Amplification of the electric area of a pulse during reflections from a moving mirror

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An analysis was made of the transformation of the electric area of an electromagnetic pulse upon reflection from a moving mirror. Conditions are found under which reflection leads to an increase (in absolute value) of the electric area. For a dynamic resonator, one of the mirrors of which periodically oscillates, an expression is obtained for the threshold for generating pulses with an ever-increasing area. Nonlinear factors that limit the exponential growth of the pulse electric area are discussed.

Keyword: pulse electric area, unipolar pulse, dynamic resonator.

Introduction

Extremely short unipolar or half-cycle electromagnetic pulses are of considerable interest in the problems of impact on micro-objects [1–3]. This is due to the unidirectional nature of such an impact with the instantaneous transmission of a mechanical momentum to the object charges, while „pushes“ of the object on different half-cycles of bipolar pulses are directed to opposite directions and thereby compensate each other.

Extremely short pulses can be obtained by transition to the X-ray region of the spectrum or by phasing pulses of various optical harmonics [4–6]. However, such pulses will be bipolar, with a limited impact on micro-objects. At the same time, shortening of pulses is also possible without a transition to the region of high frequencies due to a decrease in the number of field oscillations during the pulse, up to subcycle one. Thus, it is unipolar pulses that serve as the limiting stage of pulse shortening.

Apparently, the first publication about electromagnetic pulses with non-zero electric area

$$\mathbf{S}_E = \int \mathbf{E}\,dt, \qquad (1)$$

where $\mathbf{E}$ — electric field strength and t — time, was an article by Bessonov [7]. Although the statements about the impossibility of electromagnetic pulses with a nonzero area are often found in the literature, see, for example, [8], the arguments of these works cannot, in our opinion, be considered consistent and correct. At the same time, it should be recognized that in practice the generation of extremely short unipolar or quasi-unipolar pulses presents a serious problem, while it is relatively easy to obtain longer unipolar pulses. An overview of various methods for generating such pulses is contained in [2,3].

Another problem is that it is impossible to amplify (increase) the electrical area of pulses using standard methods. Indeed, directly from Maxwell's equations of electrodynamics of continuous media [9] for one-dimensional (plane-wave) geometry with radiation propagation along the axis z follows the electric area conservation rule [10]:

$$\frac{d}{dz}\mathbf{S}_E = 0. \qquad (2)$$

This rule is valid regardless of the type of constitutive relations for the medium that is, for laser (amplifying) media too. Therefore, laser amplification of the electric area of pulses is impossible.

In this paper we analyze the possibility of amplifying and generating the electric area of pulses by using their reflections from moving mirrors or inhomogeneities of the medium.

Reflection from the moving mirror

Let an electromagnetic pulse propagate in vacuum along z axis from the region $z = -\infty$ to the mirror moving with a speed V along the same axis. If the mirror is ideal with an amplitude reflection coefficient $r = -1$ for all frequencies, then the reflected radiation pulse will have the same shape as the incident one, but with the amplitude changed by $-(1 - V/c)/(1 + V/c)$ times [11,12] (c — speed of light in vacuum).

Since the electric pulse area (1) is the zero-frequency component of pulse spectrum, this also applies to the ratio of the areas of the incident $\mathbf{S}_{E,\text{in}}$ and the reflected $\mathbf{S}_{E,\text{r}}$ pulses:

$$\mathbf{S}_{E,\text{r}} = -\frac{1 - V/c}{1 + V/c}\,\mathbf{S}_{E,\text{in}}. \qquad (3)$$

Therefore, for the counter movement of the mirror towards the incident pulse ($V < 0$), the electric area of the reflected pulse will exceed in absolute value the value of incident pulse area, that is, the electric area will be amplified. This is a relativistic effect that takes place in the laboratory coordinate system, while in the coordinate system associated with the mirror, the areas of the incident and reflected pulses differ only in sign.

Real mirrors, unlike ideal ones, have frequency dispersion. With regard to the issue relating to the amplification of the electric area, the value of the reflection coefficient of a fixed mirror in the limit of zero radiation frequency r_0 is important; this coefficient can be calculated by Fresnel-type formulas [9,13], in which static values of optical constants are used. Then, instead of (3) we get

$$\mathbf{S}_{E,r} = r_0 \frac{1 - V/c}{1 + V/c} \mathbf{S}_{E,in}. \qquad (4)$$

Now, the amplification of the area during reflection will take place under the condition

$$|r_0| > \frac{1 - |V|/c}{1 + |V|/c} \qquad (5)$$

or

$$\frac{|V|}{c} > \frac{1 - |r_0|}{1 + |r_0|} \qquad (6)$$

(counter motion). For a high reflection coefficient $1 - |r_0| \ll 1$ these conditions are simplified

$$|r_0| > 1 - |V|/c, \quad \frac{|V|}{c} > \frac{1}{2}(1 - |r_0|). \qquad (7)$$

Mirrors with a reflectance very close to 100% are currently available. In particular, there is an intense flow of publications on media with „colossal" dielectric constant, including more than 10^7 [14] which provides almost perfect reflection and, accordingly, low mirror speed requirements. Possible options are reflection from plasma mirrors [15] or from fast-moving inhomogeneities of the medium. We also note that in the course of reflections the shift of zero frequency does not happen; for it there is no Doppler effect (but the component amplitude changes).

Dynamic resonator

The above possibility of amplifying the electric area of a pulse also implies the possibility of generating pulses with a large electric area under the condition of multiple reflections available in resonator circuits. Resonator schemes with a moving mirror are traditionally considered in relation to the dynamic Casimir effect, see the review [16] (its classical counterpart was qualitatively analyzed in [17] even before the first quantum consideration [18]) and in the resonator optomechanics problems, see review [19]. We will focus on the classical approach, since it allows us to quantitatively take into account many factors that is difficult to do in the quantum consideration of the electromagnetic field [20].

Let us consider, also in the plane-wave approximation, a two-mirror resonator, one mirror of which periodically oscillates around the average position z_{right}. The other mirror is considered by us as fixed with the coordinate z_{left}. With fine tuning of the average resonator length $L_1 = z_{right} - z_{left}$, the light travel time in the resonator $2L_0/c$ coincides with the mirror oscillation period . Then the most favorable for the emergence situation (from a seed with a nonzero electric area) corresponds to the reflection of a short pulse from the right mirror at the moments, when the mirror speed is maximum in absolute value and is directed to the resonator center, $V = V_{max}$. The condition for the generation threshold with amplification of the electric area of the pulse can be considered to coincide with (5) or (6), if by V we mean the value V_{max}, and by r_0 — the product of the amplitude reflection coefficients of the resonator mirrors (for $V = 0$) at zero frequency (static limit). Let us stipulate that under the conditions of periodic generation of pulses, the concept of their electric area can be used only approximately, considering the duration of the pulses to be much shorter, than the time of light travel through the resonator and, when integrating in (1), be limited to an interval equal to the time of this travel.

The increase in the pulse electric area under the conditions above the generation threshold at the initial stage is exponential, it is limited by nonlinear factors. In optomechanics, this factor is usually the light pressure on the mirror [19]. Another factor may be the change in the optical characteristics of the mirror material in strong electromagnetic fields. The analysis of these factors requires concretizing of the method of mirror movement and mirror material that is beyond the scope of this paper.

Conclusions

Relativistic effects in the reflection of a pulse from a moving mirror allow to achieve an increase in the electric area of the pulse, when the threshold value of the mirror speed or its reflection coefficient (in the absence of movement) is exceeded. The latter is determined by the static characteristics of the mirror material (the values of the static dielectric constant and conductivity). The use of a quasi-one-dimensional (plane-wave) approximation for unipolar pulses is justified, for example, in the case of coaxial waveguides, which do not have a cutoff frequency [21].

The transition from amplification to generation of pulses with a high electric area is achievable in a resonator scheme with a periodically moving mirror, when seeded with a pulse with a nonzero area. The generation threshold value is obtained by a natural generalization of the expression for the area amplification threshold at a single pulse reflection.

The limitation of the initial exponential growth of the electric area of the pulse with an increase in the number of its reflections from a moving mirror occurs due to such nonlinear effects as light pressure on the mirrors and a change in the properties of the mirror material in a

strong electromagnetic field. It is experimentally easier to implement relatively long pulses with a significant electric area, since this does not require a large width of the spectral contour of the mirror reflection coefficients.

Conflict of interest

The author declares that he has no conflict of interest.

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