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Energy of electron-hole relative motion in exciton in external electric field in thick GaAs plate

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The dependence of the energy of the relative electron-hole motion in an exciton on the magnitude of applied electric field for various thicknesses of an ideal flat semiconductor plate is theoretically calculated. It is shown that the variation of the plate thickness significantly affects the dependence of the energy on the electric field. The effect should be observed in plates whose thickness exceeds the Bohr exciton radius by two orders of magnitude.

Keywords: exciton, electrick field, thick plate.

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1. Introduction

Exciton states in semiconductor layered heterostructures, such as quantum wells (QW), have been widely studied by now by optical spectroscopy methods [1–3]. The influence of QW layer thickness on exciton states manifests itself in size quantization of exciton energy. If QW width is by an order greater than the exciton Bohr radius (the so-called wide wells), size quantization of exciton motion as a unity takes places [4–13]. If QW width is comparable to or less than the Bohr radius (the so-called narrow wells), a restriction of well sizes has an impact on the relative motion of the electron and hole in the exciton, see, e.g., [14–17].

External fields applied to a QW lead to additional effects. For instance, uniaxial pressure in narrow and wide QWs leads to a monotonous increase of exciton bond energy [18], to an energy shift of levels of exciton size quantization [19,20], effect of approach of masses of heavy and light hole excitons and phase inversion of excitonic peculiarities in optical spectra [21,22]. A magnetic field applied in the Vogt geometry causes an effective increase of the exciton mass [23–30]. In the Faraday geometry, an effect of dependence of the excitonic g-factor on the exciton wave vector is observed [31–35].

The present paper is dedicated to studying the influence of an electric field on the states of electron and hole relative motion in thick GaAs layers. Most earlier papers are devoted to studying the influence of an electric field more detailed field on the exciton in narrow QWs. An electric field in such wells leads to the Stark quantum effect, see, e. g., [36–44]. The influence of an electric field on the exciton in wide QWs has also been studied in recent years. It was shown in [45] that an electric field causes a modification of the wave function of the electron and hole relative motion inside the exciton. Paper [46] analyzed the influence of an electric field on light-excitonic interaction in a wide QW. Papers [47–49] are dedicated to studying the effect of phase inversion of exciton spectral oscillations in a wide QW under the action of an electric field.

As already stated, optical spectroscopy methods usually reveal peculiarities (resonances), related to size quantization of exciton motion, in wide QWs. However, the wider the QW, the lower the amplitude of the observed resonances, and they cannot be distinguished on the background of spectral noises in case of sufficiently large thickness of the QW layer. Optical spectra of such thick layers have only a resonance whose energy is close to the energy of the main excitonic transition in a bulk material. The motional energy of excitons, which correspond to this resonance, is low as compared to its bond energy and that's why it need not be considered. In the present paper we will restrict ourselves to a consideration of only such exciton states. The layers where only the main excitonic resonance is seen will be hereinafter called semiconductor plates. As far as the authors of the present paper know, the first attempt at a theoretical description of the influence of an electric field on the states of electron and hole relative motion in a semiconductor plate was made in paper [50]. It was devoted to studying the influence of thickness of a gallium arsenide layer, containing an exciton, on the dependence of light-excitonic interaction on the applied electric field. It was shown that this influence is most vivid for GaAs layer thicknesses over 1000 nm, that is, in the layers designated above as semiconductor flat plates. In the present paper we consider the influence of the semiconductor plate thickness on the energy of electron and hole relative motion in the exciton.

2. Exciton Hamiltonian in a perfect semiconductor plate in an external homogeneous electric field

We will consider an exciton in a wide plane GaAs layer. We assume that the layer boundaries are perpendicular to the z axis, the direction of which coincides with the crystallographic axis [001]. We assume that the interfaces are impermeable for electrons and holes, that is, a plate is a potential well with infinitely high walls. The electric field vector $\mathbf{F}$ is assumed to be also directed along the z axis, that is, $F = F_z$, $F_x = F_y = 0$. The x and y coordinate axes are directed along the tetragonal axes [100] and [010], respectively. The exciton states, observed in an optical experiment in a cubic crystal, are formed by the states of a doubly degenerate conduction band of symmetry Γ_6 and a fourfold degenerate valence band of symmetry Γ_8. The exciton Hamiltonian is formed by Hamiltonians of free electrons and holes (H_c and H_v) in these bands, their Coulomb interaction and the external field potential energy:

$$\hat{H}_X = E_g + \hat{H}_c + \hat{H}_v - \frac{e^2}{\varepsilon_0 r} - eFz. \tag{1}$$

Here E_g is the semiconductor band gap, e is the electron charge, ε_0 is the semiconductor background permittivity, $r = |\mathbf{r}_e - \mathbf{r}_h|$ is the distance between electron and hole ($\mathbf{r}_e, \mathbf{r}_h$ are radius-vectors of electron and hole), and value $z = z_e - z_h$ is the projection of this distance onto the z axis.

The Hamiltonians of free electrons and holes in the cubic-lattice crystal have the form [51,52]:

$$\hat{H}_e(\hat{k}_e) = \frac{\hbar^2 \hat{k}_e^2}{2m_e}, \tag{2}$$

$$\hat{H}_h(\hat{k}_h) = \left(\frac{\hbar^2}{2m_0}\right)\Bigg[\gamma_1 \mathbf{I} \hat{k}_h^2 - 2\gamma_2 \sum_{\alpha=x,y,z} J_\alpha^2 \left(\hat{k}_{h\alpha}^2 - \frac{\hat{k}_h^2}{3}\right) - 4\gamma_3 \sum_{\alpha\neq\beta} \{J_\alpha, J_\beta\} \hat{k}_{h\alpha}\hat{k}_{h\beta}\Bigg]. \tag{3}$$

Here the indices α and β take on values x, y, z; dimensionless constants γ_1, γ_2 and γ_3 are the Luttinger parameters; m_e is the effective electron mass in the conduction band; m_0 is the electron mass in vacuum. Matrices J_α describe the hole spin states in the valence band.

It should be noted that the optical spectroscopy methods for high-quality GaAs/AlGaAs heterostructures make it possible to observe the spectral peculiarities related to size quantization of exciton motion, in the layers having a thickness up to 1000 nm [9]. However, in the present paper we restrict ourselves to an analysis of the exciton ground state only (its motion as a unity is neglected) and study the influence of the layer boundaries only on the states of relative motion of the electron and hole which make up the exciton. Therefore we do not consider the kinetic energy of exciton motion. Operators of the wave vector of a free electron and a hole can be expressed in this case via operators of relative motion in an exciton using the formulas

$$\hat{k}_{h\alpha} = -\frac{1}{\hbar}\hat{p}_\alpha, \quad \hat{k}_{e\alpha} = \frac{1}{\hbar}\hat{p}_\alpha. \tag{4}$$

Here $\hat{p}_\alpha = -i\hbar\partial/\partial\alpha$ is the operator of the relative motion momentum where $\alpha = x_e - x_h,\ y_e - y_h,\ z_e - z_h$ are coordinates of electron and hole relative motion; x_e, y_e, z_e and x_h, y_h, z_h are coordinates of the free electron and hole. By substituting expressions (4) in exciton Hamiltonian (1) with account of (2) and (3), we can obtain the exciton Hamiltonian in the notation of relative motion coordinates.

It follows from expression (3) that the tensor of effective hole mass for a semiconductor with a degenerate valence band, such as GaAs, is anisotropic [52], and the heavy and light exciton states cannot be considered independently. The exciton state energy in such cases is calculated by the perturbation theory method, see, for instance, [53]. A zero-order Hamiltonian is chosen so that its intrinsic energies are identical for states of heavy and light excitons. This is achieved by including a spherically symmetrical part of the Luttinger Hamiltonian, which is common for a heavy hole and a light hole, into the excitonic Hamiltonian in the zero approximation. The first addend in expression (3) is usually chosen as such a spherically symmetrical part of a hole Hamiltonian. All the other operators of the Luttinger Hamiltonian are considered as perturbations, see [53].

It will be demonstrated below that in our paper the zero-order Hamiltonian of the exciton includes, in addition to the other operators, the term $-eFz$ from expression (1). Our analysis shows that the inclusion of only the first addend of the Luttinger Hamiltonian into the zero-order Hamiltonian provides an incorrect result in this case. It arises during calculation of the contribution to the energy from the perturbation, which is obtained from second term (3) after substitution of (4) in it and has the form

$$\hat{V} = -2\gamma_2\left(\frac{1}{2m_0}\right)\left[J_z^2\frac{2}{3} - J_x^2\frac{1}{3} - J_y^2\frac{1}{3}\right]\hat{p}_z^2. \tag{5}$$

An analysis shows that the absolute magnitudes of the matrix elements of such a perturbation at a sufficiently large preset external field increase proportionally to the increase of plate width L. Upon a transition to the bulk crystal $L \to \infty$, consequently, the matrix element of operator (5) must also tend to infinity.

Such non-physical behavior of perturbation matrix element (5) is due to the fact that it is a correction for the kinetic energy of electron and hole relative motion along the field direction. The kinetic energy of this motion changes due to the corresponding change in potential energy. That's why if the zero-order Hamiltonian includes $-eFz$, all the operators in expression (5) must also be included in this Hamiltonian. Otherwise, the energy will have the above-mentioned non-physical dependence on plate width. Obviously, all the terms included in (5) cannot be included in the zero-order Hamiltonian simultaneously for light and heavy excitons. To do this for a heavy exciton only, the following value should be chosen in the zero-order approximation as a spherically symmetrical hole mass which is identical for heavy and light holes

$$m_h = \frac{m_0}{\gamma_1 - 2\gamma}. \tag{6}$$

Then, after substituting expressions (2) and (3) into expression (1) with account of (4), the zero-order Hamiltonian both the for the heavy and for the light excitons is as follows:

$$\hat{H}_X^{(0)} = E_g + \frac{\hat{p}^2}{2\mu} - \frac{e^2}{\varepsilon_0 r} - eFz. \tag{7}$$

Here $\mu = m_e m_h/(m_e + m_h)$ is the reduced mass in which m_h is spherically symmetrical hole mass (6). Thereat, the complete exciton Hamiltonian will be represented as

$$\hat{H}_X = E_g + \hat{H}_X^{(0)} + \hat{V}_1 + \overline{V}_2. \tag{8}$$

The third term in the right member of this expression is the combination of the terms, obtained from the second sum in (3), which are not included in zero-order Hamiltonian (7). The matrix operator of this perturbation is:

$$\hat{V}_1 = \frac{\gamma_2}{m_0}\left[\frac{1}{2}\begin{pmatrix} 3 & 0 & -\sqrt{3} & 0 \\ 0 & 1 & 0 & -\sqrt{3} \\ -\sqrt{3} & 0 & 1 & 0 \\ 0 & -\sqrt{3} & 0 & 3 \end{pmatrix}\hat{p}_x^2 + \frac{1}{2}\begin{pmatrix} 3 & 0 & \sqrt{3} & 0 \\ 0 & 1 & 0 & \sqrt{3} \\ \sqrt{3} & 0 & 1 & 0 \\ 0 & \sqrt{3} & 0 & 3 \end{pmatrix}\hat{p}_y^2 + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\hat{p}_z^2\right]. \tag{9}$$

The upper and lower lines of these matrices describe heavy excitons, and the other lines describe the states of light excitons.

The fourth term in expression (8) is obtained from the third sum in expression (3) after substitution of (4) in it and has the form

$$\hat{V}_2 = -2\,\frac{\gamma_3}{m_0}\left(\sum_{\alpha\neq\beta}\{J_\alpha, J_\beta\}\hat{p}_\alpha\hat{p}_\beta\right). \tag{10}$$

Perturbations (9) and (10) are calculated on the eigen functions of zero-order Hamiltonian (7). In order to find them, it is convenient to change over to parabolic coordinates, as was done in papers [54,55]:

$$\xi = r + z, \quad \eta = r - z, \quad \varphi = \arctan(x/y). \tag{11}$$

An equation for eigen values and eigen functions of Hamiltonian (7) is broken down into two independent differential equations which have the form [54,55]:

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\,\frac{df_\nu(\eta)}{d\eta}\right) + \left(-\frac{m^2}{4\eta^2} - \frac{\nu'}{\eta} + \frac{\mu R}{2\hbar^2} - \frac{\mu e F\eta}{4\hbar^2}\right)f_\nu(\eta) = 0,$$

$$\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\,\frac{dg_\nu(\xi)}{d\xi}\right) + \left(-\frac{m^2}{4\xi^2} + \frac{\nu'}{\xi} + \frac{\mu R}{2\hbar^2} + \frac{\mu e F\xi}{4\hbar^2}\right)g_\nu(\xi) = 0. \tag{12}$$

In the equations, R is the exciton energy minus E_g, while ν and ν' are the parameters of separation of variables. The exciton wave function can be written down as

$$\phi = A f_\nu(\eta) g'_\nu(\xi) e^{\pm im\varphi}, \ \nu' = \nu + \mu e^2/(\varepsilon_0\hbar^2), \tag{13}$$

where A is the normalization constant. It should be noted that our consideration is limited only to the states which most efficiently interact with light. In case of GaAs these are the states with $m = 0$. Therefore we will omit in Hamiltonian (12) the term, which contains m, and the exponent in (13).

An analysis shows that function f_ν quickly decreases with an increase of η. Then a sufficiently large value of $\eta_{\max}$, for which $f_\nu(\eta_{\max}) \approx 0$, can be chosen for this decaying part of the wave function. If thickness L of the semiconductor layer, where an exciton is located, exceeds the value $\eta_{\max}$, then this layer can be considered infinite for this decaying part of the wave function [50]. An analysis shows that the following can be chosen for GaAs

$$\eta_{\max} = 120\,\text{nm}. \tag{14}$$

At the same time, the function $g_\nu(\xi)$ at a sufficiently large value of F has a long oscillating „tail“, the amplitude of which remains finite at any finite value of ξ, see papers [46,50]. This means that the oscillating part of wave function (13) in a perfect semiconductor plane layer must become zero only on the layer boundary. It follows from expressions (11) that this point corresponds to the values

$$\xi_{\max} = 2L + \eta_{\max}. \tag{15}$$

Thus, the eigen value problem (12) for an exciton in a perfect QW or a semiconductor plate in an external electric field must be solved with the boundary conditions

$$f_\nu(\eta_{\max}) = 0, \tag{16}$$

$$g'_\nu(\xi_{\max}) = 0,$$

where $\eta_{\max}$ and $\xi_{\max}$ are described by expressions (14) and (15).

Fig. 1 schematically shows the profiles of potential energy in the space of relative motion of an electron-hole pair along the coordinate $z = z_e - z_h$ in fields $F = 0$ and $F = 1\,\text{kV/cm}$. The central region highlighted in the figure approximately corresponds to a Coulombic well of mutual attraction between an electron and a hole. In the absence of a field, the potential profile for the wave function of electron and hole relative motion is a symmetrical infinitely deep potential well, tunneling outside which is virtually impossible. An external field leads to a skew of the Coulomb potential. Thereat, the potential barrier in the right part of the figure lowers down, and the Coulombic well potential barrier becomes tunneling-transparent for an electron and a hole along the coordinate $z = z_e - z_h$. In the coordinates which describe the electron and hole position, z_e and z_h, this means that these particles may go beyond the Coulombic

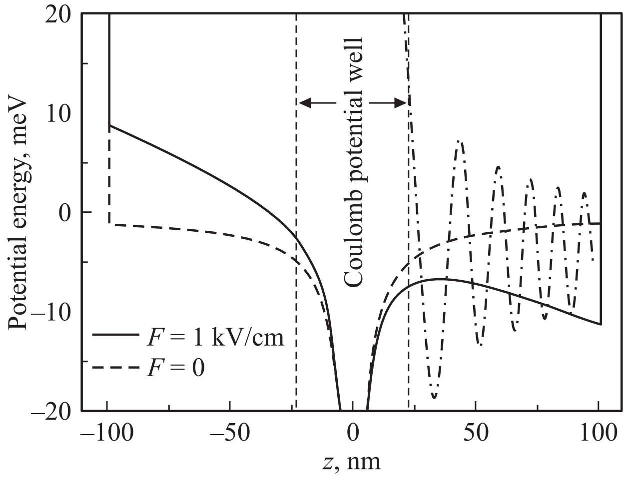

Figure 1. Potential profile of a plate with the thickness of $L = 100$ nm along the coordinate $z = z_e - z_h$ of electron and hole relative motion. The region highlighted by vertical dashed straight lines approximately corresponds to a Coulombic potential well of mutual attraction between an electron and a hole. The thin dashed-and-dotted line schematically shows the oscillating wave function of electron and hole relative motion at $F = 1$ kV/cm.

well and go away from it in opposite directions. However, the electron and hole which left the Coulombic well of their mutual attraction under the action of an external field, always return to it due to reflection from potential barriers on the plate boundaries. In other words, the states of electron and hole relative motion in a layer with an infinitely high potential barrier on the boundaries are stationary in case of any field.

Having calculated the eigen wave functions of zero-order Hamiltonian (7), let us consider matrix elements (9) and (10). An analysis shows that the matrix elements of the operators of squared momentum components $p_x^2 = \langle\phi|\hat{p}_x^2|\phi\rangle$ and $p_y^2 = \langle\phi|\hat{p}_y^2|\phi\rangle$, calculated on wave functions (13), are equal to each other. Due to this reason, the non-diagonal elements of matrix operator (9) are mutually canceled out and do not contribute to the exciton energy. The calculations of the matrix elements of operators $\hat{p}_\alpha \hat{p}_\beta$, included in (10), on ground state wave function (13) result in $\langle\phi|\hat{p}_x\hat{p}_y|\phi\rangle = \langle\phi|\hat{p}_x\hat{p}_z|\phi\rangle = \langle\phi|\hat{p}_y\hat{p}_z|\phi\rangle = 0$. From this it follows that operator (10) in the first approximation of the perturbation theory does not contribute to exciton energy. Thus, all the non-diagonal matrix elements of complete Hamiltonian (8), which describe the mixing of the heavy and light exciton states, turn out to be equal to zero. This makes it possible to further consider the heavy and light excitons independently from each other in the first approximation. We will restrict ourselves to consideration of a heavy exciton only, because this type of states manifests itself most vividly in optical experiments, see, e.g. [4–12].

It should be noted that the third term in perturbation (9) contains some operators from (5), which are not included in zero-order Hamiltonian (7). This term leads to the above-mentioned non-physical dependence of energy on plate width for a light-hole exciton only, because all its matrix elements, which describe the heavy-hole exciton states, are zero. More specifically, the non-physical dependence of energy on layer width for a light-hole exciton can be also eliminated by choosing a spherically symmetrical mass of a hole in the zero approximation. To do so, the value $m_h = m_0/(\gamma_1 + 2\gamma_2)$ should be substituted instead of value (6) as the hole mass included in quantity μ in expression (7). Thereat, the problem of non-physical behavior of energy from L arises for the states of a heavy-hole exciton in this case.

It is also necessary to consider the contribution to the heavy-hole exciton energy made by the diagonal matrix elements of the first two terms in expression (9). After omitting the light-hole exciton from consideration, this perturbation for a heavy-hole exciton can be written down in the one-dimensional form

$$\hat{V}_1 = \frac{3\gamma_2}{2\mu}\,(\hat{p}_x^2 + \hat{p}_y^2). \tag{17}$$

An analysis shows that the energy correction, described by this perturbation, does not depend on the applied field and plate width. It reduces the absolute magnitude of energy of the ground excitonic state by a constant magnitude equal approximately to 20% of the value of its bond energy in the absence of a field. Since this correction does not have a direct impact on the effects discussed here, we do not consider it below.

3. Results and discussions

We have calculated the energies of relative motion of the electron and hole of the heavy exciton ground state in electric fields from 0.5 to 4 kV/cm in plane semiconductor GaAs layers from 200 to 4000 nm. The following values of the constitutive parameters were used in the calculations: $m_e = 0.067m_0$, $\gamma_1 = 6.8$, $\gamma = \gamma_2 = 2.3$ [52] and $\varepsilon_0 = 12.56$ [56]. The results are shown in Fig. 2. The figure shows that exciton energy R in fields $F \geq 0.5$ kV/cm decreases in case of field increase, due to the Stark effect in the exciton. Moreover, as can be seen in the figure, the exciton energy depends not only on the applied field, but also on the plate thickness. The greater the plate thickness L, the less steeper is the dependence of excitonic state energy on F.

The dependences shown in Fig. 2 can be approximated by functions which have the form

$$R = A(L)F^2 + B(L)|F| + C(L). \tag{18}$$

All three coefficients A, B, C here depend on the semiconductor layer thickness. Their dependences are shown in Fig. 3.

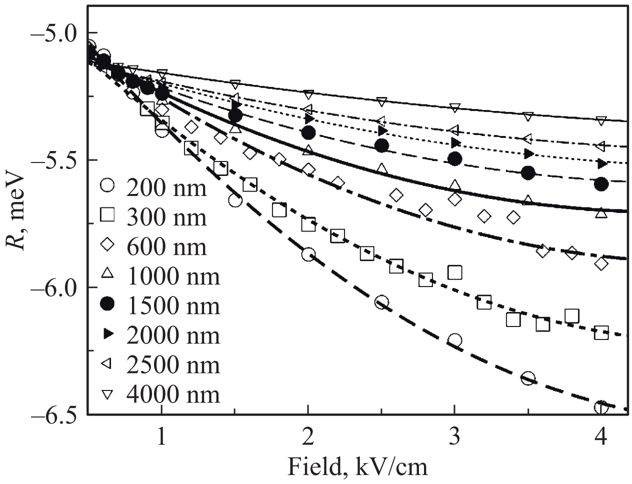

Figure 2. Dependence of exciton energy R on electric field for fields of $0.5 \leq F \leq 4\,\mathrm{kV/cm}$ in plates $L = 200$, 300, 600, 1000, 1500, 2500 and 4000 nm thick.

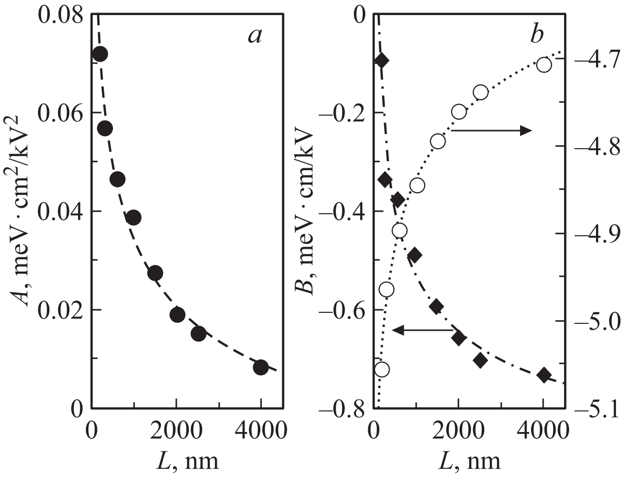

Figure 3. Dependences of coefficients *a*) A and *b*) B and C in expression (18) on plate thickness L. The horizontal arrows in panel *b* show the axes to which the corresponding curves pertain; the dependences of coefficients B and C are shown by rhombic and round empty points, respectively. The smooth curves were plotted using formulas (19).

The dependences on L for the coefficients in expression (18) can be also approximated by the following expressions:

$$A = \frac{\alpha_A}{L^{1/6}} + \beta_A,$$

$$B = \frac{\alpha_B}{L^{1/6}} + \beta_B,$$

$$C = \frac{\alpha_C}{L^{1/3}} + \beta_C. \tag{19}$$

Here

$\alpha_A = 0.38\,\mathrm{meV \cdot cm^{13/6}/kV^2}$, $\beta_A = -0.087\,\mathrm{meV \cdot cm^2/kV^2}$,

$\alpha_B = -3.73\,\mathrm{meV \cdot cm^{7/6}/kV}$, $\beta_A = -0.85\,\mathrm{meV \cdot cm/kV}$

and

$$\alpha_C = 3.16\,\mathrm{meV \cdot cm^{1/3}}, \quad \beta_C = -5.26\,\mathrm{meV}.$$

Thus, numerical modeling provides expressions which can be used to describe the exciton energy in GaAs plates in the field of $F \geq 0.5\,\mathrm{kV/cm}$.

To analyze the nature of flattening of the $R(F)$ dependences in case of plate thickness increase (see Fig. 2), we calculated the matrix elements from the last three operators in Hamiltonian (7) separately for the field of $F = 4\,\mathrm{kV/cm}$ and different values of L. Let us introduce the following notations:

$$T = \left\langle \phi \middle| \frac{\hat{p}^2}{2\mu_h} \middle| \phi \right\rangle,$$

$$U_\mathrm{Q} = \left\langle \phi \middle| \frac{e^2}{\varepsilon_0 r} \middle| \phi \right\rangle,$$

$$U_\mathrm{F} = \langle \phi | eFz | \phi \rangle. \tag{20}$$

Exciton energy is the sum of these matrix elements

$$R = T + U_\mathrm{Q} + U_\mathrm{F}. \tag{21}$$

The dependences of T, U_Q, U_F and R on plate thickness L are shown in Fig. 4. Contributions of kinetic energy of electron and hole relative motion T and external field potential energy U_F greatly depend on L, approximately according to the linear law. But these two contributions are opposite in signs, close in absolute magnitude and to a great degree compensate each other. This is an expected result since the kinetic energy of relative motion outside a Coulombic hole increases chiefly due to a decrease of the electron-hole pair potential energy in an external field. However, mutual

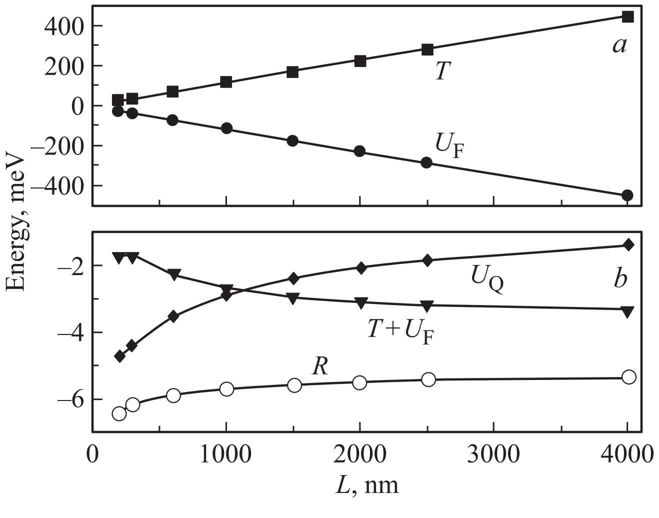

Figure 4. *a*) Dependences of matrix elements T (black squares) and U_F (black circles) on plate thickness L for the field of $F = 4\,\mathrm{kV/cm}$. *b*) Similar dependences of the contribution to energy U_Q (black rhombi), sum $T + U_\mathrm{F}$ (black triangles) and exciton energy $R(L)$ (hollow circles).

compensation is not complete, so that their sum $T + U_{\mathrm{F}} < 0$ makes a non-zero contribution to exciton energy. The difference of sum $T + U_{\mathrm{F}}$ from zero is conditioned by the fact that energy T also increases due to a decrease of potential energy of the Coulombic interaction between the electron and hole U_{Q}. The contribution to the Coulomb potential energy decreases in modulus with an increase of L, as shown in Fig. 4, *b*. Consequently, the kinetic energy of electron and hole relative motion T must be decreased by the same value. Therefore the total negative contribution to exciton energy $T + U_{\mathrm{F}}$ increases in absolute magnitude with an increase of the plate thickness, as shown in Fig. 4, *b*.

A decrease of the contribution of U_{Q} with an increase of L is due to the fact that the wave function of electron and hole relative motion (13) in a strong field is weakly localized in a Coulombic well, see Fig. 1. The greater the plate thickness, the longer the oscillating tails of the wave function and the weaker such localization. Consequently, an increase of L reduces the contribution of the Coulombic interaction between the electron and hole U_{Q} into the exciton energy, as shown in Fig. 4, *b*.

Since the decrease of the contribution of U_{Q} in modulus is greater than the increase of the contribution of $T + U_{\mathrm{F}}$ in modulus, their total sum $R < 0$, according to expression (21), must decrease in absolute magnitude with an increase of L under a fixed field. This is observed as a flattening of the $R(F)$ dependences with a plate thickness increase, see Fig. 2.

It should be noted that our results seemingly contradict the previous calculations of exciton bond energy in a bulk material. According to [55], the absolute magnitude of negative energy of exciton bond in a bulk crystal in weak fields increases proportionally to the squared F in compliance with the Stark effect theory. However, the negative value of energy of electron and hole relative motion in strong fields must decrease in modulus and become zero at a certain moment. This occurs because the electric field separates the electron and hole, due to which the average distance between them increases. This weakens their Coulombic interaction and, consequently, the modulus of exciton bond energy decreases and becomes equal to zero at $F \approx 4.2\,\mathrm{kV/cm}$ [55]. This corresponds to exciton ionization under which the electron and hole, which have left the Coulombic well of their attraction, cannot return to it anymore. Nothing of the kind takes place in our model.

To explain this discrepancy between our results and the results obtained in [55], it is to be recalled that we consider an exciton in a wide plate. The electron and hole, which have left the Coulombic well, will always return to it due to reflection from the plate boundaries. Exciton ionization does not occur under any field due to the presence of boundaries having an infinitely high potential. As a consequence, an irreversible breakdown of the exciton into an electron and a hole, accompanied with a decrease of the modulus of energy R with field strengthening, does not occur in a perfect plate. On the other hand, a limiting transition to the case of a non-stationary problem must occur at $L \to \infty$ [55]. Let us briefly discuss this transition.

An electron and a hole in a real crystal have a finite mean free path length l_{mfp} due to the presence of scattering centers and scattering by phonons. As discussed in [50], the value of l_{mfp} can be less than L for a thick semiconductor layer. An irreversible breakdown of the excitonic state will occur as a result of electron and hole scattering outside the Coulombic well, see, e.g., [57,58]. The presence of exciton decomposition makes the problem non-stationary, because the exciton lifetime is limited by the processes of electron and hole scattering. Therefore, the results of solving of stationary problem (12) with boundary conditions (16) become incorrect at $L \gg l_{mfp}$. In our calculations, it is assumed that the following condition is met for the largest considered layers

$$L \le l_{mfp}. \tag{22}$$

If it is met, the problem of an exciton in an electric field in a semiconductor plate can be considered stationary with a good accuracy. It should be noted that the value of l_{mfp} in the simplest model does not depend on the applied field [59]. The mean free path length in high-quality GaAs-based heterostructures can reach $3\,\mu$m at low temperatures [60]. The value is slightly greater for a plate having the maximum considered thickness $L = 4000\,\mathrm{nm}$, but it can be taken approximately equal to $3\,\mu$m. Therefore all the considered values of L meet condition (22). On the other hand, there is a fundamental process of charge carrier scattering related to phonon emission [3]. This scattering channel cannot be eliminated either by improving the plate quality or by reducing the sample temperature. It is assumed within our model that this fundamental scattering process can be neglected.

When criterion (22) is violated, the exciton states become non-stationary, and our approach cannot be used. A non-stationary approach should be used at $L \gg l_{mfp}$, e.g., as described in [55].

It should be noted that the dependence of Stark shift magnitude was previously discussed for narrow wells having a width approximately equal to or less than the exciton Bohr radius in a bulk crystal [39–41]. Our results mean that such a dependence occurs even for semiconductor plates the thickness of which exceeds the exciton Bohr radius by two decimal orders. However, this dependence in narrow wells is opposite to the one described here, namely, the Stark shift in narrow QWs increases quickly with an increase of L, see [39–41]. The physical sense of an increase of the Stark shift value is that the average distance between an electron and a hole in an exciton in narrow wells is smaller than in a bulk crystal, because it is limited by potential barriers of the QW. Thanks to this limitation, the narrower the well, the stronger the Coulombic interaction between an electron and a hole in an exciton. That's why an exciton in narrow QWs is more resistant to the action of an external electric field than an exciton in a bulk material. An increase of the width of a narrow well causes a decrease of the Coulombic interaction between an electron and a hole and, consequently, a stronger Stark effect under the application of an electric field. Such behavior of the Stark effect will be observed until the exciton bond energy in the absence of an external field depends on QW width. As demonstrated in [6,9], the well sizes do not affect the energy value if the QW width is by an order greater than the exciton Bohr radius in a bulk crystal, a_B. Therefore, our model is applicable only to wells having a width which meets the condition

$$L \geq 10a_B. \tag{23}$$

Expression (23) is another criterion of applicability of our model.

4. Conclusion

Influence of GaAs plate thickness on the energy of an exciton in an external electric field was analyzed. It is shown that energy in relatively strong fields must depend not only on the applied electric field, but also on the thickness of the semiconductor plate where the exciton is being considered. This effect must be observed in plates having a thickness which is by two decimal orders greater than the value of the exciton Bohr radius in bulk GaAs. Thereat, the dependence on the field is the weaker, the greater the semiconductor plate thickness. A comparison with the case of an exciton in an electric field in a bulk crystal was performed. A criterion for a limiting transition from the case of a thicker plate to the case of an exciton in a bulk semiconductor is given.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] E.L. Ivchenko. Optical Spectroscopy of Semiconductor Nanostructures. Alpha Science, Harrow (2005).
[2] A.V. Kavokin, J.J. Baumberg, G. Malpuech, F.P. Laussy. Microcavities. Oxford University, N.Y. (2007).
[3] C.F. Klingshirn. Semiconductor Optics. 4th ed. Springer, Berlin (2012).
[4] V.A. Kiselev, B.S. Razbirin, I.N. Uraltsev. Pisma v ZhETF **18**, *8*, 504 (1973) (in Russian).
[5] V.A. Kiselev, I.V. Makarenko, B.S. Razbirin, I.N. Uraltsev. FTT **19**, *8*, 1348 (1977) (in Russian).
[6] N. Tomassini, A. D'Andrea, R. Del Sole, H. Tuffigo-Ulmer, R.T. Cox. Phys. Rev. B **51**, *8*, 5005 (1995).
[7] E. Ubyivovk, Yu.K. Dolgikh, Yu.P. Efimov, S.A. Eliseev, I.Ya. Gerlovin, I.V. Ignatiev, V.V. Petrov, V.V. Ovsyankin. J. Luminesc. **102**–**103**, 751 (2003).
[8] S.A. Markov, R.P. Seisyan, V.A. Kosobukin. FTP **38**, *2*, 230 (2004) (in Russian).
[9] D.K. Loginov, E.V. Ubyivovk, Yu.P. Efimov, V.V. Petrov, S.A. Eliseev, Yu.K. Dolgikh, I.V. Ignatyev, V.P. Kochereshko, A.V. Selkin. FTT **48**, *11*, 1979 (2006) (in Russian).
[10] E.V. Ubyivovk, D.K. Loginov, I.Ya. Gerlovin, Yu.K. Dolgikh, Yu.P. Efimov, S.A. Eliseev, V.V. Petrov, O.F. Vyvenko, A.A. Sitnikova, D.A. Kirilenko. FTT **51**, *9*, 1818 (2009) (in Russian).
[11] A.V. Trifonov, S.N. Korotan, A.S. Kurdyubov, I.Ya. Gerlovin, I.V. Ignatiev, Yu.P. Efimov, S.A. Eliseev, V.V. Petrov, Yu.K. Dolgikh, V.V. Ovsyankin, A.V. Kavokin. Phys. Rev. B **91**, *11*, 115307 (2015).
[12] E.S. Khramtsov, P.A. Belov, P.S. Grigoryev, I.V. Ignatiev, S.Yu. Verbin, Yu.P. Efimov, S.A. Eliseev, V.A. Lovtcius, V.V. Petrov, S.L. Yakovlev. J. Appl. Phys. **119**, *18*, 184301 (2016).
[13] E.S. Khramtsov, P.S. Grigoryev, D.K. Loginov, I.V. Ignatiev, Yu.P. Efimov, S.A. Eliseev, P.Yu. Shapochkin, E.L. Ivchenko, M. Bayer. Phys. Rev. B **99**, *3*, 035431 (2019).
[14] U. Ekenberg, M. Altarelli. Phys. Rev. B **35**, *14*, 7585 (1987).
[15] M.H. Degani, O. Hipólito. Phys. Rev. B **35**, *17*, 9345(R) (1987).
[16] B. Gerlach, J. Wusthoff, M.O. Dzero, M.A. Smondyrev. Phys. Rev. B **58**, *16*, 10568 (1998).
[17] T. Makino, Y. Segawa, M. Kawasaki, H. Koinuma. Semicond. Sci. Technol. **20**, *4*, S78 (2005).
[18] T. Hiroshima. Phys. Rev. B **36**, *8*, 4518(R) (1987).
[19] J. Lee, C. Jagannath, M.O. Vassell, E.S. Koteles. Phys. Rev. B **37**, *8*, 4164 (1988).
[20] A.R. Glanfield, G. Rau, P.C. Klipstein, G.W. Smith. Phys. Status Solidi B **211**, *1*, 269 (1999).
[21] D.K. Loginov, A.V. Trifonov, I.V. Ignatiev. Phys. Rev. B **90**, *7*, 075306 (2014).
[22] D.K. Loginov, P.S. Grigoryev, Yu.P. Efimov, S.A. Eliseev, V.A. Lovtcius, V.V. Petrov, E.V. Ubyivovk, I.V. Ignatiev. Phys. Status Solidi B **253**, *8*, 1537 (2016).
[23] Yu.E. Lozovik, I.V. Ovchinnikov, S.Yu. Volkov, L.V. Butov, D.S. Chemla. Phys. Rev. B **65**, *23*, 235304 (2002).
[24] D. Loginov, V.P. Kochereshko, A. Litvinov, L. Besombes, H. Mariette, J.J. Davies, L.C. Smith, D. Wolverson. Acta Physica Pol. A **112**, *2*, 381 (2007).
[25] D.K. Loginov. FTT **52**, *1*, 58 (2010) (in Russian).
[26] D.K. Loginov, V.P. Kochereshko, R.T. Cox, L. Besombes, H. Mariette, J.J. Davies, D. Wolverson, L.C. Smith. Phys.Status Solidi B **247**, *6*, 1528 (2010).
[27] J. Wilkes, E.A. Muljarov. New J. Phys. **18**, 023032 (2016).
[28] J. Wilkes, E. Muljarov. Superlattices. Microstruct. **108**, 32 (2017).
[29] S.Yu. Bodnar, P.S. Grigoryev, D.K. Loginov, V.G. Davydov, Yu.P. Efimov, S.A. Eliseev, V.A. Lovtcius, E.V. Ubyivovk, V.Yu. Mikhailovskii, I.V. Ignatiev. Phys. Rev. B **95**, *19*, 195311 (2017).
[30] A. Chavesab, F.M. Peeters. Solid State Commun. **334**–**335**, 114371 (2021).
[31] J.J. Davies, D. Wolverson, V.P. Kochereshko, A.V. Platonov, R.T. Cox, J. Cibert, H. Mariette, C. Bodin, C. Gourgon, E.V. Ubyivovk, Y.P. Efimov, S.A. Eliseev. Phys. Rev. Lett. **97**, *18*, 187403 (2006).
[32] D. Loginov, V.P. Kochereshko, A. Litvinov, L. Besombes, H. Mariette, J.J. Davies, L.C. Smith, D. Wolverson. Acta Physica Pol. A **112**, *2*, 161 (2007).
[33] V.P. Kochereshko, A.V. Platonov, D.K. Loginov, J.J. Davies, D. Wolverson, L.C. Smith, H. Boukari, R.T. Cox, J. Cibert, H. Mariette. Semicond. Sci. Technol. **23**, *11*, 114011 (2008).
[34] J.J. Davies, L.C. Smith, D. Wolverson, A. Gust, C. Kruse, D. Hommel, V.P. Kochereshko. Phys. Rev. B **81**, *8*, 085208 (2010).

[35] P.S. Grigoryev, O.A. Yugov, S.A. Eliseev, Yu.P. Efimov, V.A. Lovtcius, V.V. Petrov, V.F. Sapega, I.V. Ignatiev. Phys. Rev. B **93**, *20*, 205425 (2016).

[36] D.A.B. Miller, D.S. Chemla, T.C. Damen, A.C. Gossard, W. Wiegmann, T.H. Wood, C.A. Burrus. Phys. Rev. Lett. **53**, *22*, 2173 (1984).

[37] D.A.B. Miller, D.S. Chemla, T.C. Damen, A.C. Gossard, W. Wiegmann, T.H. Wood, C.A. Burrus. Phys. Rev. B **32**, *2*, 1043 (1985).

[38] R.T. Collins, L. Vina, W.I. Wang, L.L. Chang, L. Esaki, K. von Klitzing, K. Ploog. Phys. Rev. B **36**, *3*, 1531 (1987).

[39] G.D. Sanders, K.K. Bajaj. Phys. Rev. B **35**, *5*, 2308 (1987).

[40] P.W. Yu, G.D. Sanders, K.R. Evans, D.C. Reynolds, K.K. Bajaj, C.E. Stutz, R.L. Jones. Phys. Rev. B **38**, *11*, 7796 (1988).

[41] C. Morhain, T. Bretagnon, P. Lefebvre, X. Tang, P. Valvin, T. Guillet, B. Gil, T. Taliercio, M. Teisseire-Doninelli, B. Vinter, C. Deparis. Phys. Rev. B **72**, *24*, 241305(R) (2005).

[42] Y.-H. Kuo, Yong Kyu Lee, Y. Ge, S. Ren, J.E. Roth, T.I. Kamins, D.A.B. Miller, J.S. Harris. Nature **437**, *7063*, 1334 (2005).

[43] K. Sivalertporn, L. Mouchliadis, A.L. Ivanov, R. Philp, E.A. Muljarov. Phys. Rev. B **85**, *4*, 045207 (2012).

[44] S. Zielińska-Raczyńska, G. Czajkowski, D. Ziemkiewicz. Eur. Phys. J. B **88**, 338 (2015).

[45] S. Zielińska-Raczyńska, D. Ziemkiewicz, G. Czajkowski. Phys. Rev. B **97**, *16*, 165205 (2018).

[46] D.K. Loginov, A.V. Donets. FTT **62**, *2*, 208 (2020) (in Russian).

[47] D.K. Loginov, A.V. Donets. Semiconductors **52**, *5*, 562 (2018).

[48] D.K. Loginov, P.A. Belov, V.G. Davydov, I.Ya. Gerlovin, I.V. Ignatiev, A.V. Kavokin, Y. Masumoto. Phys. Rev. Res. **2**, *3*, 033510 (2020).

[49] D.K. Loginov, P.A. Belov, I.Ya. Gerlovin, I.V. Ignatyev. FTP **55**, *9*, 779 (2021) (in Russian).

[50] D.K. Loginov, A.V. Donets. FTT **63**, *4*, 457 (2021) (in Russian).

[51] J.M. Luttinger. Phys. Rev. **102**, *4*, 1030 (1956).

[52] E.L. Ivchenko, G. Pikus. Superlattices and Other Heterostructures. Springer-Verlag, Berlin (1995).

[53] A. Baldereschi, N.O. Lipari. Phys. Rev. Lett. **25**, *3*, 373 (1970).

[54] L.D. Landau, E.M. Lifschitz. Quantum Mechanics. 3rd ed. Pergamon, Oxford (1977).

[55] D.F. Blossey. Phys. Rev. B **3**, *4*, 1382 (1971).

[56] S. Adachi. GaAs and Related Materials. World Scientific Publishing, Singapore (1994).

[57] S. Haastrup, S. Latini, K. Bolotin, K.S. Thygesen. Phys. Rev. B **94**, *4*, 041401 (2016).

[58] M. Massicotte, F. Vialla, P. Schmidt, M.B. Lundeberg, S. Latini, S. Haastrup, M. Danovich, D. Davydovskaya, K. Watanabe, T. Taniguchi, V.I. Fal'ko, K.S. Thygesen, T.G. Pedersen, F.H.L. Koppens. Nature Commun. **9**, 1633 (2018).

[59] A. Jaeger, G. Weiser. Phys. Rev. B **58**, *16*, 10674 (1998).

[60] B. Brill, M. Heiblum. Phys. Rev. B **54**, *24*, R17280 (1996).