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Influence of a strong low-frequency wave on the propagation of weak ultrasonic pulses in a rod resonator made of annealed polycrystalline copper

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An experimental study of an influence of a strong low-frequency wave on the propagation of weak ultrasonic pulses in an acoustic resonator made of annealed polycrystalline copper has been carried out. The measurements were carried out with harmonic excitation of the resonator at its first four longitudinal modes in the range from 2 to 15 kHz, the frequency of ultrasonic pulses varied from 65 to 400 kHz. The analysis of the observed nonlinear effects was carried out within the framework of the polycrystal equation of state obtained on the basis of a modified string model of the Granato–Lucke dislocation. The values of the parameters of dissipative and reactive nonlinearity of dislocations in annealed copper are determined.

Keywords: dislocation dissipative and reactive nonlinearity, polycrystalline copper, elastic waves.

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Introduction

The issues related to nonlinear wave processes (NWPs) in microinhomogeneous solids [1–3] with anomalously high non-analytical and frequency-dependent nonlinearity are of urgent in current acoustics. Their topicality stems from the fact that the „classical“ five-constant elasticity theory [4,5], which corresponds to weakly nonlinear homogeneous media (with analytical and frequency-independent nonlinearity), does not provide explanations for the common patterns of nonlinear acoustical effects (NAEs) in experiments with strongly nonlinear microinhomogeneous media, and a „general“ (universal) NWP theory for such media has not been formulated yet.

Many polycrystalline metals and rocks are classified as strongly nonlinear microinhomogeneous solid media. Their nonlinearity is associated with the motion and interaction of dislocations (linear defects of polycrystals) with point defects (vacancies and impurity and interstitial atoms) [6–15]. Polycrystals manifest hysteretic nonlinearity in the low-frequency (LF) range and dissipative and reactive nonlinearity in the high-frequency (HF) range [3]. The NAE patterns related to hysteretic, dissipative, and reactive nonlinearity tend to differ, thus providing an opportunity to study separately the contributions of these nonlinearities to the manifestation of LF and HF nonlinear acoustical effects.

The nonlinear properties of a medium are manifested most vividly at relatively high amplitudes of elastic waves, which are easy to obtain in high-Q resonators. Hysteretic effects of amplitude-dependent internal friction (nonlinear losses, shift of resonance frequencies, and higher harmonic generation) and effects of damping and carrier phase delay (CPD) of weak ultrasonic pulses in the field of a strong LF wave, which are attributable to HF dissipative and reactive nonlinearity instead of LF hysteretic nonlinearity, are observed in resonators made of polycrystalline solids [3,16].

Since nonlinear effects are often manifested differently in different media, the determination of amplitude–frequency dependences of NAEs in microinhomogeneous media is one of the prime objectives in their experimental studies. The fundamental purpose of these studies has to do with the identification of mechanisms behind the anomalously high acoustical nonlinearity of various microinhomogeneous media. The applied objective of this research is the design and development of nonlinear acoustical diagnostics methods for microinhomogeneous media and materials [3,17,18].

In the present study, which is a continuation of [19], we report new experimental data on the influence of a strong LF pump wave in a rod resonator made of annealed polycrystalline copper on the propagation of weak ultrasonic pulses (UPs) in it. The amplitude–frequency dependences of nonlinear damping and CPD of ultrasonic pulses under the influence of a strong low-frequency wave were determined. The observed nonlinear effects (damping and CPD of ultrasonic pulses) were analyzed within the framework of the equation of state of a polycrystal containing dislocation dissipative and reactive nonlinearities [20]. The parameters of dissipative and reactive nonlinearity of dislocations for annealed copper were determined by comparing the experimental and analytical data.

1. Experimental diagram

A rod resonator made of annealed copper was used to examine experimentally the influence of a strong standing

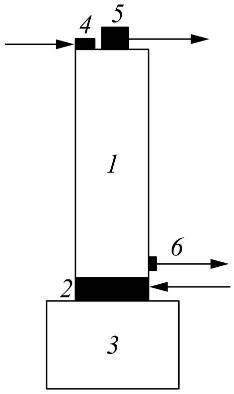

Figure 1. Diagram of the experiment.

LF wave on the propagation of weak travelling UPs. Rod length $L = 0.43$ m. The annealing temperature and time were 600°C and approximately 2 h. (The same resonator was used earlier in [19] to study the LF effects of amplitude-dependent internal friction arising due to hysteretic nonlinearity of annealed copper under LF harmonic excitation of this resonator.)

The diagram of the experimental setup is presented in Fig. 1. Piezoceramic radiator *2* was glued to the lower end of rod *1* to excite a longitudinal LF pump wave in it. The other side of the radiator was glued to massive load *3*. HF piezoceramic radiator *4* (used to generate longitudinal UPs propagating along the rod axis) and piezoreceiver *5* (detecting LF vibrations produced by radiator *2*) were glued to the upper free end of rod *1*. Piezoreceiver *6*, which responded to longitudinal HF vibrations of the rod (along its axis), was glued to the side surface of rod *1* in the vicinity of radiator *2*. This piezoreceiver was used to detect and measure the amplitude and CPD of UPs that were generated by radiator *4* and propagated along the rod. Pulse frequency f was varied from 65 to 400 kHz.

Rod *1* served in this experiment as a resonator for an LF wave and as a virtually unbounded medium for travelling UPs. Eigen frequencies F_p of LF longitudinal modes of this resonator are given by expression $F_p \cong (2p-1)C_0/4L$, where C_0 is the phase velocity of an LF longitudinal wave in the rod and p is the mode number, $p = 1, 2, 3, 4\ldots$. Resonance frequencies F_p and quality factors Q_p for the first four longitudinal modes of the resonator were $F_1 = 2204$ Hz, $F_2 = 6447$ Hz, $F_3 = 10697$ Hz, $F_4 = 14928$ Hz and $Q_1 = 450$, $Q_2 = 883$, $Q_3 = 578$, $Q_4 = 711$.

2. Measurement results

A strong LF standing pump wave and weak UPs were generated in rod *1* in the experiment. Pulses propagated along the rod, were detected by piezoreceiver *6*, and were fed to the input of an oscilloscope where their amplitude $U(\varepsilon_m)$ and CPD $\Delta\tau(\varepsilon_m)$ were measured as functions of deformation (LF wave) amplitude ε_m (in resonance). When ε_m increased, amplitude $U(\varepsilon_m)$ of detected UPs decreased, while their CPD $\Delta\tau(\varepsilon_m)$ grew (i.e., the damping of pulses intensified, and their phase velocity became lower).

Figure 2 presents the dependences of nonlinear damping coefficient $\chi(\varepsilon_m) = \ln[U_0/U(\varepsilon_m)]$ (U_0 is the amplitude of pulses at $\varepsilon_m = 0$) and carrier phase delay $\Delta\tau(\varepsilon_m)$ of UPs with frequency $f = 365$ kHz on amplitude ε_m of the LF wave at $p = 1, 2, 3, 4$. The following dependences are evident: $\chi(\varepsilon_m) \propto \varepsilon_m$, $\Delta\tau(\varepsilon_m) \propto \varepsilon_m^2$, and $\chi(\varepsilon_m)$ and $\Delta\tau(\varepsilon_m)$ at $\varepsilon_m = \mathrm{const}$ increase with LF wave frequency F_p.

Figure 3 shows the dependences of coefficient $\chi(\varepsilon_m)$ and phase delay $\Delta\tau(\varepsilon_m)$ of UPs with frequencies $f = 365$ kHz on frequency F_p at $\varepsilon_m = 2 \cdot 10^{-5}$. It can be seen from these figures that $\chi(\varepsilon_m) \propto F_p^{1/2}$ and $\Delta\tau(\varepsilon_m) \propto F_P$. It should be noted that dependences $\chi = \chi(\varepsilon_m)$ and $\Delta\tau = \Delta\tau(\varepsilon_m)$ on ε_m

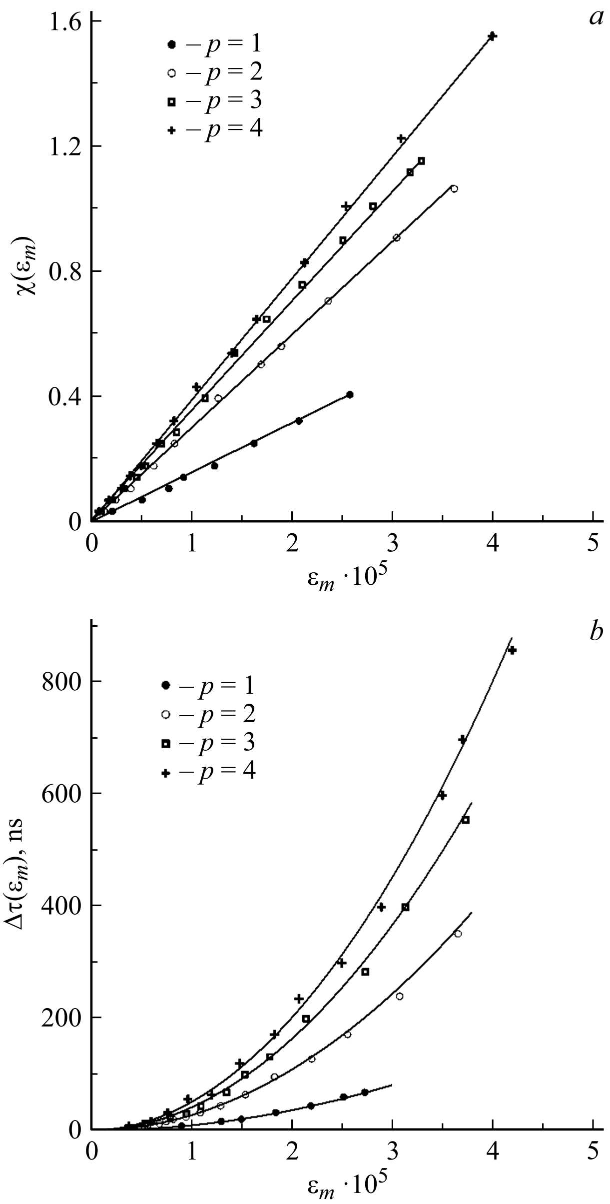

Figure 2. Results of measurement of dependences $\chi(\varepsilon_m)$ (*a*) and $\Delta\tau(\varepsilon_m)$ (*b*) on ε_m at $f = 365$ kHz. Lines correspond to dependences $\chi(\varepsilon_m) \propto \varepsilon_m$, $\Delta\tau(\varepsilon_m) \propto \varepsilon_m^2$.

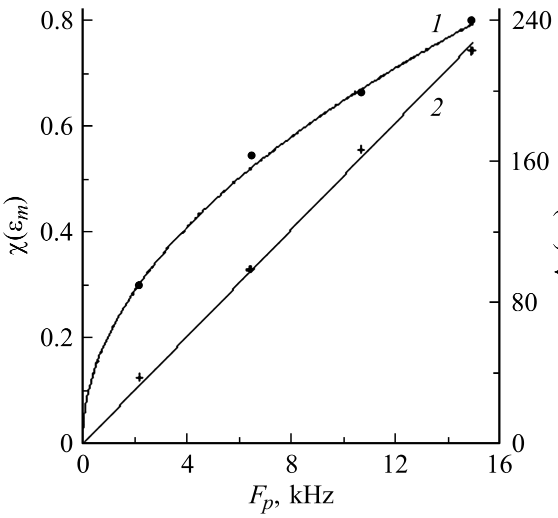

Figure 3. Results of measurement of dependences $\chi(\varepsilon_m)$ (*1*) and $\Delta\tau(\varepsilon_m)$ (*2*) on F_p at $\varepsilon_m = 2 \cdot 10^{-5}$ and $f = 365$ kHz. Lines correspond to the following dependences: *1* — $\chi(\varepsilon_m) \propto F_p^{1/2}$, *2* — $\Delta\tau(\varepsilon_m) \propto F_p$.

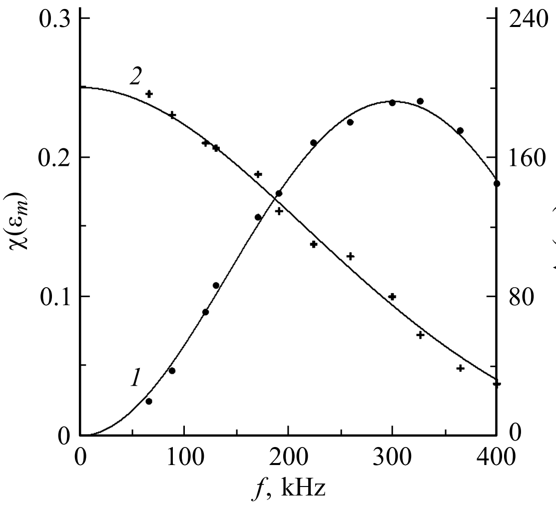

Figure 4. Dependences of $\chi(\varepsilon_m)$ (*1*) and $\Delta\tau(\varepsilon_m)$ *2*) on frequency f upon resonator excitation at the second-mode frequency ($p = 2$) at $\varepsilon_m = 6.8 \cdot 10^{-6}$. Curves *1* and *2* represent the results of calculation in accordance with formulae (1), (2); • and + denote the measurement data.

and F_p correspond neither to amplitude nor to frequency dependences of LF effects of amplitude-dependent internal friction established in [19] for the same resonator, which are induced by hysteretic nonlinearity of annealed copper.

Figure 4 shows the dependences of damping coefficient $\chi = \chi(\varepsilon_m)$ and CPD $\Delta\tau = \Delta\tau(\varepsilon_m)$ on pulse frequency f at $\varepsilon_m = 6.8 \cdot 10^{-6}$ and $p = 2$. It follows from Fig. 4 that coefficient $\chi = \chi(\varepsilon_m)$ first (at 65 kHz $< f <$ 300 kHz) increases and then (at 300 kHz $< f <$ 400 kHz) decreases as frequency f rises, while delay $\Delta\tau(\varepsilon_m)$ decreases as f grows within the 65 kHz $< f <$ 400 kHz interval.

3. Analysis and comparison of experimental and theoretical results

The theoretical description of effects observed in the present experiment was provided in [20], where the expressions for $\chi = \chi(\varepsilon_m)$ and $\Delta\tau(\varepsilon_m)$ were derived within the framework of the equation of state of a polycrystalline solid with dislocation dissipative and reactive nonlinearity:

$$\chi(\varepsilon_m) = \frac{\mu P}{\sqrt{\pi}} \frac{\Gamma[(m+1)/2]}{\Gamma[(m+2)/2]} \varepsilon_m^m L \Omega_p^q d_0 \omega^2$$
$$\times \int_0^\infty \frac{[(\Omega^2 - \omega^2)^2 - d_0^2\omega^2] l N(l) dl}{[(\Omega^2 - \Omega_p^2)^2 + d_0^2\Omega_p^2]^{m/2} [(\Omega^2 - \omega^2)^2 + d_0^2\omega^2]^2}, \quad (1)$$

$$\tau(\varepsilon_m) = \frac{\eta Q}{\sqrt{\pi}} \frac{\Gamma[(n+1)/2]}{\Gamma[(n+2)/2]} \varepsilon_m^n L \Omega_p^r$$
$$\times \int_0^\infty \frac{[(\Omega^2 - \omega^2)^2 - d_0^2\omega^2] \Omega^2 l N(l) dl}{[(\Omega^2 - \Omega_p^2)^2 + d_0^2\Omega_p^2]^{n/2} [(\Omega^2 - \omega^2)^2 + d_0^2\omega^2]^2}, \quad (2)$$

where

$$P = \frac{8R^2C_0}{\pi^{9/2}} \frac{(1+q)\Gamma[(m+3)/2]}{\Gamma[(m+4)/2]} B\left[\frac{m-q+1}{2}, \frac{q+1}{2}\right]$$
$$\times \left(\frac{4RC_0^2}{\pi^2 b^2}\right)^m \left(\frac{b}{C_\perp}\right)^q,$$

$$Q = \frac{8R^2C_0}{\pi^{9/2}} \frac{(1+n-r)\Gamma[(n+3)/2]}{\Gamma[(n+4)/2]} B\left[\frac{n-r+1}{2}, \frac{r+1}{2}\right]$$
$$\times \left(\frac{4RC_0^2}{\pi^2 b^2}\right)^n \left(\frac{b}{C_\perp}\right)^r,$$

G and $E = 2G/(1+\nu)$ are the shear modulus and the Young's modulus; $C_\perp = (G/\rho)^{1/2}$ is the shear wave velocity; $C_0 = (E/\rho)^{1/2}$; ν and ρ are the Poisson's ratio and the density, b is the Burgers vector magnitude; $A = \pi\rho b^2$ is the mass of a unit dislocation length, B and $C = 2Gb^2/\pi(1-\nu)$ are the linear friction coefficient and the linear tension coefficient of a dislocation; l, $\Omega(l) = [2/(1-\nu)]^{1/2}(C_\perp/l)$ and $d_0 = B/A$ are the length, the resonance frequency, and the damping parameter of a dislocation; μ, η and m, q, n, r are dimensionless parameters and exponents of power of dissipative and reactive nonlinearity of a dislocation, $m \geq q \geq 0$, $n \geq r \geq 0$; $N(l)$ is the function of distribution of dislocations over length l, $\int_0^\infty lN(l)dl = \Lambda$, Λ is the dislocation density, R is the orientation factor, $\Omega_p = 2\pi F_P$, $\omega = 2\pi f$.

Let us analyze the experimental and theoretical results and determine the parameters of the dislocation structure of annealed copper.

It follows directly from the comparison of expressions (1), (2) with the measurement results in Figs. 2, 3 that $m = 1$, $q = 1/2$, $n = 2$, $r = 1$.

Let us now adjust distribution function $N(l)$ and dislocation parameters in such a way as to obtain a fit between dependences (1), (2) on frequency $f = \omega/2\pi$ and the measurement results (Fig. 4). A close fit is obtained with the modified Koehler distribution function

$$N(l) = \frac{\Lambda \exp[-(l - l_0)/L_0]}{L_0(l_0 + L_0)},\ 0 \le l_0 \le l < \infty, \quad (3)$$

with the following parameters: $C_\perp = 2.3 \cdot 10^3$ m/s, $C_0 = 3.8 \cdot 10^3$ m/s, $\nu = 0.28$, $\Lambda = 10^{12}\,\mathrm{m}^{-2}$, $L_0 = 10^{-4}$ m, $l_0 = 5 \cdot 10^{-5}$ m, $b = 3 \cdot 10^{-10}$ m, $d_0 = 10^9\,\mathrm{s}^{-1}$, $R^3 = 7.7 \cdot 10^{-3}$, $R^4 = 3.03 \cdot 10^{-3}$, $\mu = 4.1 \cdot 10^{-1}$, $\eta = 1.5 \cdot 10^2$.

Length $l_0 = 0$ for the Koehler distribution function [6] obtained in the case of random arrangement of impurity atoms along the dislocation line, and $L_0 = \int_0^\infty lN(l)dl \Big/ \int_0^\infty N(l)dl$ is the average dislocation length. Modified Koehler distribution (3) has $l_0 > 0$ ($l_0 \gg b$), and average dislocation length $\langle l \rangle$ is given by expression $\langle l \rangle = L_0 + l_0$.

Conclusion

The results of examination of the influence of a strong LF wave on damping and the carrier phase delay of weak UPs in a rod resonator made of annealed polycrystalline copper were reported, and it was demonstrated that its acoustical nonlinearity features dissipative and reactive components. The key characteristics of dislocations of annealed copper (length distribution function, damping parameter, exponents of power, and parameters of dissipative and reactive nonlinearity) were determined by comparing the experimentally measured amplitude–frequency dependences with the theoretical ones [20] obtained based on the modified string model of a Granato–Lücke dislocation. It follows from the analysis of results presented above and in [19] that the manifestations of hysteretic, dissipative, and reactive nonlinearities of annealed copper differ qualitatively, since the amplitude and frequency dependences of LF and HF nonlinear effects differ. This is indicative of the fact that the mechanisms of hysteretic and dissipative and reactive nonlinearities of annealed copper are also different. The hysteretic nonlinearity of polycrystalline solids is associated with the periodic separation of dislocations from impurity atoms (and attachment to them), while dissipative and reactive nonlinearities are associated with nonlinear friction and tension of dislocations that undergo vibratory motion under the influence of an intense elastic wave in the environment of point defects of a polycrystal. The obtained results suggest that dislocation hysteretic, dissipative, and reactive nonlinearities are sensitive characteristics of polycrystalline solids that may find application in nonlinear acoustical diagnostics.

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Conflict of interest

The authors declare that they have no conflict of interest.

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