09

Excitation and ionization of a particle in a one-dimensional potential well of zero radius by an extremely short light pulse

© R.M. Arkhipov[1,2], M.V. Arkhipov[1], A.V. Pakhomov[1], N.N. Rosanov[1,2]

[1] St. Petersburg State University, 199034 St. Petersburg, Russia
[2] Ioffe Institute, 194021 St. Petersburg, Russia

e-mail: arkhipovrostislav@gmail.com, m.arkhipov@spbu.ru, antpakhom@gmail.com, nnrosanov@mail.ru

Received November 24, 2021
Revised November 24, 2021
Accepted December 03, 2021

The Migdal sudden perturbation approximation is used to solve the problem of excitation and ionization particles in a one-dimensional potential of zero radius with an extremely short pulse. There is only one energy level in such a one-dimensional the delta-shaped potential well. It is shown that for pulse durations shorter than the characteristic period of oscillations of the wave function of the particle in the bound state, the population of the level (and the probability of ionization) is determined by the ratio of the electric area of the pulse to the characteristic „scale" of the area inversely proportional to the size of localization of the particle in a bound state.

Keywords: unipolar pulses, electrical pulse area, atomic scale of electric area.

DOI: 10.21883/EOS.2022.03.53561.2959-21

Introduction

In recent years, possibility of obtaining unipolar electromagnetic pulses with non-zero electric area, defined as

$$S_E \equiv \int_{-\infty}^{\infty} E(t)dt,$$

$E(t)$ — electric field strength at a given point in space (see review [1] and cited literature) has been actively studied. Such pulses can have many different applications, e.g., for ultra-fast and efficient control of the dynamics of wave packets in matter, as compared to conventional bipolar pulses, acceleration of charges, and other applications, see [1].

As the results of various studies demonstrate, the effect of unipolar pulses on microobjects is determined by pulse electrical area, rather than its energy, if the pulse duration is less than the characteristic oscillation period of the wave packet in matter [2–7]. Some methods for experimental determination of radiation unipolarity and its electrical area were first proposed relatively recently [8].

When excitation pulse duration is shorter than the characteris.tic time T associated with the energy of the ground state the standard Keldysh photoionization theory becomes inapplicable [7]. And definition of some physical quantities, such as Keldysh parameter, which sets the criterion of a strong and weak field, requires revision.

To specify the degree of unipolar pulses effect on quantum objects, a new physical quantity has recently been introduced — an atomic scale of electric pulse area inversely proportional to the characteristic size of the system [4]. As shown in this paper, the ground state population in the simplest multilevel quantum systems (hydrogen atom, quantum oscillator, etc.) is determined by the ratio of the electrical pulse area to its atomic scale.

In the paper [7] the ionization of 3D quantum systems (hydrogen atom, spherical quantum point, 3D potential of zero radius) by a extremely short pulse was considered. It was shown that ionization probability is also determined by the ratio of the electric area of the pulse to its atomic scale, which is inversely proportional to the characteristic size of the system in the ground state.

Theoretical description of interaction between extremely short and unipolar pulses and multilevel quantum systems is a difficult problem. To model real quantum systems, model of the zero-radius potential is attractive. This model is actively used to study various processes in nuclear and atomic physics, to describe the behavior of ions in external fields, see [9–11] and the cited literature.

The problem of ionization from a 3D δ-pit by a circularly polarized monochromatic wave has been considered in [12–14]. In paper [15], the 3D zero-radius potential model was applied to the analysis of electron ejection from negative ions by unipolar pulses.

The simplest model is the 1D model of the zero-radius potential. Despite its simplicity, it has also been used to model various systems, such as water-like atoms, two-atom molecules, ions, and more complex systems [16–19].

A number of papers have considered problems of interaction of powerful laser radiation with atomic systems, which have been modeled by a 1D zero-radius potential [20–22]. See more details about use of this model in various problems in review [23] and cited literature.

As it was already noted above, recently it was demonstrated that the probability of ionization of a wide class of 3D quantum systems with extremely short pulses is determined by the ratio of the electric area of the pulse to the recently introduced atomic scale of electric area, which is inversely proportional to the specific size of the system [7].

This paper studies probability of conserving the ground state and ionization of a particle in a 1D potential pit of zero radius under the action of an extremely short pulse with duration shorter than the characteristic time T associated with the energy of the ground state, $T = 2\pi\hbar/E$ (E is the energy of the particle in the ground state).

A comparison is made with the case of a 3D potential of zero radius. It is shown that the probabilities of conserving the bound state in the 1D and 3D cases are very different in form. However, in 1D case it is also possible to introduce a characteristic measure of the area S_0, inversely proportional to the area of electron localization in the bound state.

Theoretical consideration and discussion of results

The Schrödinger equation with a delta potential in the 1D case has the appearance of

$$\psi'' + \frac{2m}{\hbar^2}(E - U(x))\psi = 0, \tag{1}$$

$$U(x) = -V_0\delta(x).$$

Such pit has only one energy level $E = -\frac{m}{2\hbar^2}V_0^2$. The normalized wave functions of the bound state are given by the expression [24]

$$\psi_0(x) = \sqrt{\alpha}e^{\alpha x}, \quad x < 0,$$

$$\psi_0(x) = \sqrt{\alpha}e^{-\alpha x}, \quad x > 0,$$

$$\alpha \equiv \frac{m}{\hbar^2}V_0. \tag{2}$$

In these expressions, the characteristic size of the electron localization region is present

$$x_0 = \frac{1}{2\alpha} = \frac{\hbar^2}{2mV_0}. \tag{3}$$

The duration of the excitation pulse τ is assumed to be shorter than the bound state „oscillation period" T, $\tau \ll T = 2\pi\hbar/E$. For example, in the case of ion H^-, for which the zero-radius potential model is actively used, time $T = 5.4$ fs in the 3D consideration [15]. Therefore, optical pulses of attosecond duration can be actively used in similar problems [25–27].

The wave function of the particle after the pulse in the Migdal sudden approximation has the well-known form [6,7,28]:

$$\Psi_+(x) = \psi_0(x)e^{i\frac{q}{\hbar}S_E x}, \tag{4}$$

where q — particle charge.

Amplitude of the bound state after the pulse is defined by the equation

$$a_0 = \int_{-\infty}^{\infty} \psi_0^2(x)e^{i\frac{q}{\hbar}S_E x}dx.$$

By integration, from equations (2) and (4) we can find the population of the particle bound state in the pit after the end of the pulse:

$$w_0 = |a_0|^2 = \frac{1}{(1 + S_E^2/S_0^2)^2}. \tag{5}$$

Population of the bound state in (5) decreases with increasing electric area, at least as S_E^{-4}. This equation introduces characteristic scale of the electric pulse area $S_0 \equiv \frac{2\alpha\hbar}{q} = \frac{\hbar}{qx_0}$, inversely proportional to specific size of electron localization region x_0. It has the meaning of characteristic value of the electric pulse area, when effective emptying of the system bound state is possible.

Thus, the population is determined by the ratio of the electric pulse area to its characteristic scale S_E/S_0. Accordingly, the probability of ionization $w_{\text{ion}} = 1 - w_0$ is also determined by electric area of pulse with characteristic scale S_0, inversely proportional to size of system x_0.

In the case of 3D potential of zero radius, however, equation for the probability of conserving the bound state differs in form from the equation in 1D case (5) and has the appearance of an arctangent [7,15]:

$$w_0 = a_{00}^2, \quad w_{\text{ion}} = 1 - w_0,$$

$$a_{00} = (S_0/S_E)\arctan(S_E/S_0). \tag{6}$$

However, in 3D case as well, the characteristic scale of area S_0 in this equation is also inversely proportional to the region of localization of the electron in the bound state in the pit [7].

Note the similarity (5) with the probability of conservation of the main state of hydrogen atom [7,29]

$$w_0 = \left[1 + (S_E/S_{at})^2\right]^{-4}. \tag{7}$$

It is only that in this case probability diminishes faster as S_E^{-8}. And value of atomic scale of area $S_{at} = \frac{2\hbar}{qa_0}$ is also inversely proportional to radius of the first Bohr orbit a_0, i.e. characteristic size of the system.

Conclusion

Thus, in the case of the 1D zero-radius potential model, the probability of preserving the bound state of the particle and its ionization is determined by the ratio of the electric pulse area to its characteristic scale, S_E/S_0. This scale of area S_0 is inversely proportional to the characteristic size of the localization region of the particle in the bound state x_0.

The concept of area scale, first introduced in [4,7], is valid for a wide class of quantum systems, both 1D and 3D ones. It can be used to estimate the value of the electric pulse area required to effectively excite and ionize quantum systems using unipolar and subcyclic pulses.

The value of the area scale must be taken into account when analyzing the interaction of extremely short pulses with quantum objects, when the pulse duration is shorter than the characteristic time T associated with the energy of the ground state.

Funding

The study was funded by the Russian Science Foundation within the framework of scientific project 21-72-10028.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] R.M. Arkhipov, M.V. Arkhipov, N.N. Rosanov, Quant. Electron., **50** (9), 801 (2020).
[2] P.H. Bucksbaum. AIP Conf. Proc., **323** (1), 416 (1994).
[3] D. Dimitrovski, E.A. Solov'ev, J.S. Briggs. Phys. Rev. A, **72**, 043411 (2005).
[4] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, N.N. Rosanov. JETP Lett., **114** (3), 129 (2021).
[5] R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, N.N. Rosanov. JETP Lett., **114** (5), 250 (2021).
[6] R.M. Arkhipov, M.V. Arkhipov, I. Babushkin, A. Demircan, U. Morgner, N.N. Rosanov. Opt. Lett., **44** (5), 1202 (2019).
[7] N. Rosanov, D. Tumakov, M. Arkhipov, R. Arkhipov. Phys. Rev. A, 104, 063101 (2021).
[8] M.V. Arkhipov, A.N. Tsypkin, M.O. Zhukova, A.O. Ismagilov, A.V. Pakhomov, N.N. Rosanov, R.M. Arkhipov, JETP Lett, 115(1), 1 (2022); DOI: 10.1134/S0021364022010015.
[9] E. Fermi. Ric. Sci., **7**, 13 (1936).
[10] Y.N. Demkov, V.N. Ostrovskii. *Zero-Range Potentials and Their Applications in Atomic Physics* (Plenum, New York, 1988).
[11] N.L. Manakov, M.V. Frolov, B. Borca, A.F. Starace. J. Phys. B, **36**, R49 (2003).
[12] I.J. Berson. J. Physics B: Atomic and Molecular Physics, **8** (18), 3078 (1975).
[13] N.L. Manakov, L.P. Rapoport. Sov. Phys. JETP, **42** (3), 430 (1976).
[14] N.B. Delone, V.P. Kraynov. *Atom v silnom svetovom pole* (M., Atomizdat, 1978) (in Russian).
[15] T.P. Grozdanov, J. Jaćimović. Phys. Rev. A, **79**, 013413 (2009).
[16] A.A. Frost. J. Chem. Phys., **25**, 1150 (1956).
[17] A.A. Frost. J. Chem. Phys., **22**, 1613 (1954).
[18] A.A. Frost, F.E. Leland. J. Chem. Phys., **25**, 1154 (1956).
[19] I. Lapidus. Am. J. Phys., **38**, 905 (1970).
[20] S. Geltman. J. Physics B, **10** (5), Article 019, 831 (1977).
[21] S. Geltman. J. Physics B, **27**, 1497 (1994).
[22] Q. Su, B.P. Irving, C.W. Johnson, J.H. Eberly. J. Physics B: Atomic, Molecular and Optical Physics, **29** (23), 5755 (1996).
[23] M. Belloni, R.W. Robinett. Physics Reports, **540** (2), 25 (2014).
[24] N. Zettili. *Quantum Mechanics Concepts and Applications* (John Wiley & Sons, 2009).
[25] M.T. Hassan, T.T. Luu, A. Moulet, O. Raskazovskaya, P. Zhokhov, M. Garg, N. Karpowicz, A.M. Zheltikov, V. Pervak, F. Krausz, E. Goulielmakis. Nature, **530**, 66 (2016).
[26] H.-C. Wu., J. Meyer-ter-Vehn. Nature Photon., **6**, 304 (2012).
[27] J. Xu, B. Shen, X. Zhang, Y. Shi, L. Ji, L. Zhang, T. Xu, W. Wang, X. Zhao, Z. Xu. Sci. Rep., **8**, 2669 (2018).
[28] A.B. Migdal, JETP 9, 1163 (1939) (in Russian).
[29] N.N. Rosanov. Opt. Spectrosc., **124** (1), 72 (2018).