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Determining the Parameters of the SIS-Mixer at an Intermediate Frequency

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The level of signal reflection at an intermediate frequency from an SIS-mixer was calculated and experimentally measured. The measurement was performed for the mixer in operation, namely, it was biased by voltage and the signal of the local oscillator was applied. It is shown that the power reflection coefficient can vary from -20 to -3 dB depending on the operating point. An experimental method for determining the impedance of the supply line of the SIS-mixer is proposed and tested.

1. Introduction

Radio astronomy is the main driver for the development of ultrasensitive mixers for heterodyne receivers of millimeter and submillimeter wavelength electromagnetic radiation. Mixers based on the superconductor–insulator–superconductor (SIS) tunneling junction [1] have record-breaking noise characteristics in this range, close to the quantum limit. Among the ground-based receivers, the mixers with the separation of side bands, which have two single SIS-mixers in their composition, have become the most widespread [2,3]. For the efficient operation of the receiver, namely to achieve maximum sensitivity and for high quality sideband separation, it is fundamentally important to have SIS mixers with low level of reflection. Low level of reflections is needed both at the mixer input, i.e. at the frequency of the received signal, at high frequency (HF) [4,5], and at the output, i.e. at intermediate frequency (IF). Presented work is devoted to determining the level of reflections from the SIS-mixer in the operating mode in order to minimize this level in the future. To solve the original problem, the theoretical calculation of the level of reflection from the SIS-mixer along the output path of the IF was carried out, as well as the experimental circuit was assembled, and direct measurement of the reflected signal was carried out.

2. Theory

The magnitude of reflections from the SIS-mixer along the IF path, characterized by the parameter S_{11} [6], can be determined if the IF output impedance of the SIS-mixer Z_{IF} and the impedance of the supply line of the IF path Z_L are known:

$$S_{11_{IF}} = \frac{Z_{IF} - Z_L}{Z_{IF} + Z_L}. \quad (1)$$

To calculate Z_{IF}, we use the 3-frequency approximation to Tucker's theory of quantum mixing [1]. There are considered signals at frequencies $f_m = mf_{LO} + f_0$, where $m = 0, \pm 1$; $f_{\pm 1}$ are upper and lower bands; f_{LO} is local oscillator frequency; f_0 is intermediate frequency. We believe that higher harmonics are shunted by the capacitance of the SIS-mixer, which in our case is about 100 fF.

Taking into account the interaction of the mixer ports (Fig. 1), the voltage and current components of weak signals are linearly related by the conductivity matrix $i_m = \sum_{m'} Y'_{mm'} v_{m'}$:

$$Y'_{mm'}(V_{gap}, R_N, f_{LO}, \alpha, V_0)$$

$$= \begin{bmatrix} Y_{11} + Y_S & Y_{10} & Y_{1-1} \\ Y_{01} & Y_{00} + Y_L & Y_{0-1} \\ Y_{-11} & Y_{-10} & Y_{-1-1} + Y_I \end{bmatrix}, \quad (2)$$

where V_{gap} is bias voltage of the tunnel current step, R_N is differential resistance of the volt-ampere characteristic (VAC) in the normal state, $\alpha = eV_{LO}/hf_{LO}$ is dimensionless pumping parameter proportional to the local oscillator voltage V_{LO}, V_0 is bias voltage of the SIS-mixer.

Each port has its own channel conductance: $Y_1 = Y_S$ is port load with received signal, $Y_0 = Y_L$ is IF port load, $Y_{-1} = Y_I$ is low band port load.
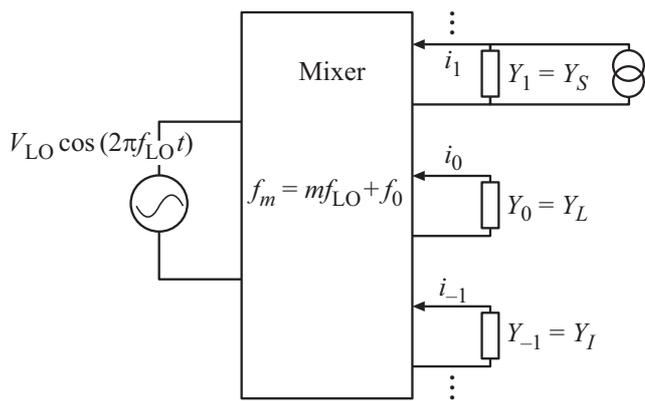
Figure 1. Equivalent circuit of a heterodyne mixer, with impressed local oscillator signal f_{LO}, received frequency signal f_m and IF signal f_0.
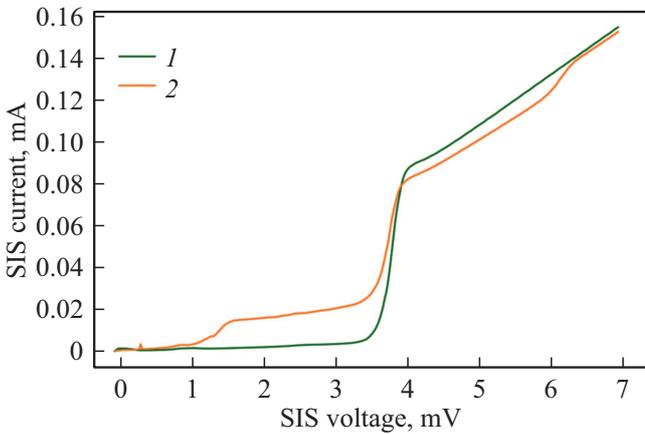

Figure 2. VAC: autonomous VAC (*1*, green curve) and VAC with local oscillator signal applied (*2*, orange curve).

For ideal two-way mixer, the impedances of the upper and low bands are: $Y_1 = Y_{-1}$. In our case, $Y_{\pm 1}$ is the result of the local oscillator conductance conversion by several microstrip lines.

Using the result [1], we shall obtain $Y_{mm'} = G_{mm'} + iB_{mm'}$, where

$$G_{mm'} = \frac{e}{2\hbar\omega_{m'}} \sum_{n,n'=-\infty}^{\infty} J_n(\alpha)J_{n'}(\alpha)\delta_{m-m',n'-n}$$

$$\times \left\{ \left[I_{DC}\left(V_0 + \frac{n'\hbar\omega}{e} + \frac{\hbar\omega_{m'}}{e}\right) - I_{DC}\left(V_0 + \frac{n'\hbar\omega}{e}\right) \right] \right.$$

$$\left. + \left[I_{DC}\left(V_0 + \frac{n'\hbar\omega}{e}\right) - I_{DC}\left(V_0 + \frac{n\hbar\omega}{e} - \frac{\hbar\omega_{m'}}{e}\right) \right] \right\},$$

$$B_{mm'} = \frac{e}{2\hbar\omega_{m'}} \sum_{n,n'=-\infty}^{\infty} J_n(\alpha)J_{n'}(\alpha)\delta_{m-m',n'-n}$$

$$\times \left\{ \left[I_{KK}\left(V_0 + \frac{n'\hbar\omega}{e} + \frac{\hbar\omega_{m'}}{e}\right) - I_{KK}\left(V_0 + \frac{n'\hbar\omega}{e}\right) \right] \right.$$

$$\left. - \left[I_{KK}\left(V_0 + \frac{n'\hbar\omega}{e}\right) - I_{KK}\left(V_0 + \frac{n\hbar\omega}{e} - \frac{\hbar\omega_{m'}}{e}\right) \right] \right\},$$

where $I_{DC}(V_0)$ is dependence of the SIS-mixer tunnel current on its voltage; $I_{KK}(V_0)$ is dependence of the Kramers–Kronig relation of current I_{DC} on SIS-mixer voltage V_0; $J_n(\alpha)$ is Bessel function of n order of the pumping parameter α.

Then in our case the IF impedance is

$$Z_{IF} =\parallel Y_{mm'} + Y_m\delta_{mm'} \parallel_{00}^{-1} . \qquad (3)$$

Figure 2 shows the measured VAC of the SIS junction, which is the dependence $I_{DC}(V_0)$ and determines I_{KK}. The
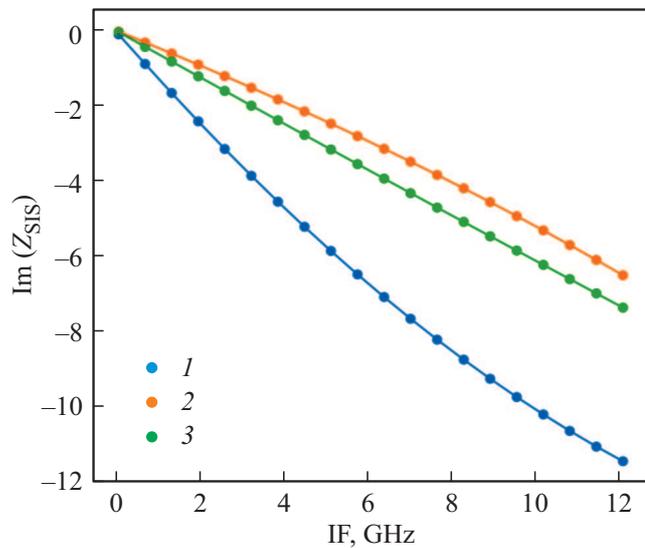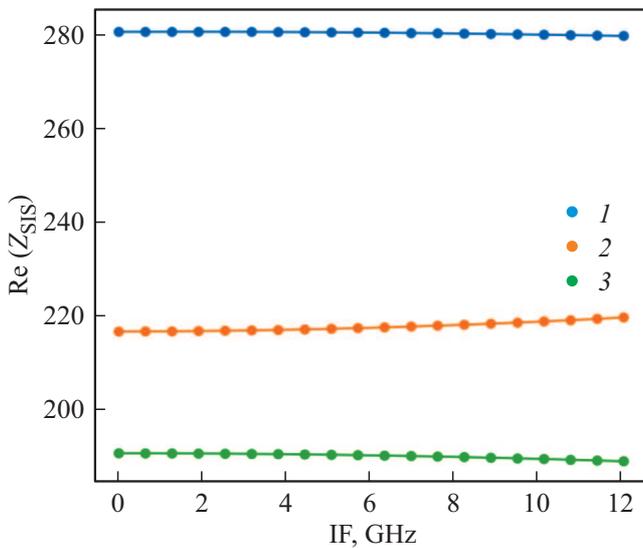

Figure 3. Real (left) and imaginary (right) parts of the calculated impedance of the SIS-mixer from the IF at different voltages on the SIS-mixer; *1* — 2.8, *2* — 3, *3* — 3.2 mV.
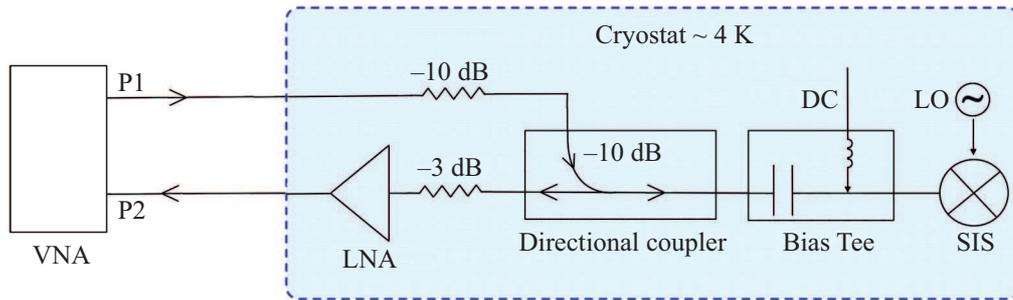
Figure 4. Scheme of the experiment on measuring the reflection from the SIS mixer for the IF output.

autonomous curve is shown by a green line, while the orange curve shows the VAC for the case when the external generator signal with a frequency of 600 GHz is applied.

Figure 3 shows the result of theoretical calculation of the impedance using the VACs (Fig. 2) in the IF range $0.1-12$ GHz at various voltages at the SIS junction: 2.8, 3, 3.2 mV, and the supply line impedance is set to $\sim 50\,\Omega$. It can be concluded that the real part changes slightly with an intermediate frequency, in contrast to the imaginary part. It can also be seen that the impedance strongly depends on the voltage at the SIS junction.

3. Experimental setup

The objective of the experiment is to measure the level of reflection from the SIS-mixer through the IF output channel when the bias voltage is applied to the mixer and the high-frequency local oscillator signal is applied. The measurements were carried out in the range of $4-8$ GHz; this range is determined by the bandwidth of the cryogenic IF amplifier used. The scheme of the experiment is shown in Fig. 4; the SIS-mixer is placed in closed cycle cryostat at temperature of about 4 K. The high-frequency local oscillator (LO) is integrated with the SIS-mixer on a single chip and is a distributed Josephson junction (DJJ) with a viscous flow of magnetic vortices. Vector network analyzer (VNA), located outside the cryostat, generates a test signal in the $4-8$ GHz range on port P1, which, passing through the -10 dB attenuator, enters the directional coupler which sends it to the SIS-mixer with a coupling coefficient of about -10 dB. Next, the signal passes through a bias tee, which allows the IF signal to pass unhindered, while setting the voltage on the SIS-mixer for direct current through a large inductance. After bouncing off the SIS-mixer, the main part of the signal passes directly through the directional coupler and is fed to the input of a cryogenic low noise amplifier (LNA), which amplifies this signal and sends it to the P2 receive port of the VNA. In fact, the measured parameter is the ratio of the VNA signals on ports P1 and P2, or rather, its spectrum.

4. Calibration

An important step in the experiment is the calibration of the VNA in order to improve the accuracy of measurements. We use the standard one-port calibration (Fig. 5), which is based on the determination of 3 circuit parameters: D are direct leakages in the circuit, R are internal reflections, M is misalignment. By simple transformations, the actual reflection coefficient Γ in terms of the measured value Γ_m and 3 calibration parameters D, R, M, can be explicitly expressed:

$$\Gamma = \frac{\Gamma_m - D}{R + M(\Gamma_m - D)}. \qquad (4)$$

The coefficients D, R, M can be determined from three calibration measurements by composing and solving a sys-
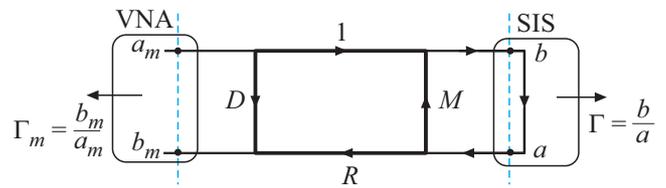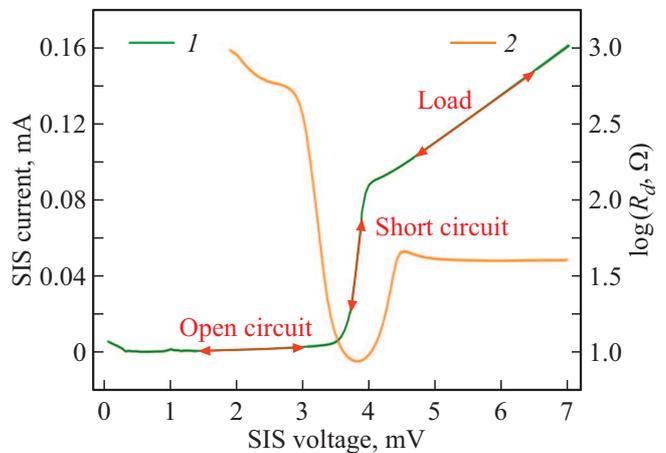

Figure 5. Schema of one-port calibration.

Figure 6. Autonomous volt-ampere characteristic of the SIS-mixer, green line *1*; the corresponding differential resistance R_d is shown as orange line *2*.

tem of three equations (4), where the reflection coefficient Γ is calculated theoretically by formula (1), and the value Γ_m is directly measured by a vector network analyzer.

In calibration measurements at 4 K the SIS junction itself is used as a calibrator [7]. This avoids errors that occur during calibration at room temperature and are associated with changes in the electrical length and impedance of circuit elements during cooling. The SIS-mixer is in an autonomous state, i.e. without the application of an external signal. Figure 6 illustrates which bias voltages are used for calibration: 1) at bias voltage of 2 mV, the differential resistance becomes about 1000 Ω, which is close to the situation of „an open circuit", since the impedance of the supply line is close to 50 Ω; 2) at bias voltage of 3.8 mV, i.e. in the middle of the tunnel current surge, its differential resistance is about 3 Ω, which corresponds approximately to the calibration „short circuit"; 3) at bias voltage of $5-7$ mV, the differential resistance becomes $\sim 41\,\Omega$, which is close to the situation of „a loaded line".

These three calibrations are enough to compose 3 equations using (4) and thereby determine the coefficients D, R and M. This allows to take into account all reflections and leaks in the circuit and correctly measure the reflection from the SIS-mixer in operating mode. It should be noted that in the given calibration, the internal capacitance of the SIS-mixer acts as a part of the external circuit and its influence is also leveled by the calibration.

5. Results

The VAC of the SIS-mixer, when applying a signal of local oscillator with a frequency of ~ 600 GHz in „working" mode, is shown in Fig. 2 by an orange curve. The range of the „operating " bias voltage lies approximately in the interval of $1.6-3.4$ mV. This voltage region corresponds to the so-called quasiparticle step caused by the application of a local oscillator signal to the SIS junction. Figure 7 shows the
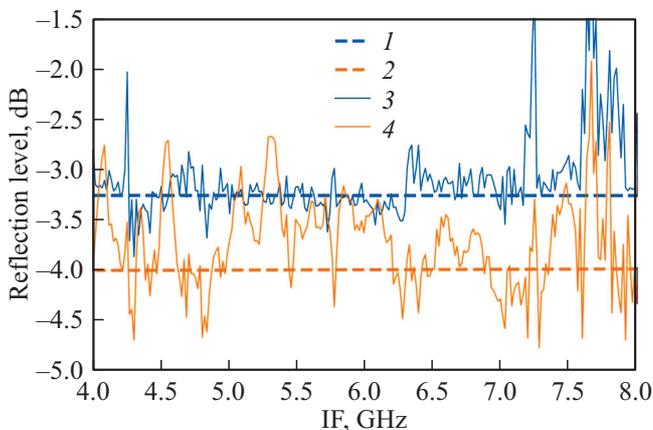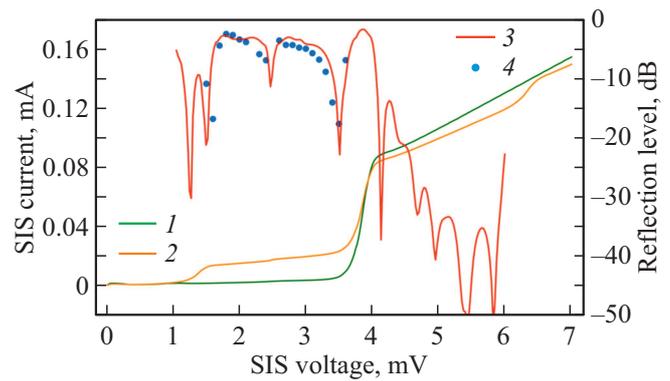

Figure 8. VAC of the SIS-mixer: autonomous (green curve *1*), loaded with local oscillator signal (orange curve *2*). Theoretical reflection calculation (red curve *3*). Results of reflection measurements (blue dots *4*). IF is equal to 6 GHz.
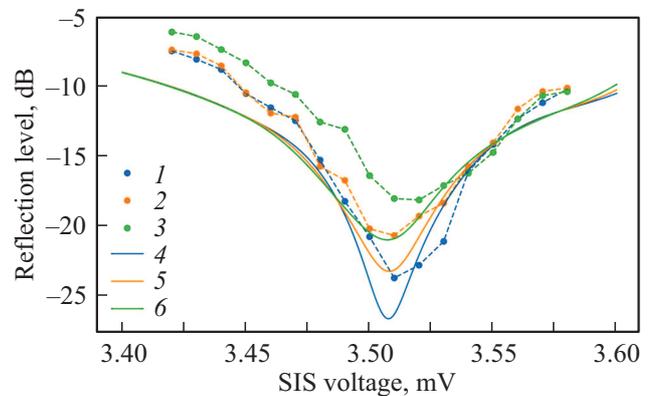

Figure 9. Reflection level in „the well". Points (*1, 2, 3*) are experimental reflection level for IF *4, 6, 8* GHz respectively; solid curves (*4, 5, 6*) are theoretical calculation of the reflection level for IF *4, 6, 8* GHz respectively.

experimental (solid curves) and theoretical (dashed curves) frequency dependences of the reflection level at different voltages across the SIS mixer. From the above results, it can be seen that in the range of $4-8$ GHz, the reflection level almost does not change with frequency, which is in good agreement with theoretical predictions.

The change in the level of reflections with varying the bias voltage of the SIS-mixer is shown in Fig. 8. Here are the results of measurements and calculation of reflection for the middle of the IF range, namely at frequency of 6 GHz. Blue points are experimental data, red solid curve is theoretical calculation. The reflection level is on average about 4 dB. At voltage of about 2 mV, there is VAC distortion caused by the Josephson's step-edge, which manifests itself due to the presence of unsuppressed critical junction current. This distortion causes the anomaly in the level of reflection. It can be seen that at a voltage of 3.5 mV, the reflection level is significantly reduced (details are in Fig. 9). This absorption peak can be explained by the fact that the impedance of the SIS-mixer becomes almost equal to the

Figure 7. The theoretically calculated (*1, 2*, dotted lines) and experimentally determined (*3, 4*, solid curves) reflection level from the SIS junction in „operating mode". SIS-mixer voltages *1, 3 — 2*; *2, 4 — 2.6* mV.

impedance of the supply line of the IF, formula (1). More precisely, at this point, the differential resistance of the SIS-mixer becomes equal to the real component of the supply line impedance $R_{d_{SIS}} = \text{Re}(Z_L)$, and by calculating the impedance of the SIS-mixer using formula (3), we can experimentally determine the impedance of the supply line Z_L. In this case, we observe that Z_L is $50.3 + i \cdot 1.8\,\Omega$ with a high degree of accuracy. In the Fig. 9, the comparison of the experimental level of reflection and the theoretical one is given. It is important to note that the minimum reflection level is determined by the modulus of the difference between the imaginary components of the impedances of the SIS-mixer and the supply line. The difference between the depth of the absorption peak in the measurement and in the calculation makes it possible to check the reliability of the calculation, as well as to estimate the value of the complex part of the impedance of the supply line and due to measurements at different IF frequencies. In our case, one can conclude that the imaginary part of the supply line impedance does not exceed $2\,\Omega$. Thus, the method for detecting impedance of the supply line using the features of the VACs of the SIS-mixer has been proposed and tested.

In general, one can conclude that the level of reflection is quite high and averages about $-4.5\,\text{dB}$ in the „operating" range, which forces us to use special valves in the IF channel in SIS receivers to minimize standing waves in the IF path.

6. Conclusion

The method for experimental and theoretical determination of the IF parameters of the SIS junction is presented. This makes it possible to investigate the dependence of the level of reflections from the SIS-mixer on the IF output on the bias voltage and on the power of the reference signal. Determining the parameters of the SIS junction itself, combined with simulating the elements of the IF channel, will allow in the future to calculate with high accuracy the IF characteristics of the mixer itself, as well as the entire receiver designed on its basis. Additional calibration by the absorption peak when varying the bias voltage improves the measurement accuracy, which is in good agreement with the theoretical calculation.

In the future, it is planned to study the IF parameters at frequencies of $4-12\,\text{GHz}$ and higher, as well as when varying the power of the local oscillator in a wide range.

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