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## Electron-phonon interaction in the surface dimer adsorption model

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In the scope of the model of surface dimer, formed by the adsorbed atom and below lying substrate surface atom, the problem on the electron-phonon interaction effect on the charge transfer in adsorbed system is considered. The case of adsorption on metal is examined thoroughly.

**Keywords:** adsorbate, substrate, dimer's vibrations, charge transfer.

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### 1. Introduction

To the best of the authors' knowledge, the question of the adatom vibrations effect on its electronic state was first considered in articles [1–3]. In this case, the adsorption standard model (ASM) was used on Anderson–Newns metals [4,5]. Recently, to describe adsorption on solid substrates, the surface dimer model (SDM) was proposed, which, unlike the standard model, considers not an adsorbed particle, but a surface molecule formed by this particle and the nearest atom of the substrate surface [6]. In this article, we consider how vibrations of a surface dimer affect the electronic characteristics of an adsorption system and, most importantly, charge transfer within a dimer and between a dimer and a substrate. Although in the case under consideration it is more correct to speak of electron-vibrational interaction, we will use the standard term „electron-phonon“ interaction, comparing the eigenfrequency of  $\Omega$  dimer vibrations with a phonon with a zero wave vector.

Let us represent the Hamiltonian of a free (not bound to the substrate) dimer  $H_{\text{dim}}$ , consisting of atoms  $a$  (adsorbed atom) and  $s$  (surface atom of the substrate), which orbitals are characterized by energies  $\varepsilon_a$  and  $\varepsilon_s$ , and the occupation numbers are 1, as the sum of  $H_{\text{dim}} = H_{el} + H_{vib} + H_{\text{int}}$ . The Hamiltonian of the subsystem of spinless electrons has the form

$$H_{el} = \varepsilon_a \hat{n}_a + \varepsilon_s \hat{n}_s - t(s^+ a + a^+ s), \quad (1)$$

where  $t$  is energy of electron hopping between atoms  $a$  and  $s$ ,  $\hat{n}_a = a^+ a$  and  $\hat{n}_s = s^+ s$  are atomic number operators for  $a$  and  $s$ ,  $a^+(a)$  and  $s^+(s)$  are electron creation (annihilation) operators,  $n_{s(a)}$  are atomic occupation numbers  $a$  and  $s$  [6]. The Hamiltonian  $H_{vib}$  describing the vibrations of a free surface dimer can be represented in the classical form

$$H_{vib} = \frac{1}{2} M \dot{l}^2 + \frac{1}{2} k (l - l_0)^2, \quad (2)$$

where  $M$  is reduced mass of dimer atoms,  $k = M\Omega^2$  is force constant of dimer with equilibrium length  $l_0$ . The Hamiltonian of the interaction  $H_{\text{int}}$  between the electronic and vibrational subsystems of a free dimer is

$$H_{\text{int}} = \nu (\hat{n}_a + \hat{n}_s) (l - l_0), \quad (3)$$

where  $\nu$  is deformation constant of the surface dimer. Using the relation  $\partial \langle H_{\text{dim}} \rangle / \partial l = \langle \partial H_{\text{dim}} / \partial l \rangle$ , we get  $l - l_0 = -\nu (n_a + n_s) / k$ . Then the Hamiltonian  $H_{\text{dim}}$  can be written as

$$H_{\text{dim}} = w_a \hat{n}_a + w_s \hat{n}_s - t(s^+ a + a^+ s) + H'. \quad (4)$$

Here  $w_{a(s)} = \varepsilon_{a(s)} - \lambda n_{a(s)}$ , where  $\lambda = \nu^2 / k$  is the constant of the electron-phonon interaction of the surface dimer and the term  $H'$  contains all the contributions that do not have an operator structure. Using the results of [6] and neglecting the metallicity of the bond, we obtain

$$\nu = (\partial \sqrt{t^2 + \delta^2} / \partial l)_{l=l_0} = 4t\alpha_c / l_0,$$

$$k = 4\alpha_c t (1 - 2\alpha_p^2) / l_0^2 \quad \text{and} \quad \lambda = 4\alpha_c t / (1 - 2\alpha_p^2),$$

where bond covalency  $\alpha_c = t / \sqrt{t^2 + \delta^2}$ , bond polarity  $\alpha_p = \sqrt{1 - \alpha_c^2}$  and  $\delta = (\varepsilon_a - \varepsilon_s) / 2$  [7].

Hamiltonian (4) corresponds to the Green's function of the free dimer  $g_{\text{dim}}^{\pm}(\omega) = (\omega - \omega_{\pm} + i0^+)^{-1}$ , where  $\omega$  is energy variable and energies of local levels are  $\omega_{\pm} = w \pm R$ ,  $R = \sqrt{\Delta^2 + t^2}$ ,  $w = (w_a + w_s) / 2$  and  $\Delta = (w_a - w_s) / 2$ . The densities of atom states of a free dimer are

$$\rho_{a,s}(\omega) = D_{\pm} \delta(\omega - \omega_{+}) + D_{\mp} \delta(\omega - \omega_{-}),$$

where  $D_{\pm} = (1 \pm \Delta / R) / 2$  (upper signs refer to atom  $a$ , lower refer to atom  $s$ ),  $\delta(\dots)$  is Dirac delta function. The state density of the free dimer itself is  $\rho_{\text{dim}}(\omega) = \delta(\omega - \omega_{+}) + \delta(\omega - \omega_{-})$ , and the

occupation number  $n_{\text{dim}} = 2[\Theta(\omega - \omega_+) + \Theta(\omega - \omega_-)]$ , where  $\Theta(\dots)$  is Heaviside function. Then, we set  $\varepsilon_a + \varepsilon_s - 2\lambda = 0$ , where we take into account that  $n_a + n_s = 2$  in a free dimer. The charge transfer between atoms of a free dimer is  $m = n_a - n_s$ , so  $\Delta = \delta - \lambda m/2$ , where  $\delta = (\varepsilon_a - \varepsilon_s)/2$ . It is easy to show that  $\partial m/\partial \lambda = -m^2/R^3$ , i.e. as  $\lambda$  increases, the value of  $|m|$  decreases.

Let's take into account the presence of a substrate. In the weak coupling limit ( $\Gamma(\omega) \ll t$ ), density of states of the epitaxial dimer densities of the epitaxial dimer (epidimer)  $\bar{\rho}_{\text{dim}}(\omega)$  and its constituent atoms  $\bar{\rho}_{a(s)}(\omega)$  can be represented as

$$\begin{aligned}\bar{\rho}_{a(s)}(\omega) &= D_{\pm}\rho_+(\omega) + D_{\mp}\rho_-(\omega), \\ \bar{\rho}_{\text{dim}}(\omega) &= \rho_+(\omega) + \rho_-(\omega), \\ \rho_{\pm}(\omega) &= \frac{1}{\pi} \frac{\Gamma(\omega)}{(\omega - \omega_{\pm} - \Lambda(\omega))^2 + \Gamma^2(\omega)}.\end{aligned}\quad (5)$$

Here  $\Gamma(\omega) = \pi V^2 \rho_{\text{sub}}(\omega)$  is the local level broadening function  $\omega_{\pm}$ , the function

$$\Lambda(\omega) = V^2 P \int_{-\infty}^{\infty} \rho_{\text{sub}}(\omega') (\omega - \omega') d\omega'$$

determines the shift of these levels,  $\rho_{\text{sub}}(\omega)$  is density of substrate states,  $V$  is matrix element of the interaction of  $\omega_{\pm}$  levels with substrate states,  $P$  is principal value symbol. For further analysis, it is necessary to specify the substrate by setting the function  $\rho_{\text{sub}}(\omega)$ .

Let us consider adsorption on metal. Setting  $\rho_{\text{sub}}(\omega) = \rho_m = \text{const}$  (Anderson model [5]), we find  $\Gamma(\omega) = \Gamma_m = \pi V^2 \rho_m$  and  $\Lambda(\omega) = \Lambda_m = 0$ . Then the densities of states of the epidimer atoms  $\bar{\rho}_{a(s)}^m(\omega)$  are the sums of two Lorentz profiles  $\rho_{\pm}^m(\omega) = \Gamma_m/\pi[(\omega - \omega_{\pm})^2 + \Gamma_m^2]$  with weight factors  $D_{\pm}$  [6,7]. At zero temperature, the level occupation numbers  $\omega_{\pm}$  are equal to

$$n_{\pm}^m = 2 \int_{-\infty}^{\varepsilon_F} \rho_{\pm}^m(\omega) d\omega,$$

where  $\varepsilon_F$  is the Fermi level of the system, whence, taking into account (5), we find  $n_{\pm}^m = (2/\pi) \text{arccot}[(\omega_{\pm} - \varepsilon_F)/\Gamma_m]$ . For the epidimer, the occupation number is

$$\bar{n}_{\text{dim}}^m = n_+^m + n_-^m \quad \text{and} \quad \bar{m}^m \equiv \bar{n}_a^m - \bar{n}_s^m = (\bar{\Delta}/\bar{R})(n_+^m - n_-^m),$$

where  $\bar{\Delta} = \delta - \lambda \bar{m}^m/2$ ,  $\bar{R} = \sqrt{\bar{\Delta}^2 + t^2}$ . At  $\varepsilon_F = 0$  there is no charge transfer between the dimer and the substrate, so  $\bar{n}_{\text{dim}}^m = 2$ . The interatomic charge transfer inside the dimer is

$$\bar{m}^m = (2\bar{\Delta}/\pi\bar{R}) [\arctan(\omega_-/\Gamma_m) - \arctan(\omega_+/\Gamma_m)].$$

In the weak coupling regime, the dimer-substrate  $\Gamma_m/|\omega_{\pm}| \ll 1$  and  $\Gamma_m/R \ll 1$  in the first approximation, we have

$$n_+^m(\varepsilon_F = 0) \approx 2\Gamma_m/\pi\omega_+ \quad \text{and} \quad n_-^m(\varepsilon_F = 0) \approx 2 - 2\Gamma_m/\pi\omega_-.$$

Therefore,

$$(\bar{m}^m)_{\varepsilon_F=0} \approx -2(\bar{\Delta}/\bar{R})(1 - 2\Gamma_m/\pi\bar{R}).$$

Thus, at  $\varepsilon_F = 0$ , the value of the interatomic charge transfer in the epidimer is smaller than in the free dimer. In the limit  $|\varepsilon_F| \ll |\omega_{\pm}|$  we have  $n_{\pm}^m \approx n_{\pm}^m(\varepsilon_F = 0) - 2\rho_{\pm}^m(\omega_{\pm})\varepsilon_F$ , where  $\rho_{\pm}^m(\omega_{\pm})\varepsilon_F \sim \Gamma_m\varepsilon_F/R^2$ , i.e. it is a quantity of the second order infinitesimal. In the same limit, the charge transfer between the dimer and the substrate is  $\bar{m}_{\text{dim}}^m = 2 - \bar{n}_{\text{dim}}^m \approx 2\Gamma_m/\bar{R}$ , or a quantity of the first order infinitesimal. At the same time  $\partial \bar{m}_{\text{dim}}^m/\partial \lambda = (\Gamma_m\bar{\Delta}/\bar{R}^3)\bar{m}_{\text{dim}}^m$ , so that the electron-phonon interaction increases the value of  $|\bar{m}_{\text{dim}}^m|$  at  $\bar{\Delta} > 0$  and decreases it at  $\bar{\Delta} < 0$ .

Above, we considered the case of a weak bond between the dimer and the substrate. It is easy to show (e.g. see [8]) that in the strong bond regime in the zeroth order in  $t^2/\Gamma_m^2$  the problem of epidimer is reduced to the problem of adsorption of two isolated atoms  $a$  and  $s$ . If, as in the article [8], we take into account the Coulomb repulsion  $G$  of electrons of atoms  $a$  and  $s$ , then expression (4) will be valid after replacing  $w_{a(s)} = \varepsilon_{a(s)} - \lambda n_{a(s)}$  with  $w'_{a(s)} = \varepsilon_{a(s)} + G n'_{s,a} - \lambda n'_{a(s)}$ .

In this article, we considered adsorption on a metal with a constant state density (Anderson model), which is applicable to the description of simple (non-transitional) metals with a wide  $sp$ -band. For  $d$ - and  $f$ -metals, it is more correct to use the Friedel state density model of „pedestal“ type [9], shift function  $\Lambda(\omega)$  for which is given in [10]. Note that the Friedel model is also suitable for describing two-dimensional metals [11]. In case of adsorption on semiconductor and dielectric substrates, it is convenient to use the Haldane–Anderson state density model [5,12]. Adsorption on graphene-like compounds is considered in [13].

Unfortunately, it should be noted that the question of the role of the electron-phonon interaction in adsorption has not been practically studied (e.g. see [4,14]), although, on the other hand, there is growing interest in electron-phonon effects in epitaxial [15,16] and free [17–19] two-dimensional layers and topological insulators [20,21]. Therefore, in the adsorption problem, due to the lack of information, we used the simplest model of electron-phonon interaction, similar to the widely used one-phonon Holstein model [22,23].

## Conflict of interest

The authors declare that they have no conflict of interest.

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