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Measuring the effect of gravitational frequency shift doubling using a hydrogen clock-based quantum level

© V.F. Fateev, F.R. Smirnov, E.A. Rybakov

All-Russian Research Institute of Physical-Technical and Radiotechnical Measurements, Mendeleevo, Moscow oblast, Russia
E-mail: frsmirnov@vniiftri.ru

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The effect of gravitational frequency shift doubling in a system of stationary and transported hydrogen quantum clocks in the Earth's gravitational field has been measured for the first time with an instability of $1 \cdot 10^{-15}$. The clocks were separated in height by 34 m and connected by a radio channel based on optical fiber. The measured relative doubled „redshift" effect at this altitude was $\Delta f_{GR}/f_{ref} = (-7.73 \pm 1.61) \cdot 10^{-15}$.

Keywords: time dilation gravitational effect, gravitational frequency shift, quantum hydrogen clock.

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Experiments with quantum levels measuring the difference in gravitational potentials between points on the Earth's surface with the use of transported hydrogen quantum clocks [1–3] based on the known Einstein's gravitational time dilation [4] have already been performed. In the present study, we propose to switch to the direct measurement of the gravitational frequency shift.

A quantum level model, which included highly stable stationary and mobile (transported) hydrogen quantum clocks, was used to measure the gravitational frequency shift. A Ch1-1003M active hydrogen frequency and time standard located in a thermal chamber with daily temperature variations no greater than $0.1°$C was used as the stationary (reference) quantum clock (RQFTS). This clock was mounted in a specially equipped room on the 6th floor of an engineering building and had a relative daily instability of $(0.5-0.6) \cdot 10^{-15}$.

A PKCh-N active hydrogen frequency and time standard produced by ZAO „Vremya-Ch" with a daily instability of about $1 \cdot 10^{-15}$ served as the transported quantum clock. A Ch7-315 frequency comparator was positioned on the 6th floor in the immediate vicinity of the RQFTS. The SMF-28 single-mode optical fiber was used to transmit the measurement radio signal with frequency f_{TR}^{down} (about 10 MHz) to the 6th floor from the PKCh-N transported to the underground floor. Modulation and demodulation of the optical carrier by the PKCh-N radio signal were performed using an OTS-1RefT-10/S5-1307-E2-IC optical transmitter unit and an OTS-1RefR-10/S5-E2-IC optical receiver unit, respectively. Both units were produced by Emcore. The experiment was carried out in four stages.

The first stage is the initial calibration: the PKCh-N clock was positioned on the 6th floor in the immediate vicinity of the stationary Ch1-1003M RQFTS, and their signals with frequencies f_{TR}^{up} and f_{ref}, both of which were close to 10 MHz, were fed to two inputs of the Ch7-315 comparator. The measurement result at the comparator output is given by

$$\frac{\Delta f_1^{meas}}{f_{ref}} = \frac{f_{TR}^{up} - f_{ref}}{f_{ref}} = \frac{\Delta f^0}{f_{ref}}, \tag{1}$$

where Δf^0 is the initial technological frequency difference.

A total of 258 498 measurements were carried out in 72 h. The average value of the measured initial relative shift (with the PKCh-N frequency instability and the average RQFTS instability taken into account) determined by statistical processing of the measurement data was $\Delta f_1^{meas}/f_{ref} = (-128.14 \pm 1.14) \cdot 10^{-15}$.

At the second stage, PKCh-N was transported from the 6th floor to the underground floor to an elevation of -34 m and kept there for 72 h. Let us denote the frequency of its clock generator in the lower clock position as f_{TR}^{down}. To determine how frequency f_{TR}^{down} changes relative to f_{TR}^{up}, we consider the relations between proper time intervals at different points of the gravitational field in general relativity (GR) [4]. Let us denote the proper time scale of the transported PKCh-N clock in the lower and upper positions as τ_{TR}^{down} and τ_{TR}^{up}, respectively. Taking into account the fact that proper (measured) frequencies f_{TR}^{down} and f_{TR}^{up} of clock generators of a pair of quantum clocks are related in direct proportion to their proper time intervals [5], we may write the following for a stationary clock in GR:

$$\frac{\tau_{TR}^{up}}{\tau_{TR}^{down}} = \frac{f_{TR}^{up}}{f_{TR}^{down}} = 1 - \frac{\varphi_{TR}^{up} - \varphi_{TR}^{down}}{c^2} \approx 1 + \frac{gH}{c^2}, \tag{2}$$

where c is the speed of light, $\varphi_{TR}^{up} = \mu/R_E$, $\varphi_{TR}^{down} = \mu/(R_E - H)$ are the gravitational potentials at the upper and the lower points, R_E and H are the Earth's radius and the difference in orthometric elevation between the upper and the lower clocks, and $\mu = 3.986 \cdot 10^{14}\,\text{m}^3/\text{s}^2$ is the geocentric gravitational constant. The centrifugal potentials are neglected. It follows from formula (2) that the lower clock, which is located in the gravitational field of the Earth with a higher absolute potential value, is slowed down. This effect was measured both on the Earth (see, e.g., [1–3]) and in space by observing the onboard time scales of GALILEO navigational satellites [6,7].

This relation yields the expression for the sought-for gravitational shift of the PKCh-N clock generator frequency f_{TR}^{up} occurring when this clock is transported „down":

$$f_{TR}^{down} \approx f_{TR}^{up}\left(1 - \frac{gH}{c^2}\right). \quad (3)$$

Thus, the clock generator frequency undergoes a „red" (negative) gravitational shift as the elevation above ground of a quantum clock decreases. It also follows from (3) that the shift is positive („blue") when the elevation increases. The first study of the positive gravitational shift of the clock generator frequency of quantum clocks in space was published by Ashby [8]. This effect is now used widely to enhance the accuracy of satellite navigation. In order to compensate the „blue" shift, an opposite-sign correction is introduced prior to launch into the frequency of clock generators of onboard satellite clocks of global navigational satellite systems. Specifically, the relative value of this correction is $-4.36 \cdot 10^{-10}$ for GLONASS [9] and $-4.4647 \cdot 10^{-10}$ for GPS [8].

The third stage is the transmission of the radio signal with frequency f_{TR}^{down} via a fiber with the use of modulation/demodulation and the measurement of the frequency difference at the comparator inputs. The radio signal frequency at the fiber input (in the lower position) is $f_{FOCL}^{down} = f_{TR}^{down}$, while the frequency at the fiber output (in the „upper"position) is f_{FOCL}^{up}. The relation between proper frequencies and proper time intervals at the ends of the wave path is known from GR [4]:

$$\frac{\tau_{FOCL}^{up}}{\tau_{FOCL}^{down}} = \frac{f_{FOCL}^{down}}{f_{FOCL}^{up}} = 1 - \frac{\varphi_{FOCL}^{up} - \varphi_{FOCL}^{down}}{c^2} \approx 1 + \frac{gH}{c^2}, \quad (4)$$

where the expressions for potentials from formula (2) were used.

The corresponding frequency shift of an electromagnetic wave along the path of its propagation from the lower position to the upper one is negative (classical „red" shift):

$$f_{FOCL}^{up} \approx f_{FOCL}^{down}\left(1 - \frac{gH}{c^2}\right) = f_{TR}^{donw}\left(1 - \frac{gH}{c^2}\right). \quad (5)$$

The existence of „red"and „blue" gravitational frequency shifts in an electromagnetic wave is undisputed at present: this effect has already been verified experimentally in a considerable number of studies (see, e.g., reviews [10,11]). Among the recent studies, worthy of mention are the successful experiments with optical frequency standards (with an instability of 10^{-17}–10^{-18}) and a fiber-optic communication line (FOCL) [12,13]. The effect of gravitational shift in an electromagnetic wave in a FOCL has also been proposed to be used for the measurement of optometric elevations [14–16].

Taking into account that $\Delta f^0 \ll f_{ref}$, using formulae (1) and (3), and neglecting products $\Delta f^0 \frac{2gH}{c^2}$, $f_{ref}\left(\frac{gH}{c^2}\right)^2$ due to their smallness, we rewrite formula (5) in the following way:

$$f_{FOCL}^{up} = \left(f_{ref} + \Delta f^0\right) - f_{ref}\frac{2gH}{c^2}. \quad (6)$$

Thus, the absolute and relative values of the measured frequency difference of signals fed to two inputs of the comparator are

$$\Delta f_2^{meas} = f_{FOCL}^{up} - f_{ref} = \Delta f^0 - f_{ref}\frac{2gH}{c^2},$$

$$\frac{\Delta f_2^{meas}}{f_{ref}} = \frac{f_{FOCL}^{up} - f_{ref}}{f_{ref}} = \frac{\Delta f^0}{f_{ref}} - \frac{2gH}{c^2}. \quad (7)$$

A total of 255 030 measurements were performed at the third stage within 72 h. The average value of the measured relative frequency shift (with the PKCh-N frequency instability taken into account) determined by statistical processing of the measurement data was $\Delta f_2^{meas}/f_{ref} = (-135.87 \pm 1.14) \cdot 10^{-15}$.

At the fourth stage, „red" gravitational frequency shift Δf_{GR} was calculated based on the results of measurements performed at stages 1 and 3. Using formulae (1) and (7), we find

$$\frac{\Delta f_{GR}}{f_{ref}} = \frac{\Delta f_2^{meas} - \Delta f_1^{meas}}{f_{ref}} = -\frac{2gH}{c^2}. \quad (8)$$

The result of calculations at the fourth stage was

$$\Delta f_{GR}/f_{ref} = (-7.73 \pm 1.61) \cdot 10^{-15}.$$

The sought-for orthometric elevation was $H = -\frac{c^2}{2f_{ref}g}\Delta f_{GR} \approx 35.49 \pm 7.38$ m. The actual elevation of 34 m falls within the measurement uncertainty interval.

To conclude, the doubled effect of „red" gravitational frequency shift in the Earth's gravitational field has been measured experimentally for the first time with the use of a quantum level based on stationary and transported hydrogen quantum clocks separated by a vertical distance of 34 m. The measured elevation was $H \approx 35.49 \pm 7.38$ m.

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Conflict of interest

The authors declare that they have no conflict of interest.

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