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Model of non-spherical particles' rebound and scattering at high speed interaction with a streamlined surface

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A new model is proposed for a non-spherical solid particles' impact interaction with a surface as applied to gas-particle flows over surfaces. The range of a particle impact speed typical for a vehicle flight in a dusty atmosphere is considered. The model is based on the laws of mechanics and experimental data on the restitution coefficient of the particle normal velocity. The tangential force impulse during a particle-surface interaction is assumed to be proportional to both the normal impulse and the mean tangential velocity of the particle contact point. The space orientation of incident particles relative to the surface is considered as random. The rebound of particles of different shape with varying shape parameters and of a particle mixture is studied numerically. Statistical characteristics of particles' rebound and scattering are obtained. Comparison with the known experimental data of numerical results for the mean values of the normal and tangential velocities of the particles' gravity centers in the plane of impact after rebound showed their good agreement.

Keywords: dispersed particles, 3D model of impact, numerical investigation, statistical characteristics of rebound and scattering, comparison with experimental data.

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Introduction

Modeling the impact interaction of dispersed particles with a streamlined surface is one of the fundamental problems of gas suspension mechanics. On impact particles can be deposited on a streamlined surface, bounce (reflect) from it, cause abrasive erosion, force heat transfer, and affect the flow of the carrier gas. Reflected particles significantly affect the structure and properties of the flow of a dispersed phase. In the classical mechanics of multiple-phase flows [1] and in the vast majority of modern papers, in which the gas flow around bodies is reviewed, taking into account the rebound (reflection) of particles, their form is assumed to be spherical, and their rebound from the surface is considered regular (see, for example, review [3]). However, in real flows, particles such as sand, volcanic ash or synthetic corundum always have a non-spherical shape, sometimes very complex form. The rebound of such particles has a stochastic nature, which is associated with their random orientation at the instant of collision. Along with the potential roughness of the streamlined surface, in experiments this leads to a significant data spread on the parameters of particle rebound [3–5].

Models of regular impact interaction of particles with a surface, based directly on the recovery coefficients of the normal and tangential vector components of center-of-mass velocity of average particles, which are considered to be spherical ([6–8]), are widely used in the mechanics of gas suspensions. Another approach was based on modeling the tangential force at the point of particle contact with a surface using Coulomb's friction law (the tangential force is proportional to the normal force and, accordingly, the tangential impulse is proportional to the normal impulse) and introducing the coefficient of friction [9–12]. Models of this kind are valid at relatively low impact velosities (up to several tens of meters per second). In addition, the coefficient of dynamic friction is generally unknown. In problems of two-phase aerodynamics, when the motion of an aircraft in a dusty atmosphere is considered, the particles impact speeds with the surface of an aircraft or with elements of its engine range from several tens to several hundreds of meters per second. In this range of speeds, the most reliable at present are the experimental data on the average restitution coefficients of the components of center-of-mass velocity of average particles. These data were obtained for a large number of usually non-spherical particles with a noticeable spread in size. Meanwhile, the spread of coefficient values can be quite significant [5], which makes it difficult to assess the adequacy of one or another impact model. Recently, papers have appeared in which the rebound parameters are generalized or modeled taking into account the physical properties of particle and surface materials [13–18].

However, due to the great difficulties in modeling the impact of arbitrary particles' shape (in particular, particles with sharp edges, which are typical, for example, for particles of silicon dioxide or synthetic corundum), this approach has been used so far only for spherical or ellipsoidal particles.

To solve problems of two-phase flows of gas with particles, when it is required to calculate a very large number of particles in a flow, semi-empirical impact models

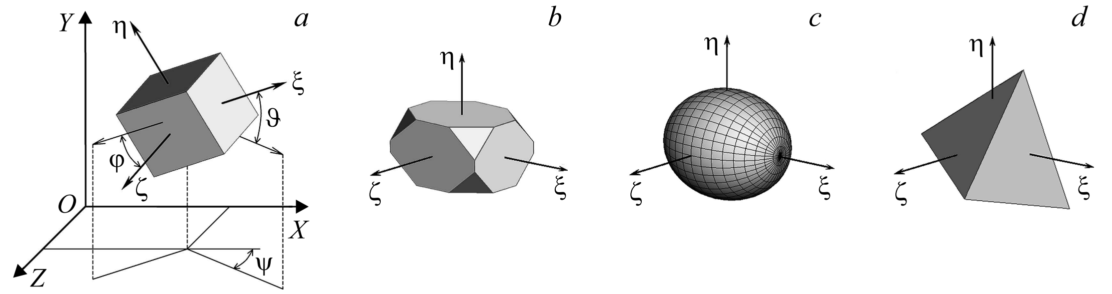

Figure 1. Particle configuration and angles φ, ψ, ϑ defining particle orientation in space: a — rectangular prism, b — rectangular flat-topped prism , c — ellipsoid, d — tetrahedron.

that take into account experimental data on restitution coefficients are currently the most preferable.

The aim of this paper is to develop a model of the impact interaction of a dispersed non-spherical particle with a surface that would adequately describe the rebound parameters and scattering characteristics of reflected particles at high impact speeds, typical in problems of aircraft aerodynamics, and which would be quite effective when calculating gas flows with a large number of particles (approximately 10^4 and more).

Earlier, the authors proposed a semi-empirical model of particle collision with a surface [19], in which it was assumed that the point of contact of the particle after the collision has a tangential velocity equal to zero. In this study, the possible sliding of a particle in the process of collision is considered, including the moment of rebound, and the impulse of the tangential force at the point of contact is modeled. Unlike the papers cited above [9–12], in which Coulomb's friction law was used to describe the tangential impulse, this model assumes that the momentum of the tangential force is proportional not only to the normal momentum, but also to the average tangential velocity of the particle's contact point, which to a certain extent takes into account the resistance to particle motion due to deformation of the surface material in the process of impact interaction. Using the proposed model, the statistical characteristics of the rebound and scattering of particles of various types of forms are studied in detail with varying form parameters and a mixture of these particles. To assess the adequacy of the model, the obtained results are compared with experimental data on the normal and tangential velocities of the center of mass of reflected particles in the impact plane.

1. Model of collision of a non-spherical particle with a surface

We will consider the impact interaction of solid rigid particles (such as sand or synthetic corundum) with the wall surface of an elasto-plastic yielding material. At high impact velocities, the particle penetrates into the wall during the impact and a contact surface is formed. The size and form of this surface are functions of time, which are determined by the impact velocity, the angle of incidence, the physical and mechanical properties of the wall material, as well as the form and the initial orientation of the particle relative to the surface. Modeling the impact interaction of an individual particle with a surface using the methods of deformable solid body mechanics requires computational resources comparable to solving the problem of a gas suspension flow around a body; therefore, such an approach for problems of two-phase aerodynamics is currently practically unrealizable.

The impact model proposed in this paper is based on the momentum and angular momentum conservation laws for a particle in integral form, experimental data on the restitution coefficient of the particle center of mass velocity normal to the surface, and additional heuristic assumptions that will be formulated below.

Let us first consider a separate collision of a particle with a flat wall. Let us introduce a local Cartesian coordinate system $OXYZ$, in which the XZ plane coincides with the wall surface, while Y axis is directed along the normal to it (we call the XY plane the impact plane). Let us represent the coordinates of the center of mass of the particle X_p, Y_p, Z_p. Let us introduce the coordinate system $O_\mathrm{p}\xi\eta\zeta$ associated with the particle with axes directed along the main axes of inertia of the particle. The angles φ, ψ, ϑ, which determine the orientation of the particle relatively to the $OXYZ$ coordinate system, and the particle forms considered in this paper are shown in Fig. 1.

Let us assume that the particle is not deformed in the process of impact with the surface (we will take the model of a rigid particle [11]).

Let the translational and angular velocity vectors $\mathbf{V}_\mathrm{p}$ and $\boldsymbol{\Omega}_\mathrm{p}$ and also angles φ, ψ, ϑ be given before the impact. It is required to determine the translational and angular velocities after the impact.

In the process of collision the point position of application of the master vector of forces on the contact surface between the particle and the wall surface and its direction change. Further, the equations of the laws of variation of momentum and angular momentum of the particle in integral form will be used. These equations include the impulse of the master vector of forces, which has one application point for each collision. We will associate this point with the contact point from the beginning of the

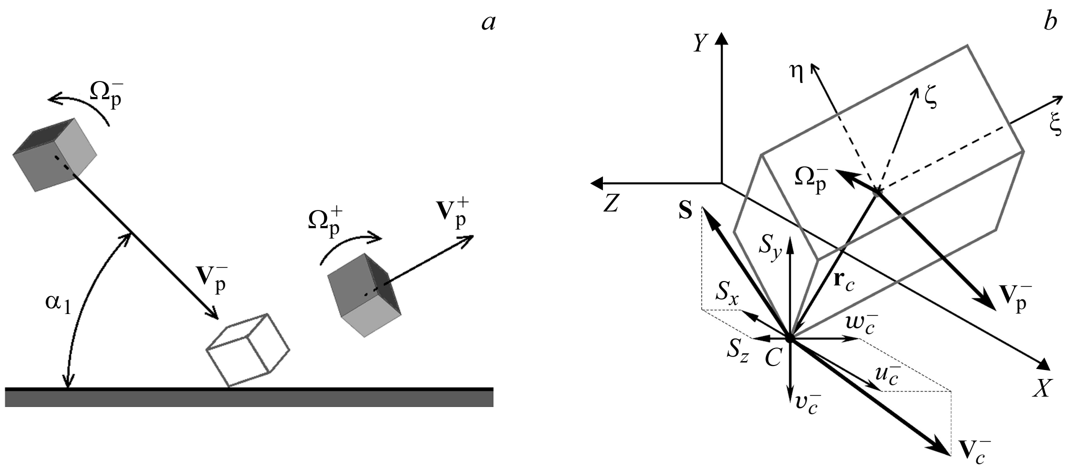

Figure 2. Impact of a non-spherical particle with a wall.

collision process (point C in Fig. 2). If the contact surface is an edge or face of a prismatic particle, then the geometric center of the edge or face is taken as the contact point to eliminate uncertainty in the calculation.

The position of the contact point relative to the center of mass of the particle is determined by the vector $\mathbf{r}_c$. We assume that the vector $\mathbf{r}_c$ does not vary during the collision process. From physical point of view this is justified by the fact that changes in the position and orientation of the particle during the impact interaction are very small. This assumption is similar to that accepted in the classical theory of impact, in which it is assumed that the impact interaction time tends to zero, the forces in the contact zone tend to infinity, and the impact impulse is a finite value [20]. Note that this assumption applies only to a single local collision during the rebound, in which the particle can experience several collisions. The possibility of several collisions is explained by the fact that after the first impact the particle starts to rotate and, having turned, again collisions with the surface by a different vertex.

The equations for the variation of momentum and angular momentum of a particle can be written in the following integral form:

$$m_p(\mathbf{V}_p^+ - \mathbf{V}_p^-) \equiv m_p\Delta\mathbf{V}_p = \int_0^{\delta t} \mathbf{f}_c(t)\mathrm{d}t \equiv \mathbf{S},$$

$$\|J_p\|\,(\mathbf{\Omega}_p^+ - \mathbf{\Omega}_p^-) \equiv \|J_p\|\,\Delta\mathbf{\Omega}_p = \mathbf{r}_c\,\mathbf{S}, \qquad (1)$$

where m_p, $\|J_p\|$ — mass and inertia tensor of the particle, $\mathbf{f}_c, \mathbf{S}$ — force and impulse acting on the particle at the contact point, δt — time interval during which the force $\mathbf{f}_c$ acts on the particle, superscripts „–" and „+" refer to the parameters of the particle before and after the impact (Fig. 2).

The particle velocity at the contact point $\mathbf{V}_c$ is related to the translational and angular velocities $\mathbf{V}_p$ and $\mathbf{\Omega}_p$ by kinematic relation

$$\mathbf{V}_c = \mathbf{V}_p + \mathbf{\Omega}_p\mathbf{r}_c, \qquad (2)$$

from which we get the following:

$$\Delta\mathbf{V}_c \equiv \mathbf{V}_c^+ - \mathbf{V}_c^- = \Delta\mathbf{V}_p + \Delta\mathbf{\Omega}_p\mathbf{r}_c. \qquad (3)$$

The combination of equations (1) and (3) gives

$$\|J_p\|\,\Delta\mathbf{\Omega}_p/m_p = \mathbf{r}_c\Delta\mathbf{V}_c - \mathbf{r}_c[\Delta\mathbf{\Omega}_p\mathbf{r}_c]. \qquad (4)$$

Equation (4) contains two unknown vectors: $\Delta\mathbf{V}_c$ and $\Delta\mathbf{\Omega}_p$.

Let u_p, v_p, w_p and u_c, v_c, w_c — components of vectors $\mathbf{V}_p$ and $\mathbf{V}_c$ in the $OXYZ$ coordinate system, and Δu_p, Δv_p, Δw_p and Δu_c, Δv_c, Δw_c — components of vectors $\Delta\mathbf{V}_p$ and $\Delta\mathbf{V}_c$ respectively.

The main difficulty in modeling the impact interaction of a particle with a surface lies in describing the impulse of the force acting on the particle from the wall at the point of contact C. Preliminary numerical simulation of the impact interaction of a solid non-spherical particle with an elasto-plastic and plastic wall using the methods of mechanics of solid deformable body showed that the particle slides during the impact interaction and its parameters after the impact (translational and angular velocities) depend significantly on the spatial orientation of the particle before impact (on positions of the vector $\mathbf{r}_c$).

We assume that the impulse of the force acting on the particle tangential to the surface is proportional to both the normal impulse and the average tangential velocity of the contact point. We note that in the case of low impact velocities (up to 20–30 m/s) it is usually assumed that the tangential impulse is proportional only to the normal impulse [9–12]. However, at high impact velocities (more than 50 m/s), the wall experiences, as a rule, elastic and plastic deformations, which depend on the impact velocity (it is this case that is considered in this paper). The accepted assumption implies that this dependence is linear.

To determine the normal impulse, we introduce into consideration the restitution coefficiennt for the normal velocity of the particle's contact point at the collision a_{nc}.

Then we can write

$$\Delta u_{\rm p} = -C_f \Delta v_{\rm p}(u_c^- + 0.5\Delta u_c)/|\mathbf{V}_c^-|,$$

$$\Delta w_{\mathbf{p}} = -C_f \Delta v_{\rm p}(w_c^- + 0.5\Delta w_c)/|\mathbf{V}_c^-|,$$

$$\Delta v_c = -v_c^-(a_{nc} + 1). \tag{5}$$

The coefficient C_f in these relations can be interpreted as the coefficient of dynamic resistance to particle sliding in the tangent direction. From a physical point of view, the coefficient of resistance C_f and the restitution coefficient of the particle velocity normal to the surface at the contact point a_{nc} in relations (5) are determined by the velocity and angle of impact, the orientation of the particle relative to the surface, and the physical and mechanical properties of the particle and wall materials.

For a_{nc} any experimental data are not available. However, it can be assumed that the value of a_{nc} is close to the value of the similar restitution coefficient for the center of mass of the particle $a_n = -v_p^+/v_p^-$. As it was specified experimentally in [21], where the rebound of synthetic corundum particles from plates of elasto-plastic materials was studied (steel, copper and lead were considered), the average values of the restitution coefficients of the normal a_n and tangential a_τ (in the impact plane) of the velocities of the center of mass of particles depend significantly on the velocity and angle of impact. The material of the plates also affects these coefficients, but its influence on a_n is much weaker than on a_τ.

The impact interaction of corundum particles with the surface of a wall made of steel St3 is studied in the present paper by numerical calculations, the results of which are given below. The dependence of a_{nc} on the velocity and impact angle for this pair is taken from [21] and has the form:

$$a_{nc} = 1 - [1 - \exp(-0.1|\mathbf{V}_c^-|^{0.61})](v_c^-/|\mathbf{V}_c^-|). \tag{6}$$

The relation (6) is valid in the range of impact velocities from 50 to 300 m/s.

The slide resistance coefficient C_f essentially depends on the orientation of the particle relative to the surface before the impact. Based on heuristic considerations and preliminary numerical experiments, the following dependence is proposed for this coefficient:

$$C_f = \exp[(\mathbf{r}_c \cdot \mathbf{V}_{\rm p}^-)/|\mathbf{r}_c|\,|\mathbf{V}_{\rm p}^-|]. \tag{7}$$

For other particles and materials, a different dependence for a_{nc} and, possibly, a slightly different relation for C_f should be used.

The set of equations (3)−(7) is closed. It is non-linear and can be solved numerically by an iterative method with respect to the components of the vectors $\Delta\mathbf{\Omega}_{\rm p}$, $\Delta\mathbf{V}_c$ and $\Delta\mathbf{V}_{\rm p}$, knowing which one can determine the translational and rotational velocities of the particle after the impact as follows:

$$\mathbf{V}_{\rm p}^+ = \mathbf{V}_{\rm p}^- + \Delta\mathbf{V}_{\rm p}, \quad \mathbf{\Omega}_{\rm p}^+ = \mathbf{\Omega}_{\rm p}^- + \Delta\mathbf{\Omega}_{\rm p}.$$

This model of impact of the particle with the wall is semi-empirical. It includes the experimental dependence (6) for a_{nc} and the relation (7) obtained as a result of heuristic considerations and selection for the coefficient C_f.

Since the proposed model, along with the laws of mechanics, is based on assumptions that cannot be strictly justified a priori, it is required to study its adequacy by comparing the calculation results with experimental data that are not used in formulating the model. Further, this model is applied to the calculation of the rebound parameters of a large number of particles of different variable forms, and the results are compared with the experiment on the average values of the normal and tangential velocities of the center of mass of particles after rebound for a corundum/steel pair over the entire range of impact angles $0 \le \alpha_1 \le 90°$.

2. Numerical study of particle rebound from a surface and comparison with experimental data

Rebound (reflection) of a particle is a complex process in which an incident particle may collide with the surface several times before it flies off. The model described above was used to determine the translational and angular velocities of the particle after each collision during the rebound.

As already noted, non-spherical particles, in contrast to spherical ones, can experience several collisions due to twisting as a result of the first impact. The translational and rotational motion of particles between collisions with the wall is considered as purely inertial without taking into account the environment (carrier gas). This assumption is valid if the characteristic time of impact interaction of particles with the surface is much less than the time of dynamic relaxation of particles in the medium. This means that the impulse of the force acting on the particle during the impact from of the carrier medium will be much less than the impulse of the force of the impact interaction of the particle with the wall.

All the results presented below were obtained for non-rotating particles before the impact and the impact velocity $V_{\rm p}^- = 200$ m/s, which corresponds to the experimental conditions [21]. Each time, the orientation of the particle before the first impact with the surface was assumed to be random and equally probable.

We introduce the parameters a, b and c, which are related to the particle sizes in the directions of the principal axes of inertia L_ξ, L_η and L_ζ as follows: $L_\xi = 2a$, $L_\eta = 2b$ and $L_\zeta = 2c$ (for ellipsoidal particles a, b and c – are the values of the semi-axes). For particles of the same form, we fix L_ξ, and the ratios b/a and c/b are assumed to be independent random variables from the range [0.5, 1] distributed according to the normal law with ensemble average 0.8 and standard deviation 0.2. A series of calculations were carried out in which the rebound of particles of each individual form with varying parameters b/a and c/b was numerically

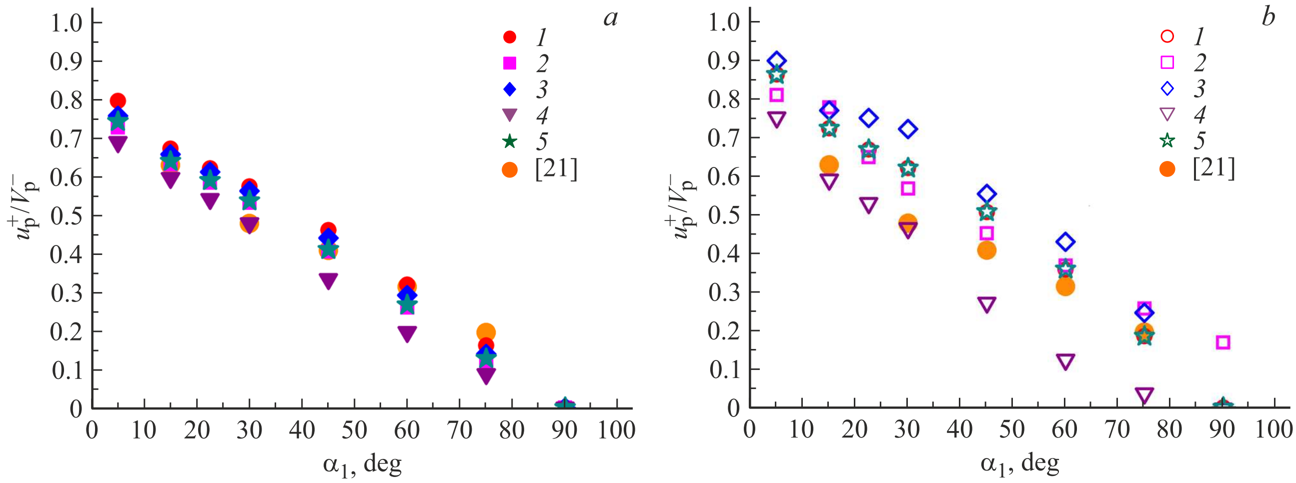

Figure 3. Average (a) and most probable (b) values of u_p^+/V_p^- as functions of impact angle α_1 for particles of various forms *1* — ellipsoids, *2* — prisms, *3* — flat-topped prisms, *4* — tetrahedra) and for a homogeneous mixture of these particles (*5*) in comparison with the experimental data [21].

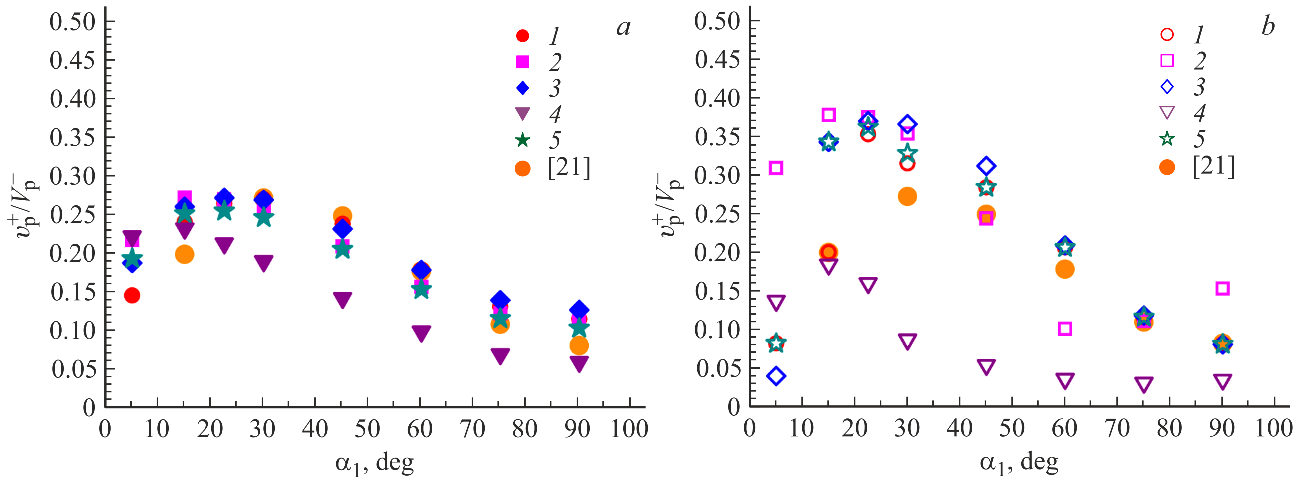

Figure 4. Average (a) and most probable (b) values of υ_p^+/V_p^- as functions of impact angle α_1 for particles of various forms (*1* — ellipsoids, *2* — prisms, *3* — flat-topped prisms, *4* — tetrahedra) and for a homogeneous mixture of these particles (*5*) in comparison with the experimental data [21].

simulated for 10^8 particles at each angle of incidence α_1, and the statistical characteristics of reflected particles were found. The specified number of particles in each series of calculations ensured the statistical stability of the results.

To validate the proposed model of impact interaction of particles with a surface, a mixture of particles of different forms (Fig. 1) in equal proportions was considered, meanwhile, the form parameters of particles of individual types were varied. Such a mixture of particles of various forms with variable parameters b/a and c/b, in our opinion, is closer to that used in experiments [21] than a mixture of particles of the same form. After calculating the statistical characteristics of the reflected particles of various forms, the statistical characteristics for a homogeneous mixture of particles were calculated (the proportions of particles of various forms in the mixture were assumed to be the same).

On the basis of statistical processing of the calculations results, the distributions of particle rebound velocities were obtained at various angles of incidence α_1 and the average and most probable values of the rebound velocities and their normal and tangential components in the impact plane (XY) were found. Normally, the numerical results are compared with the experimental data on the restitution coefficients of the tangential $a_\tau = u_p^+/u_p^-$ and normal $a_n = -\upsilon_p^+/\upsilon_p^-$ velocities of the particle center of mass. However, since $u_p^- \to 0$ for $\alpha_1 \to 90^\circ$ and $\upsilon_p^- \to 0$ at $\alpha_1 \to 0$ it is more convenient to relate the velocities u_p^+ and υ_p^+ not to these values before the impact, but to the total velocity of the particle V_p^-. Results for the average and most probable values of the tangential u_p^+/V_p^- and normal υ_p^+/V_p^- velocities of the particles after the rebound are shown in Fig. 3 and 4. It can be seen that the average and

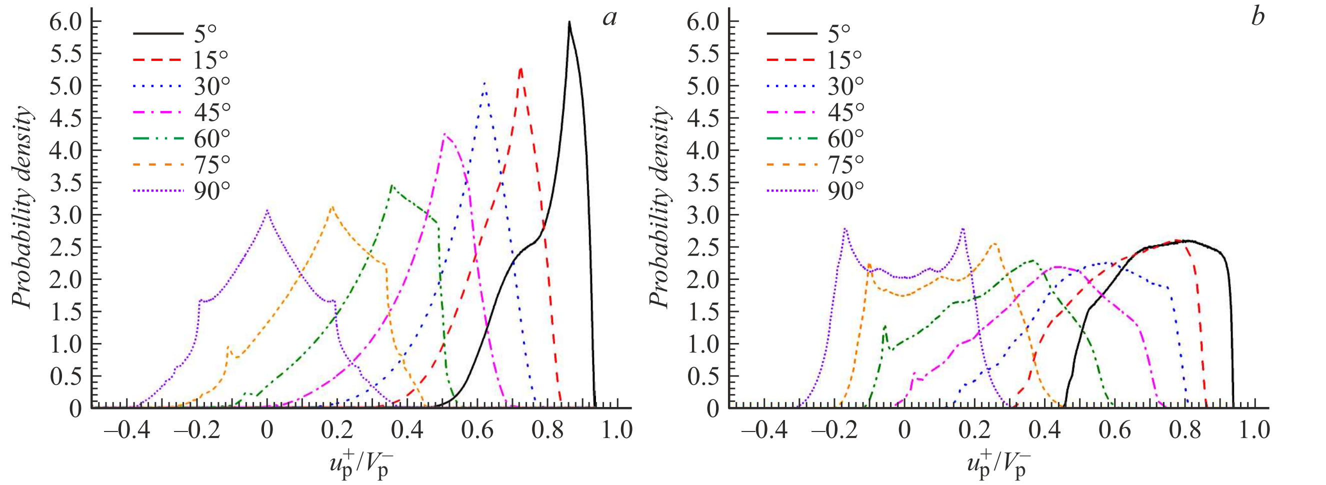

Figure 5. Density functions of the tangential rebound velocity u_p^+/V_p^- for ellipsoidal (a) and prismatic (b) particles at different impact angles α_1.

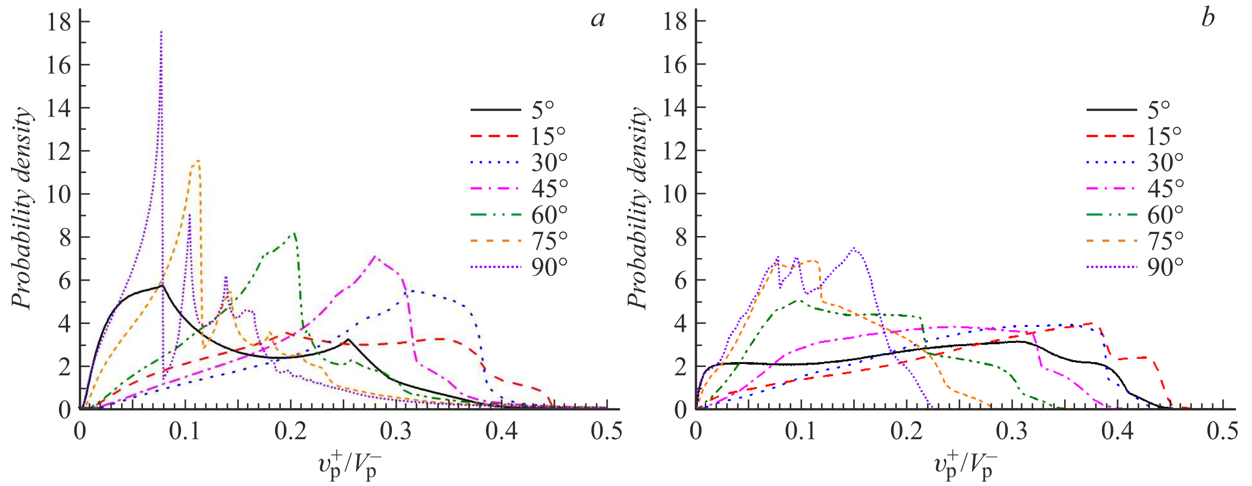

Figure 6. Density functions of the normal rebound velocity v_p^+/V_p^- for ellipsoidal (a) and prismatic (b) particles at different impact angles α_1.

most probable values differ significantly from each other for all impact angles. The experimental data from [21] are shown in both figures.. Dependences obtained for a mixture of particles of various forms are generally in better agreement with experimental data than for particles of individual forms. The presented results show that, firstly, as expected, a mixture of particles of different types of forms better corresponds to a real dispersed phase, and secondly, the proposed impact model allows calculating the average velocity of reflected particles with a sufficiently high accuracy.

The significant difference between the average values of the tangential and normal velocities u_p^+ and v_p^+ from their most probable values is explained by the form of the distribution density functions of these parameters. As an example, the distribution density functions of tangential (Fig. 5) and normal (Fig. 6) velocities for ellipsoidal and prismatic particles are presented.

Distribution of reflected particles by the number of collisions with the surface in the process of rebound for the normal impact %

Form of the particles	Number of collisions in reflection					
	1	2	3	4	5	> 5
Ellipsoids	42.2	31.2	8.9	4.3	2.6	10.8
Prisms	13.3	40.1	19.5	14.6	5.4	7.1
Flat-topped prisms	25.9	37.7	17.8	9.7	4.8	4.1
Tetrahedrons	5.2	12.8	9.3	7.9	8.3	56.5

Let us focus on the fact that for all particles, except for prismatic ones, the most probable value of the tangential velocity u_p^+/V_p^- at normal impact ($\alpha_1 = 90°$) is zero. The specified difference for prismatic particles is due to the

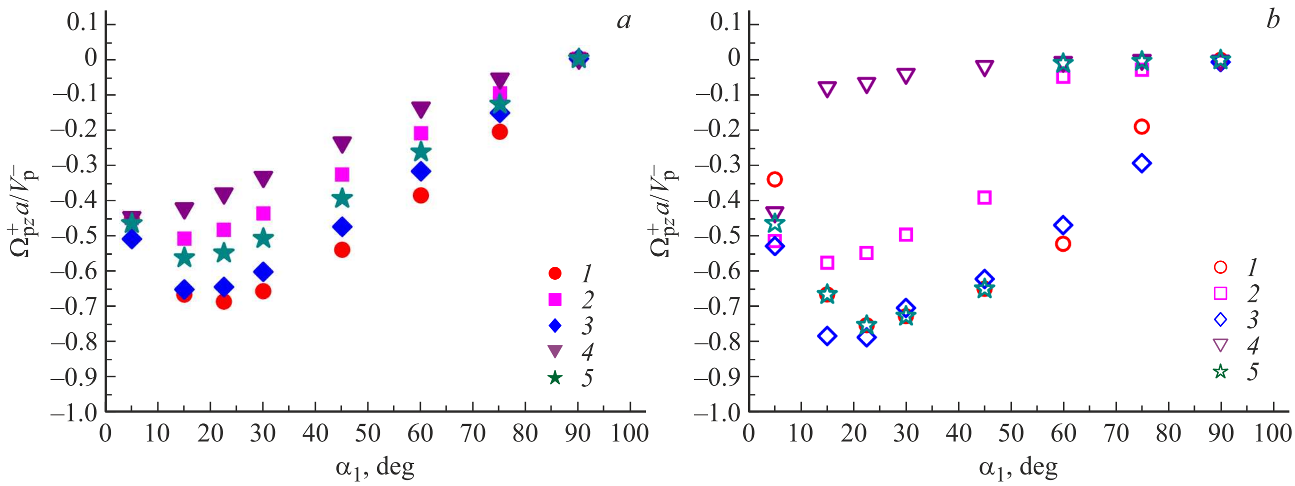

Figure 7. Average (a) and most probable (b) values of Z-components of the angular velocity vector of particles at different impact angles α_1 for particles of various forms (*1* — ellipsoids, *2* — prisms, *3* — flat-topped prisms, *4* — tetrahedra) and for a homogeneous mixture of these particles (*5*).

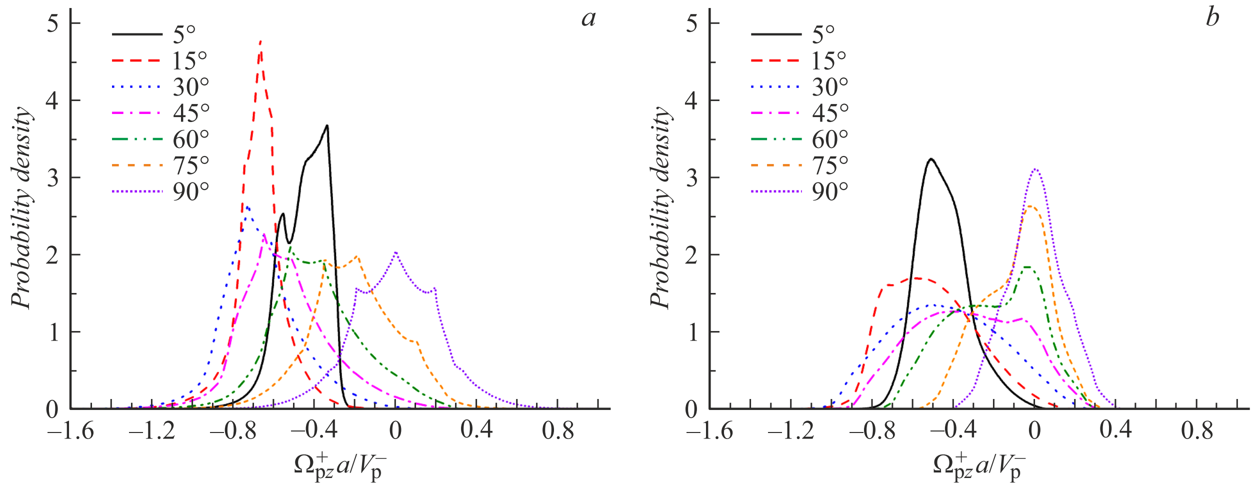

Figure 8. Density functions $\Omega^+_{pz}a/V^-_p$ at the rebound of ellipsoidal (a) and prismatic (b) particles at different impact angles α_1.

fact that the distribution density function for them has a minimum at $\alpha_1 = 90°$ (Fig. 5) and two identical maxima symmetric with respect to zero. Figure 3 shows the most probable value of u^+_p/V^-_p, corresponding to a positive maximum.

The table illustrates the statistics of the number of collisions of particles of various forms during the rebound in the case of a normal impact ($\alpha_1 = 90°$). It can be seen from the presented results that the fraction of particles that experienced only one collision with the surface is less than 50% even for ellipsoidal particles; for other forms, double and multiple collisions prevail. As the impact angle α_1 decreases, the fraction of particles of all considered forms that experienced one collision during the rebound does not change monotonically, and in all cases a noticeable fraction of particles (from 25 to 70%) experience two or more collisions.

An important parameter of the rebounded particles is their angular velocity, since it affects the Magnus shear force and, consequently, the paths of the reflected particles. Experimental data on the determination of the angular velocity of particles at the moment of rebound are not known to the authors. Figure 7 shows the results of calculations of the non-dimensional average angular velocity $\Omega^+_{pz}a/V^-_p$ and its most probable value during the rebound of particles of various forms and for a mixture of particles.

Although the average and most probable values of $\Omega^+_{pz}a/V^-_p$ for all particle forms do not exceed zero, the fraction of particles with $\Omega^+_{pz}a/V^-_p > 0$ increases with the angle α_1. As an example, Fig. 8 shows the distribution density functions of the Z-component of the non-dimensional angular velocity for reflected ellipsoidal and prismatic particles at different impact angles α_1. As expected, for $\alpha_1 = 90°$ the distribution densities of $\Omega^+_{pz}a/V^-_p$

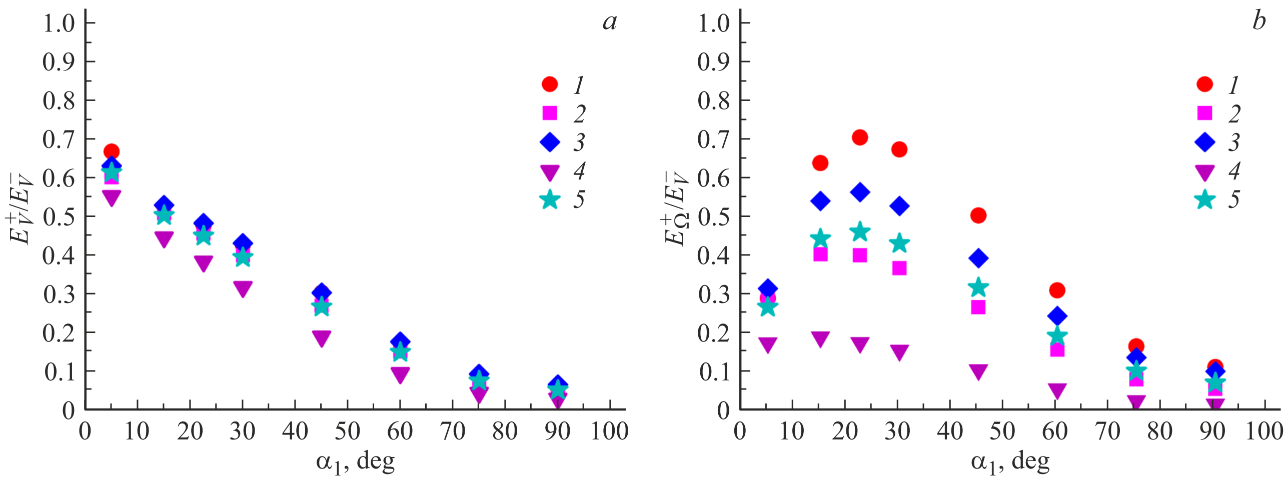

Figure 9. Average values of relative translational (a) and rotational (b) kinetic energies of reflected particles (*1* — ellipsoids, *2* — prisms, *3* — flat-topped prisms, *4* — tetrahedra, *5* — mixture).

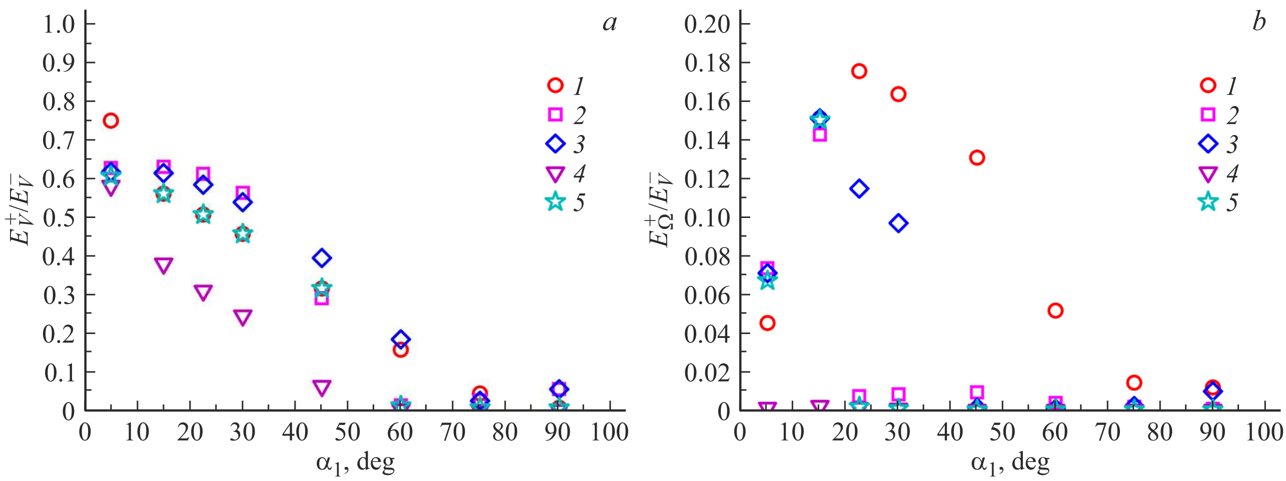

Figure 10. Most probable values of relative translational (a) and rotational (b) kinetic energies of reflected particles (*1* — ellipsoids, *2* — prisms, *3* — flat-topped prisms, *4* — tetrahedra, *5* — mixture).

are symmetric with respect to zero. Distribution densities $\Omega_{px}^+ a/V_p^-$ and $\Omega_{py}^+ a/V_p^-$ are symmetric with respect to zero at any angles α_1.

When the particles bounce, their kinetic energy decreases significantly due to the fact that the impact is inelastic and the deceleration of the particle in the tangential direction is taken into account. Determination of the particle energy loss is important for evaluating heating and abrasive surface erosion. At the rebound, the particle energy is redistributed between the translational and rotational components. Fig. 9 shows the dependences of the average relative kinetic translational E_V^+/E_V^- and rotational E_Ω^+/E_V^- energy of reflected particles on the impact angle α_1. The translational energy decreases monotonically as α_1 increases. The rotational energy has a significant maximum at $\alpha_1 \simeq 15{-}20^\circ$ for all individual particle forms and for their mixture. The greatest rotational energy is obtained for ellipsoidal particles and particles in the form of flat-topped prisms.

The distribution densities of the translational and rotational energies are extremely unequal, as a result of which the most probable values of these energies (Fig. 10) differ significantly from the average ones.

It should be noted that all the above non-dimensional rebound parameters do not depend on the particle size.

3. Scattering of non-spherical particles upon rebound from a surface

Non-spherical particles colliding with a surface at a given angle α_1 (Fig. 2) with the same translational and angular velocities V_p^- and Ω_p^- bounce in different directions. This phenomenon is called the scattering of reflected particles. It is caused by the random orientation of particles in space before the first collision. The rebound process is rather complicated, since the particle may collide with the surface

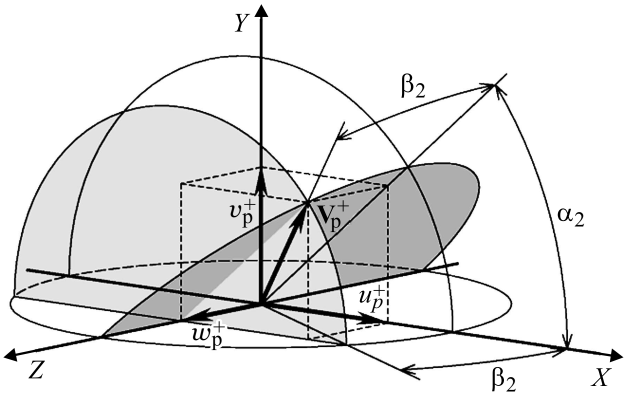

Figure 11. Angles α_2 and β_2 that determine the direction of particle rebound.

several times before it flies off (see the table in Sec. 2). Consider the impact plane XY (the plane containing the vector V_p^- and the normal to the surface). The direction of particle rebound is determined by two angles α_2 and β_2 (Fig. 11). Angle α_2 changes from 0 to π, and angle β_2 from $-\pi/2$ to $\pi/2$.

Let N be a number of incident particles with fixed $\mathbf{V}_{p1}$, $\mathbf{\Omega}_{p1}$ and α_1, while $dN(\alpha_2, \beta_2, d\alpha_2, d\beta_2)$ be a number of particles reflected in the direction determined by the angle intervals $[\alpha_2, \alpha_2 + d\alpha_2]$ and $[\beta_2, \beta_2 + d\beta_2]$. Let us introduce the distribution density function $I(\alpha_2, \beta_2)$ of reflected particles over the angles α_2 and β_2 using the relation $I(\alpha_2, \beta_2)\, d\alpha_2\, d\beta_2 = dN(\alpha_2, \beta_2, d\alpha_2, d\beta_2)/N$. The surface $I(\alpha_2, \beta_2)$ constructed in spherical coordinates is a spatial $(3D)$ scattering indicatrix, which describes the probability density of particle reflection in the direction determined by the angles (α_2, β_2). For prismatic particles and flat-topped prismatic particles the form of the spatial scattering indicatrices obtained at normal impact are shown in Fig. 12.

For two-dimensional flows, the two-dimensional $(2D)$ scattering indicatrix, which describes the distribution of reflected particles along the angle α_2, i.e., in the projection onto the impact plane is of great interest. Two-dimensional scattering indicatrices for a mixture of particles of various forms are shown in Fig. 13. As can be seen, the reflected particles are significantly scattered at all impact angles. The strongest scattering is observed at normal impact. Scattering indicatrices that are unimodal in the range of angles $0 \leq \alpha_1 < 70°$ become bimodal at $\alpha_1 \geq 70°$. Spherical particles are always reflected regularly at the same angle α_2.

Conclusion

A model of impact of solid dispersed non-spherical particles with a smooth surface is proposed. The model is based on the equations of momentum and angular momentum and the assumption that the impulse of the tangential force at the point of contact of the particle with the surface is proportional not only to the impulse of the normal force, but also to the average tangential velocity of the point of contact. For the proportional constant the relation is proposed that takes into account the orientation of the particle relative to the surface at the moment of collision.

It has been found that the process of rebound of a non-spherical particle is rather complicated and can be accompanied by several collisions of the particle with the surface before the particle flies off. The average values of the normal and tangential rebound velocities obtained as a result of direct statistical modeling for a mixture of particles of

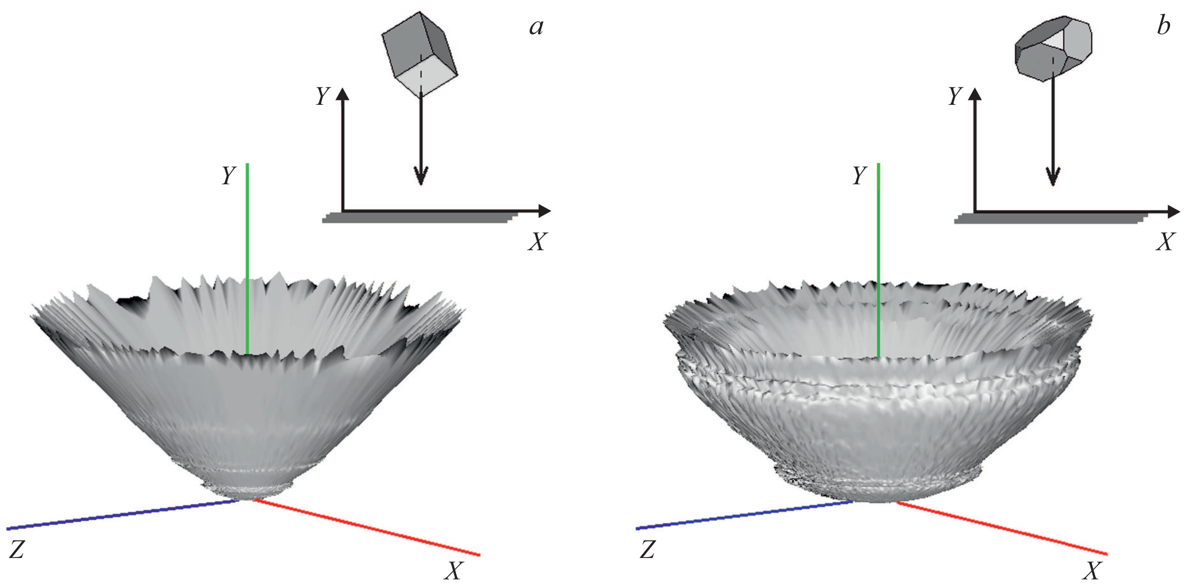

Figure 12. Spatial scattering indicatrices of prismatic particles (a) and flat-topped particles (b) at normal impact.

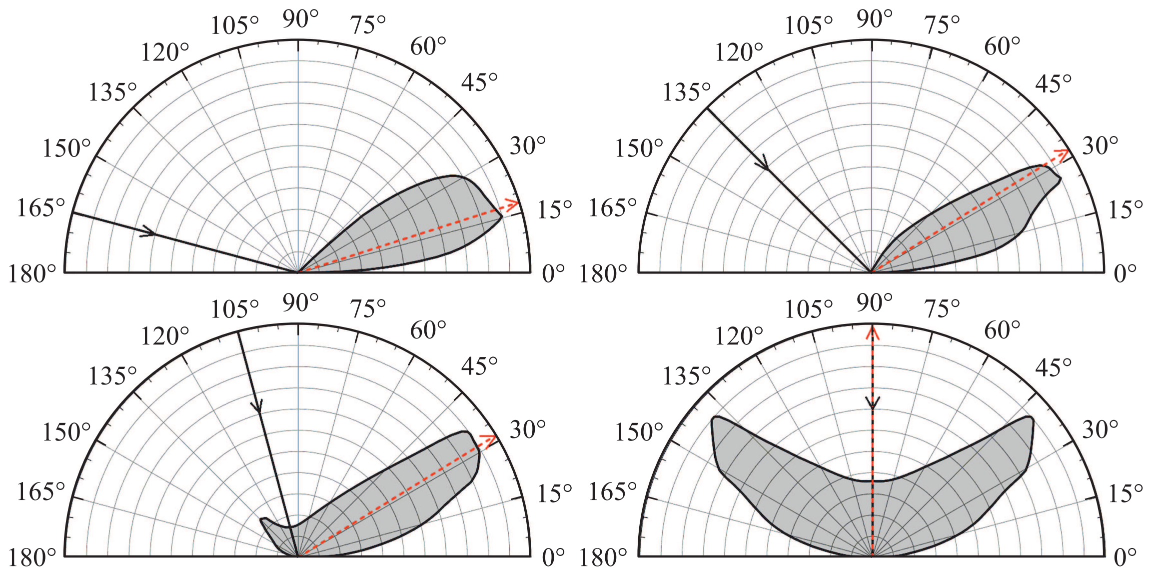

Figure 13. Two-dimensional scattering indicatrices of reflected particles in the plane of impact (XY). The direction of motion of the incident particles is shown by the black arrow; the direction of reflection of spherical particles is shown by the red dashed line.

different variable shapes using the proposed impact model demonstrated good agreement with the known experimental data for impact velocities typical of aerodynamic problems during aircraft motion in a dusty atmosphere. It is important to note that for particles of individual forms, the agreement with the experimental data turned out to be worse than for a mixture of particles of different forms.

The scattering characteristics of reflected particles are studied in detail. Dependences on the angle of impact of the average and most probable values of the rebound parameters of particles of individual variable forms and a homogeneous mixture of such particles are obtained. The distribution density functions of these parameters are found. Spatial and two-dimensional (in the plane of impact) scattering indicatrices of reflected particles are obtained. It should be underlined that, in models of two-phase flows, particles are almost always assumed to be spherical. Such particles bounce off the surface regularly, and no scattering occurs upon reflection. At the same time, scattering of reflected particles is always observed in experiments.

Meanwhile, it should be noted that this model is not universal for any materials of particles and surfaces. It can be seen from the results given in [21] for corundum particles and different materials of target plates that at close values of the coefficient a_n, the values a_τ can differ significantly. Since the coefficient a_τ is not included in the proposed model, it can be assumed that for a surface made of another material, in addition to using a different empirical dependence for a_{nc}, it will also be necessary to modify the relation for C_f.

When constructing a more universal model of the impact interaction of a non-spherical particle with a smooth surface, it is required to take into account the dependence of the impact impulse components on the particle orientation, the geometry of the contact surface, and the physical and mechanical properties of materials, that is impossible without an in-depth analysis of the impact interaction using models and methods of mechanics of a deformable solid body.

The main problem in validating models of rebound of non-spherical particles from a solid surface is the lack of reliable experimental data on the characteristics of their scattering.

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Conflict of interest

The authors declare that they have no conflict of interest.

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