^2 - 4\omega_+\omega_-(\omega_+\omega_- + 8\pi\gamma_s^2 M_s H_A).$$

The suggested model takes into account only the time dispersion of the medium. It does not take into account the spatial dispersion of the wire structure [18], as well as EMW reflection from the periodic structure, as done in [5].

Fig. 2, *a*, *b* shows the dispersion characteristics (DCh) of EMW (solid lines), calculated on the basis of (3) taking into account (4)-(6) and (2). Fig. 2, *c*, *d* shows the frequency dependences of the medium's efficient constitutive parameters. The results presented in Fig. 2, *a*, *c* show that three extraordinary EMWs exist in the AFM in the absence of a 2D-periodic structure ($\omega_{p\perp} = 0$). Two of them are slow EMWs (curves *2* and *3*), while the third one is a fast EMW (curve *1*). All the three EMWs are at frequencies where $\mu_{eff\perp} > 0$. Cut-off frequency of a fast EMW is determined by frequency ω_{ar1}. The low-frequency slow EMW does not have a cut-off frequency, while cut-off frequency for the high-frequency slow EMW corresponds to ω_{ar2}. Both slow EMWs are characterized by the limit frequencies $\omega_{\perp 1,2}$ located in the terahertz range. Thus, slow EMWs have a normal dispersion.

Fig. 2, *b* shows the EMW DCh obtained in the presence of a 2D-periodic structure of perfectly-conducting wires ($\omega_{p\perp} \neq 0$ and $\alpha_\perp = 0$), the radius of which is less than copper skin layer depth at the frequency of 0.3 THz. In this case, we can see the degeneracy of slow direct EMWs (curves *2*, *3*) into oscillations at frequencies $\omega_{\perp 1,2}$, since $\mu_{eff\perp} > 0$ in the frequency band of existence of such waves, while $\varepsilon_{eff\perp} < 0$ (Fig. 2, *c*, *d*). The cut-off frequency of the fast EMW does not correspond to frequency ω_{ar1} anymore and shifts to a higher-frequency region (> 1.5 THz), where $\mu_{eff\perp} > 0$ and $\varepsilon_{eff\perp} > 0$ (Fig. 2, *c*, *d*). The most significant result is in the formation of two backward slow EMWs (curves *4*, *5*), located in different frequency ranges, where $\mu_{eff\perp} < 0$ and $\varepsilon_{eff\perp} < 0$ (Fig. 2, *c*, *d*). It must be noted that the effective dielectric constant of the medium where $\mu_{eff\perp} < 0$ can be both less than and more than zero. Due to this, each backward EMW occupies only a part of the frequency range where $\mu_{eff\perp} < 0$. Fig. 2, *b* shows the

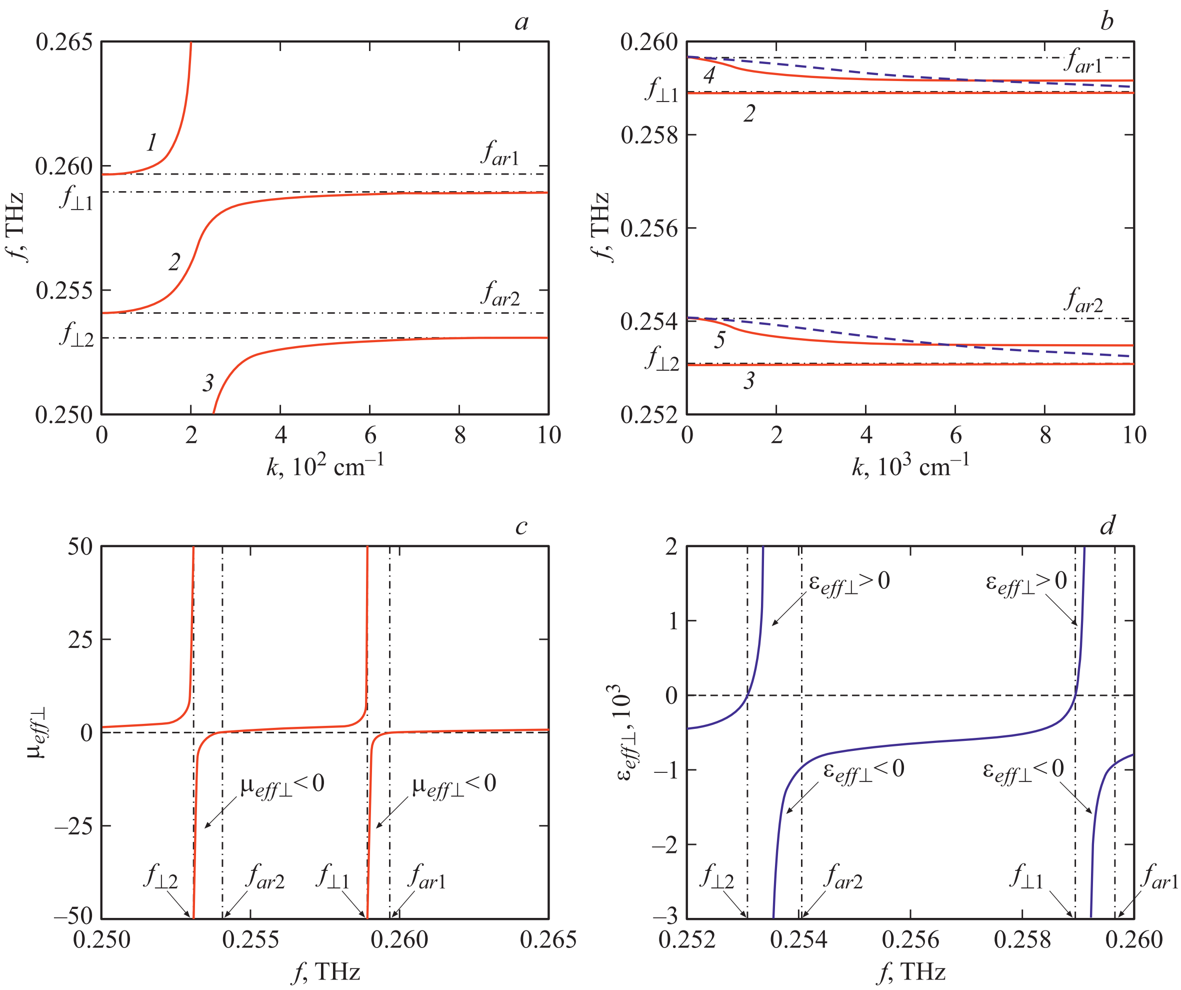

Figure 2. Dispersion characteristics of extraordinary fast EMW (*1*) and slow EMW (curves *2-5*) (a, b), as well as frequency dependences $\mu_{eff\perp}$ (c) and $\varepsilon_{eff\perp}$ (d), calculated for transversely magnetized AFMs (a, c) and AFM-metamaterial (b, d). The dashed lines in the part b show the dispersion characteristics of backward EMWs, calculated for a transversely magnetized AFM-metamaterial with an effective dielectric constant that corresponds to isotropic plasma, — $\varepsilon_{eff} = \varepsilon_r(1-\omega_p^2/\omega^2)$, where $\omega_p^2 = 2\pi c^2/[a^2\ln(a/r_1)]$. The calculations have been made for $\omega_{p\perp} = 0$ (a, c) and for $a = 10^{-3}$ cm, $r_1 = 10^{-5}$ cm, $r_2 = 10^{-4}$ cm (b, d). In all cases $H_0 = 79.58$ kA/m, $M_s = 0.0560$ T, $H_E = 40.98$ MA/m, $H_A = 636.64$ kA/m and $\varepsilon_r = 16$. The AFM parameters corresponded to MnF_2 [17].

DCh of two backward EMWs (dashed lines), calculated on the basis of (2)-(4) for the AFM-metamaterial with an effective dielectric constant that corresponds to isotropic plasma (see expression given in the caption for Fig. 2) [19]. Backward EMWs in this approximation (the insulators have an infinitely large radius) are slower and occupy a wider frequency band that is determined by expressions (7a) and (7b).

We would like to conclude by noting that the obtained results can be used in development of functional materials for terahertz magnionics.

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Conflict of interest

The authors declare that they have no conflict of interest.

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