07

Highly efficient generation of squeezed states of light based on Laguerre-Gaussian modes in a cavity

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Today, the efficient generation of squeezed states of light seems to be a significant practical problem for various quantum-optical and information applications. In this paper, we investigate the possibility of increasing the efficiency of the generation of states based on the Laguerre-Gaussian light modes in the parametric down conversion due to the optimal choice of the cavity configuration. Analyzing the Heisenberg-Langevin equations for the eigenmodes of the system, we estimate the influence of the geometric parameters of the pump beam and the idler and signal beams on the efficiency of generation of squeezed states and on the degree of squeezing. The calculation for a finite number of modes has shown that the highest theoretically possible degree of squeezing in the system is 15.85 dB.

Keywords: squeezed light, Laguerre-Gaussian modes, orbital angular momentum, optimization of PDC geometric parameters.

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1. Introduction

Currently quantum squeezed states [1] proved to be an important resource for many areas of quantum optics and quantum informatics [2,3], the importance of these states for metrology and quantum computing cannot be overestimated [4,5]. The use of quantum squeezed states is of particular interest in the building of many-particle entangled (cluster) systems for carrying out one-way calculations in continuous variables [6,7]. To build continuous cluster quantum systems the physical objects of various nature are used. For example, cluster states on spin waves inside an ensemble of atoms [8,9], in optomechanical systems [10], etc. were studied. At the same time, the light cluster states are of particular interest, for example, states based on sequences of light pulses from an optical parametric generator [11], or obtained during the generation of an optical frequency combo [12]. Light quantum systems are attractive primarily because they are highly resistant to decoherence. An essential factor is also the presence of a developed element base for such states control for the implementation of logical transformations.

Having quantum squeezed states as a resource, one can build a cluster state and, accordingly, perform an arbitrary Gaussian transformation to continuous variables using only linear optics [13]. However, the requirements for the compression level are quite high: in the paper [14] it is shown that in order to plot error-resistant schemes of quantum computing, the compression of the initial oscillators used to build clusters must reach 20.5 dB. Unfortunately, this level of squeezing is currently not achievable by experiments. Without using the procedure of results post-selection (transforming conversions from deterministic to probabilistic), by now it is experimentally possible to achieve a squeeze of 15 dB [15]. Therefore, the study of the properties of the generation of light squeezed states in terms of the possibility of obtaining high degrees of squeeze remains an urgent challenge for quantum optics.

One promising way to obtain a set of squeezed states is to use Laguerre-Gaussian modes, which have orbital angular momentum (OAM) [16,17]. Pairs of photons formed as a result of spontaneous parametric downconversion (SPDC) are entangled in their OAM projection [18,19], which is determined by the OAM conservation law. The OAM projection can take any integer values, which makes it possible to work in a high-dimensional Hilbert space. All these characteristic properties of many-particle states of light based on Laguerre-Gaussian modes make them an attractive basis for use in information and computer applications.

In our paper we are based on the results obtained in the paper [20], where the problem of generation of many-particle quantum states of light was considered under two-component pumping of a parametric crystal in a resonator. As in the previous paper we study the scheme for generating a many-particle entangled state in a spontaneous parametric downconversion scheme. All analysis is carried out in the supermode language proposed in [21]. However, our approach differs from the work [20] in that we study the features of the quantum properties of multimode light depending on the change in the resonator configuration. We analyze how the control of the ratio between the overdraw width of the generated signal mode and the pump mode affects the efficiency of generation of many-particle entangled state, and also obtain the optimal parameters

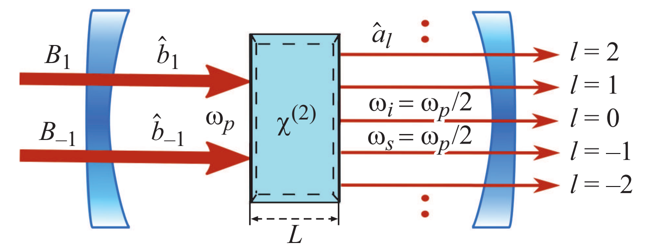

Figure 1. Crystal with quadratic nonlinearity, which provides I-type phase synchronism, is placed in the resonator with spherical mirrors. The resonator pumping consists of two spatial LG modes with OAM equal to $+1$ and -1. A field with a rich mode structure is generated in the resonator according to the values of the OAM projections, but at one frequency ω_p.

of the resonator configuration for generating the squeezed states.

2. Optical parametric transformation of Laguerre-Gauss modes below the generation threshold

2.1. Theoretical model

Let's briefly formulate the statement of the problem under consideration. A thin crystal with quadratic susceptibility $\chi^{(2)}$, which provides I-type synchronism, is placed in the resonator with spherical mirrors (Fig. 1). The eigenmodes of the spherical resonator are the set of Laguerre-Gauss (LG) modes [22]. Recall that the choice of spatial LG modes is dictated by OAM presence of such modes, which makes it possible to single out an infinite number of quantum degrees of freedom corresponding to different numbers of the orbital momentum. The system is pumped with two LG beams U_1^{LG} and U_{-1}^{LG} with OAM projections equal to $+1$ and -1 respectively, which propagate in direction z at frequency ω_p. The spatial profile of such modes is determined by the equality:

$$U_l^{LG} \propto \left(\frac{\rho\sqrt{2}}{w(z)}\right)^{|l|} \exp\left(-\frac{\rho^2}{w^2(z)}\right)$$
$$\times \exp\big(i(k_p z - \omega_p t)\big) \exp(il\varphi), \quad l = \pm 1, \tag{1}$$

where ρ, φ, z is cylindrical coordinates, the integer index $l = \pm 1$ is responsible for the projection of the orbital angular momentum onto the wave propagation axis, $w(z) = w(0)\sqrt{1 + (z/z_R)^2}$ is overdraw width of the transverse field distribution inside the resonator, $w(0)$ is beam overdraw width in the plane $z = 0$, z_R is Rayleigh radius.

Within the framework of this problem, we will consider the case of a collinear degenerate in frequency but nondegenerate in OAM process under conditions of ideal phase synchronism.

In this paper, we will be interested in the subthreshold operation of an optical parametric of generator. Such operation mode makes it possible to neglect the process of pump depletion in the theoretical description of spontaneous parametric scattering. Then the interaction Hamiltonian describing the process of parametric generation has the following form:

$$\hat{H}_I = i\hbar \sum_l (\chi_{l,1-l} B_1 \hat{a}_l^\dagger \hat{a}_{1-l}^\dagger$$
$$+ \chi_{l,-1-l} B_{-1} \hat{a}_l^\dagger \hat{a}_{-1-l}^\dagger) + H.c., \tag{2}$$

where $B_{\pm 1}$ are the numerical amplitudes of the classical pump wave and $\hat{a}_l^\dagger$ ($l = 0, \pm 1, \pm 2, \ldots$) is photon creation operators in the idler and signal modes of the LG, indexed by index l, which comply with the canonical commutation rules:

$$[\hat{a}_l, \hat{a}_{l'}^\dagger] = \delta_{l,l'}. \tag{3}$$

The effective coupling constants $\chi_{l,\pm 1-l}$ are proportional to the overlap integral between the spatial profiles of three modes: the pump mode (with index $+1$ and -1), the signal mode (with index l) and idle mode (with index $\pm 1 - l$). We will discuss in detail the properties of the $\chi_{l,\pm 1-l}$ coupling constants and explore their effect on the optimal choice of system configuration in Sections 2.2 and 2.4.

To construct the Heisenberg equations and study the degree of squeezing of various modes, we will use the approach proposed in the paper [21] to describe the optical frequency comb in a synchronously pumped optical parametric oscillator (SPOPO). We write the interaction Hamiltonian in matrix form:

$$\hat{H}_I = i\hbar \sum_{i,s} M_{i,s} \hat{a}_i^\dagger \hat{a}_s^\dagger + H.c., \quad i + s = \pm 1, \tag{4}$$

where the coupling matrix M has the following form:

$$M = \begin{pmatrix} 0 & 2\chi_{0,1}B_1 & 2\chi_{0,1}B_{-1} & 0 & \cdots \\ 2\chi_{0,1}B_1 & 0 & 0 & 0 & \cdots \\ 2\chi_{0,1}B_{-1} & 0 & 0 & 2\chi_{1,-2}B_1 & \cdots \\ 0 & 0 & 2\chi_{1,-2}B_1 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix}. \tag{5}$$

The matrix elements $M_{i,s}$ contain both information about the correlation between two modes $\hat{a}_i$ and $\hat{a}_s$, and information about the geometry of the modes, expressed in coupling constants $\chi_{l,\pm 1-l}$, and also carry a dependence on the classical pump amplitude $B_{\pm 1}$.

The paper [23] shown, that it is convenient to analyze a many-particle entangled quantum state in the resonator in the language of the system's eigenmodes, called by the authors as supermodes. The quantum states of these modes turn out to be uncorrelated squeezed states.

Let?s represent the Hamiltonian (4) in diagonal form, i.e., in terms of uncorrelated squeezed eigenmodes of the Hamiltonian $\hat{s}_n$:

$$\hat{H}_I = i\hbar \sum_n \Lambda_n (\hat{s}_n^\dagger)^2 + H.c., \tag{6}$$

$$\hat{s}_n^\dagger = \sum_i m_{n,i}\hat{a}_i^\dagger, \tag{7}$$

where Λ_n are eigenvalues, and $m_{n,i}$ is the n-th element of the i-th eigenvector of the coupling matrix M. The Λ_n spectrum determines the number of uncorrelated degrees of freedom and gives information about the degree of squeeze in $\hat{s}_n$ modes.

In the paper [21] the authors considered a multimode system based on a frequency comb consisting of 10^6 modes, i. e. analyzed the quasi-continuous spectrum. At the same time, numerical analysis and experimental results showed that out of the entire continuum of SPOPO radiation modes, effective squeeze was detected only in the first six of them [23]. In our problem, we want to investigate the properties of quantum modes analytically. Therefore, we will confine ourselves to discussing of only five highest modes $\hat{s}_i$ $(i = 1, \ldots, 5)$. In terms of the theoretical description this means that we need to cut off the infinite coupling matrix M, limiting ourselves to only first five rows and columns. The theoretical justification of this approach is given in [20]. From the experimental point of view, this can be realized using special holograms that allow selective screening of the mode with a certain OAM [24,25].

Later in Section 2.3 we will analyze the features of the system of Heisenberg-Langevin equations for supermodes $\hat{s}_n$.

2.2. Effective constants of interaction between modes

The overlapping of the transverse profiles naturally depends both on the overdraw width $w(z)$ of these profiles and on the values l of the OAM projections of three waves: the pump beam, the signal beam, and the idler beam. The OAM conservation law imposes a relation on the projections l. This leaves only two degrees of freedom to vary. We will be interested in the possibility to control effective coupling constants by changing the geometry of the fields. We will examine how a change in the beam overdraw ratio affects the efficiency of generation of a many-particle entangled state. Paper [26] shown that by choosing the resonator circuit it is possible to ensure the equality of the overdraw of the idler and signal modes $w_i(0) = w_s(0)$. For simplicity, we choose just such configuration of the resonator.

We define the effective coupling constants $\chi_{l,\pm1-l}$ as

$$\chi_{l,\pm1-l} = \int d^3\mathbf{r}\, U_{\pm1}^{LG}(\mathbf{r}) U_l^{LG*}(\mathbf{r}) U_{\pm1-l}^{LG*}(\mathbf{r})$$

$$= g_l \lambda^2 \left(\frac{1}{2+\lambda^2}\right)^{\frac{2+|l|+|1-l|}{2}}, \tag{8}$$

where the control parameter $\lambda = w_i(0)/w_p(0)$ is determined by the ratio of the overdraw width of detected idler mode $w_i(0)$ to the overdraw width of pump mode $w_p(0)$ in the plane $z = 0$. In the coefficients g_l we included all other constants that do not depend on the control parameter λ.

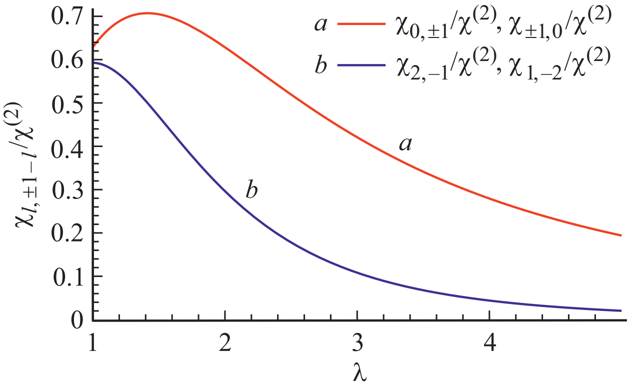

Figure 2. Normalized coupling constants $\chi_{l,\pm1-l}$ vs. parameter λ. On the graph: coefficient $\chi^{(2)}$ is non-linear susceptibility.

Efficient signal detection during spontaneous parametric scattering is directly related to the geometric relationship between the pump mode and the generated mode. For any given set of generation parameters the detecting device is prepared in such a way that the overdraw width of detected mode cannot exceed the overdraw mode of pump mode, hence parameter $\lambda \geq 1$.

Note that the coupling constants determine the efficiency of the spontaneous parametric process. Therefore, when constructing a many-particle entangled state, the coefficients $\chi_{l,\pm1-l}$ will determine the efficiency of generation of squeezed states and the correlation properties of the system. Further we will study the effect of the coefficients $\chi_{l,\pm1-l}$ upon varying the parameter λ on the limit squeeze ratio for the system under consideration.

Fig. 2 shows the dependences of the normalized coupling constants on the λ parameter for the pump wave indices $l = \pm1$. As can be seen from Fig. 2, for different projections l the maxima of the coupling constants $\chi_{l,\pm1-l}$ do not coincide, but do not exceed the value 1. For example, note that the coupling constant $\chi_{0,1}$ reaches its maximum at $\lambda = \sqrt{2}$, while the constant $\chi_{1,-2}$ is for $\lambda = 1$. Studies of the system parameters for two values $\lambda = \sqrt{2}$ and $\lambda = 1$ were done in paper [20].

2.3. Heisenberg-Langevin equations for supermodes in subthreshold mode

The Hamiltonian (6) allows us to construct an infinite system of independent Heisenberg-Langevin differential equations for modes $\hat{s}_n$:

$$\dot{\hat{s}}_n = -\gamma\hat{s}_n - 2\Lambda_n\hat{s}_n^\dagger + \hat{f}_{\hat{s}_n}. \tag{9}$$

Here γ is the relaxation rate of field modes in the resonator, which is chosen to be the same for all considered modes, $\hat{f}_{\hat{s}_j}$ is Langevin stochastic sources determined by zero mean values and non-zero second-order correlation functions of the following form:

$$\langle \hat{f}_{\hat{s}_n}(t)\hat{f}_{\hat{s}_n}(t')\rangle = 2\Lambda_n\delta(t-t'). \tag{10}$$

For five considered supermodes $\hat{s}_i$ $(i = 1, \ldots, 5)$ the system of equations is solved analytically and the eigenvalues Λ_i have the form

$$\Lambda_1 = 0, \ \ \Lambda_2 = -\frac{\gamma\xi}{2} = -\Lambda_3,$$

$$\Lambda_4 = -\frac{\gamma\sqrt{2\mu^2 + \xi^2}}{2} = -\Lambda_5. \tag{11}$$

Here, the parameters μ and ξ were introduced, which are responsible for the generation threshold of modes with different indices:

$$\mu = \alpha \cdot \frac{\chi_{0,1}}{\chi_{0,1}(\lambda_1)}, \quad \alpha = \frac{2\chi_{0,1}(\lambda_1)B_1}{\gamma}, \tag{12}$$

$$\xi = \beta \cdot \frac{\chi_{1,-2}}{\chi_{1,-2}(\lambda_2)}, \quad \beta = \frac{2\chi_{1,-2}(\lambda_2)B_1}{\gamma}, \tag{13}$$

where $\lambda_1 = \sqrt{2}$ and $\lambda_2 = 1$ are the values for which the coupling constants $\chi_{0,1}(\lambda_1)$ and $\chi_{1,-2}(\lambda_2)$ reach their maximum. This form of recording of the threshold parameters μ and ξ makes it possible to separate various factors affecting the efficiency of mode generation. For simplicity, we assume that the pump wave amplitudes B_1 and B_{-1} are real and equal to each other, $B_{-1} = B_1$.

Introducing the threshold parameters μ and ξ as two cofactors, we wanted to emphasize the effect of two different factors on the generation efficiency of many-particle state: the generation threshold and the magnitude of supermode fluctuations. The coefficients α and β characterize the threshold values of the resonator system. Factors written as fractions can be considered as geometric factors of the resonator system that affect the efficiency of parametric interaction and, as a consequence, the magnitude of supermode quadrature fluctuations. Note (Fig. 2) that the parameters α and β are interrelated by a numerical factor: $\beta = 16/(9\sqrt{3})\alpha$.

Eigenvalues $\Lambda_1, \ldots, \Lambda_5$ eigenmodes $\hat{s}_1$,...,$\hat{s}\{5\}$ uniquely characterize the quantum properties of Gaussian modes. From the fact that the eigenvalue $\Lambda_1 = 0$, it can be said that the mode $\hat{s}_1$ unlike the others remains in the vacuum state during the evolution of the system. Along with this, the eigenvalues of the supermodes pair $\hat{s}_2$ and $\hat{s}_3$, and the pair $\hat{s}_4$ and $\hat{s}_5$ are equal in absolute value and opposite in sign: $\Lambda_2 = -\Lambda_3, \Lambda_4 = -\Lambda_5$. Therefore, the quadratures of the modes $\hat{s}_2$ and $\hat{s}_3$, as well as the modes $\hat{s}_4$ and $\hat{s}_5$ respectively will be equally squeezed or stretched. Therefore, further we will discuss in detail the analysis of one supermode from each pair ($\hat{s}_2$ and $\hat{s}_4$).

From the form of the Heisenberg-Langevin equations (9) one can obtain preliminary information about the generation threshold. The generation threshold for the given mode is characterized by the ratio of the effective pumping of a given mode to the relaxation of field from the resonator (the same for all modes). Studying the near-threshold region of resonator generation, we note that the eigenvalues of Λ_2 and Λ_4 turn out to be different, i. e., we can state that the modes $\hat{s}_2$ and $\hat{s}_4$ will reach the generation threshold at different intensities of effective pumping.

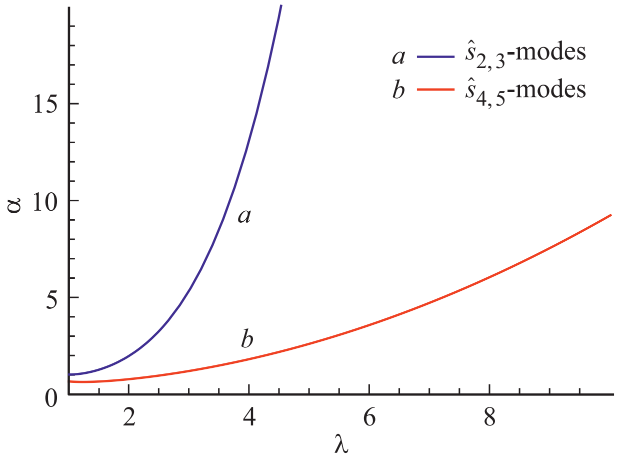

Figure 3. Generation threshold vs. λ parameter for modes $\hat{s}_{2,3}$ (a) and $\hat{s}_{4,5}$ (b). It is shown that for the supermodes $\hat{s}_{2,3}$ (a) and $\hat{s}_{4,5}$ (b) this dependence is monotonic.

Let's go to the quadrature components of the supermodes $\hat{s} = \hat{x} + i\hat{y}$, as well as to the quadrature components of the Langevin noises $\hat{f}_{\hat{s}_j} = \hat{f}'_{\hat{s}_j} + i\hat{f}''_{\hat{s}_j}$ for $j = 1, \ldots, 5$. The Heisenberg-Langevin equations for quadrature supermode components are first-order differential equations. Using the Fourier transforms, go from the system of differential equations to the system of linear algebraic equations for the Fourier transforms.

The measured values in the study of squeeze are the power spectra of quadrature fluctuations, which can be found from the solutions of these equations. We confine ourselves to the case of analyzing solutions at zero frequency, which corresponds to stationary laser generation. Since this problem is considered in the subthreshold mode, the average values of the field operators inside the resonator are equal to zero. Let?s denote the intracavity normally ordered averages of squares of fluctuations of the quadrature components at zero frequencies as $\langle: |\delta\hat{x}_j^0|^2 :\rangle$, $\langle: |\delta\hat{y}_j^0|^2 :\rangle$. Then, using the known input-output relation, they can easily be related to the corresponding averages outside the resonator $\langle|\delta\hat{X}_j^0|^2\rangle$, $\langle|\delta\hat{Y}_j^0|^2\rangle$:

$$\begin{pmatrix} \langle|\delta\hat{X}_j^0|^2\rangle \\ \langle|\delta\hat{Y}_j^0|^2\rangle \end{pmatrix} = \frac{1}{4} + 2\gamma \begin{pmatrix} \langle: |\delta\hat{x}_j^0|^2 :\rangle \\ \langle: |\delta\hat{y}_j^0|^2 :\rangle \end{pmatrix}, \quad j = 1, \ldots, 5. \tag{14}$$

The formulas obtained above make it possible to write analytical expressions for the spectral powers of quadrature fluctuations of supermodes outside the resonator:

$$\langle|\delta\hat{X}_1^0|^2\rangle = \langle|\delta\hat{Y}_1^0|^2\rangle = \frac{1}{4}, \tag{15}$$

$$\langle|\delta\hat{X}_2^0|^2\rangle = \frac{(\xi - 1)^2}{4(\xi + 1)^2} = \langle|\delta\hat{Y}_3^0|^2\rangle, \tag{16}$$

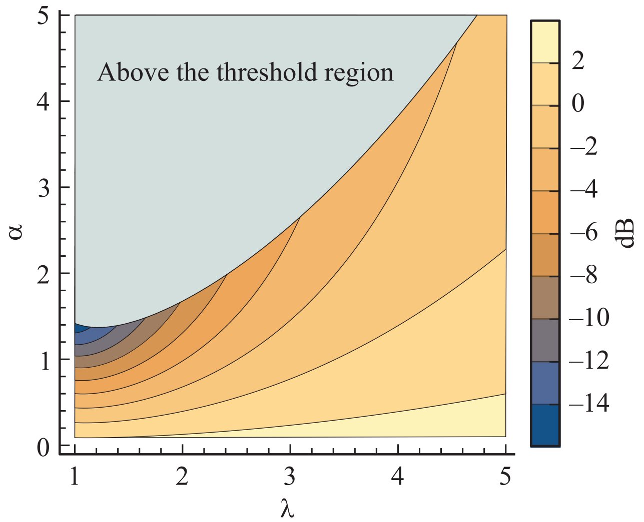

Figure 4. Sum of the quadrature power spectrum $\langle|\delta\hat{X}_2^0|^2\rangle + \langle|\delta\hat{X}_4^0|^2\rangle$ when approaching the generation threshold vs. λ parameter and α threshold parameter. The generation threshold boundary is shown. It is found that the power spectrum of quadratures monotonically depends on the parameter λ and on the threshold parameter α.

$$\langle|\delta\hat{X}_4^0|^2\rangle = \frac{(\sqrt{2\mu^2+\xi^2}-1)^2}{4(\sqrt{2\mu^2+\xi^2}+1)^2} = \langle|\delta\hat{Y}_5^0|^2\rangle. \quad (17)$$

Expression (15) confirms the earlier conclusion that the first mode is in a vacuum state, regardless of the system parameters. When constructing multimode states of light from modes with OAM, the first eigenmode should be excluded so as not to impair squeeze in the system. As can be seen, the third and fifth modes are squeezed in the Y-quadrature, while the second and fourth modes are squeezed in the X-quadrature.

To generate a five-mode field, the resonator generation threshold will be determined by the relation $2\mu^2 + \xi^2 = 1$ [20]. Analyzing expression (17), one can see that the numerator of the corresponding power spectrum tends to zero when approaching the generation threshold. That is, when approaching the threshold, the vacuum fluctuations in the fourth and fifth modes are compensated in maximum degree, and the oscillators themselves tend to ideally squeezed state. On the other hand, the second and third modes do not achieve ideal squeeze at the threshold. As follows from expression (16), for ideal squeeze of the second and third modes, the condition $\xi = 1$ shall be met. But already at $\xi^2 > 1 - 2\mu^2$ the system cannot be described by subthreshold Heisenberg-Langevin equations (9), it is necessary to consider pumping depletion and to describe the problem in terms of the above-threshold mode. Thus, when approaching the threshold part of the modes of multimode system will always remain finitely squeezed, and this fact must be taken into account when choosing the operating parameters in a multimode configuration.

2.4. Analysis of the power of quadrature fluctuations of supermodes

To analyze the quantum properties of the system, it is necessary to determine the limits of applicability of our solutions and explore the most interesting i.e. near-threshold region.

The purpose of the analysis is to find the geometric parameters of the resonator system (namely, the control parameter λ) that provide the maximum possible squeeze of all considered modes. As we saw above, the magnitude of quadrature fluctuations is determined by the proximity of each of the independent supermodes of multimode state to the common generation threshold of this state. In its turn, the value of the generation threshold itself is determined by the parameter λ.

Since the threshold constraints for the $\hat{s}_{2,3}$ and $\hat{s}_{4,5}$ supermodes are different, the degree of squeeze for these modes is also different. We want to get the highest possible squeeze level in as many modes as possible. Therefore, of greatest interest is the situation when the generation thresholds for two modes approach each other. Behavior of the generation threshold depending on the λ parameter for modes $\hat{s}_{2,3}$ (a) and $\hat{s}_{4,5}$ (b) is shown in Fig. 3. The Figure shows that as the values of λ decrease, the curves approach each other, i.e. the threshold values for different modes become close to each other. At the same time, as λ increases, the values of the threshold parameters increase, but at different rates. Accordingly, work in the region of large λ is inefficient.

At the generation threshold the power spectra of quadrature fluctuations reach the corresponding minima, which characterizes the limiting degree of mode squeeze. To study the convergence of the generation thresholds of $\hat{s}_{2,3}$ and $\hat{s}_{4,5}$ supermodes, it is necessary to analyze the minimum of the corresponding quadrature power spectrum sum $\langle|\delta\hat{X}_2^0|^2\rangle + \langle|\delta\hat{X}_4^0|^2\rangle$ considered with equal weights. As it was demonstrated above, two factors affect the compression squeeze of the multimode system: the threshold value (determined by the parameter α (12)) and the control geometric parameter λ. Fig. 4 shows the behavior of the minimum of sum of the generation power spectrum $\langle|\delta\hat{X}_2^0|^2\rangle + \langle|\delta\hat{X}_4^0|^2\rangle$ (at the generation threshold) depending on the parameters α and λ. It is shown that the minimum values of the sum of fluctuations of power spectra are reached in the limit $\lambda \to 1$. Just in this region the generation thresholds of modes $\hat{s}_{2,3}$ and $\hat{s}_{4,5}$ mode closer, and, consequently, this is the region with the maximum effective degree of squeeze of supermode quadratures. We found that the ultimate squeeze at $\lambda \to 1$ is 15.85 dB.

In the paper [20] a cluster state was constructed and studied based on the Lugger-Gauss modes at $\lambda = 1$. The results of our paper make it possible to state that specifically this configuration of a linear cluster state consisting of four nodes is optimal from the point of view of unidirectional quantum calculations.

3. Conclusion

In this paper, we analyze the quantum features of multimode field with orbital angular momentum generated in the process of spontaneous parametric downconversion under complex two-component pumping and varying field geometries.

Our goal was to build a recipe for generating a maximum squeezed multimode state based on Laguerre-Gaussian modes. This article does not discuss the issue of stability of beams with OAM, since it is discussed in detail in papers [27,28]. However, note that the presence in the spatial profile of beams with l exceeding 2 in absolute value, of phase singularity points can lead to instability of such vortex systems. This instability can be compensated by introducing l-selective losses [29]. In this case, by choosing sufficiently large transverse dimensions of the resonator, it is possible to ensure stationary generation of beams with an arbitrarily large OAM value. Note also that the presence of a nonlinear crystal in the resonator system under study can in itself be a stabilizing factor. The paper [30] shows that the stability of structures with high values of topological charge can be ensured by the presence of a nonlinear medium.

It was shown that in order to construct a protocol of generation the maximum squeezed state it is necessary to use the basis of supermodes, since these modes will have limit quantum properties. From the constructed Heisenberg-Langevin equations for eigenmodes the power spectra of quadrature fluctuations of supermodes are obtained. It is shown that the threshold constraints for $\hat{s}_{2,3}$ and $\hat{s}_{4,5}$ supermodes are different. Besides, out of the five considered modes only four modes turned out to be squeezed; the highest mode is in a vacuum state. Thus, the considered system has four true quantum degrees of freedom.

It is found that the generation threshold and the value of supermode quadrature fluctuations depend on both the control geometric parameter λ, and on the threshold parameter α. It is shown that the minimum values of the power spectrum of fluctuations are reached in the limit of $\lambda \to 1$, in this region the theoretically possible squeeze level is 15.85 dB. Thus, we can state that the most efficient region for the generation of squeezed states is a configuration with the same values of the overdraw of the pump mode and the generated resonator modes.

The construction carried out permits generalization to any given number of modes in a multimode state. However, it can be assumed that the number of modes increasing will worsen the quadrature squeeze of the supermodes, since different modes will be subject to different threshold constraints.

Acknowledgments

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Conflict of interest

The authors declare that they have no conflict of interest.

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