Peculiarities of light absorption in chirped one-dimensional photonic crystals

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The features of light absorption and localization in one-dimensional chirped photonic crystals (PCs) are studied. It is shown that although chirping leads to the loss of ideal periodicity, the integral over a wide spectral region light energy density practically does not change. We have shown that violation of the ideal periodic structure (chirping) can not only decrease the integral in the broad spectral region absorption, but also lead to an increase in this absorption in the PC, and this depends on the form of chirping and the form of modulation of the imaginary part of the dielectric permittivity. A study of the evolution of the absorption spectra when the chirping contrast is changed showed that at certain values of the chirping contrast, mini-zones of transmission appear in the photonic band gap, where significant absorption is observed.

Keywords: chirped photonic crystals, absorption, light localization, coupled modes, chirping contrast, mini zone

Introduction

Chirped photonic crystals (PCs) attract a lot of attention, since they exhibit optical analogs of the Bloch oscillation effect [1], theoretically predicted back in 1928. and experimentally confirmed only in 1992 [2], of Zener tunneling [3] and of the Wannier-Stark effect [4]. Quantum particle in a periodic potential (for example, in a crystal) is described by Bloch waves, delocalized in space and having an energy spectrum characterized by conduction bands and band gaps. If an external electric field is impressed to the crystal, then the electron located in the center of the Brillouin zone is accelerated by the field and begins to move towards the edge of the zone. Upon reaching the edge of the Brillouin zone, it is reflected, and its wave vector changes sign to the opposite one. In this case, the electron finds itself on the opposite edge of the zone and continues to move along the zone in the same direction under the influence of the electric field. Thus, in the Brillouin zone, the electron performs a periodic motion, which can be described as an interference phenomenon caused by Bragg reflection from the edges of the energy band gap. This is an oscillatory motion and is known as electronic Bloch oscillations. The Wannier-Stark ladder is a frequency analogue of Bloch oscillations in the time domain and consists in the formation of equidistant (in frequency) energy levels in the structure of the electronic band. With increase in the external electric field, the anti-crossing of adjacent passbands occurs, and the band gap between them disappears. Instead of reflecting off

the edge of the band-gap, the electrons will tunnel to a higher band with no additional energy, and thus resonant Zener tunneling occurs. In the work [5] the theory of Bloch oscillations of photons in PC was developed. Note that photons have no charge, and in the case of a PC the role of the electric field is played by the gradient of the size of the unit cell of the optical crystal, i.e.chirpization. Theoretical and experimental studies of Bloch oscillations and Zener tunneling of electromagnetic waves in various photonic structures were presented in the works [6–14].

Interest in the study of chirped PCs is also due to the following circumstance. In chirped PCs, the photonic band gap (PBG) is observed [15–18], multimode laser generation is possible in them and lasing with a broadly tunable lasing wavelength [19,20], as well as the so-called omnidirectional reflection [20,21]: in a certain wavelength range, omnidirectional reflectors completely reflect light from any polarization and at any angle of incidence. These properties have important practical applications, in particular in fiber optic communications, lasers, etc.

In chirped PCs, the so-called electromagnetic wave rainbow trapping effect [22–25] takes place. The rainbow trap consists in a strong spectrally dependent localization of electromagnetic waves in certain spatial positions in the PCs. In the visible part of the spectrum, radiation of different colors is spatially separated, forming „a rainbow", hence the name. Rainbow traps command great attention due to their wide potential applications: in optical buffers,

optical memory, optical data processing, optical filters, energy sorting and storage, enhancement of light−matter interaction etc. [22–27]. Chirped PCs are used in structural tinting-method systems [28?31]. In the present work, we investigated the features of light absorption in one-dimensional chirped PCs.

Structure of PC and method of analysis

Let us consider the reflection and transmission of light through a layer of a one-dimensional chirped PC with a permittivity having the form

$$\varepsilon(z) = \varepsilon + \Delta\varepsilon \sin\left(\frac{2\pi}{\Lambda(z)} z\right), \qquad (1)$$

where $\varepsilon = \varepsilon' + i\varepsilon''$ is a constant complex value, $\Delta\varepsilon$ and $\Lambda(z)$ are the modulation depth and period, respectively. We assume that the PC layer with thickness d is located between the planes $z = 0$ and $z = d$ and $\varepsilon_s = 1$, where $\varepsilon_s = n_s^2$ is the permittivity of the medium bordering with a layer of PC on both sides. The amplitudes of the electric field of the incident $\mathbf{E}_i$, reflected $\mathbf{E}_r$ and transmitted $\mathbf{E}_t$ plane waves are expanded in terms of orthogonal linear polarizations:

$$\mathbf{E}_{i,r,t} = E_{i,r,t}^p \mathbf{n}_p + E_{i,r,t}^s \mathbf{n}_s = \begin{pmatrix} E_{i,r,t}^p \\ E_{i,r,t}^s \end{pmatrix}, \qquad (2)$$

where $\mathbf{n}_p$ and $\mathbf{n}_s$ are the vectors of linear p- and s-polarizations, $E_{i,r,t}^p$ and $E_{i,r,t}^s$ are corresponding amplitudes of incident, reflected and transmitted waves.

To solve this problem, we will apply the theory developed in [32–34]. According to [34], the amplitude transmission and reflection coefficients for the s- and p-polarizations are $(t^{s,p} = E_t^{s,p}/E_i^{s,p}$ and $r^{s,p} = E_r^{s,p}/E_i^{s,p})$ are defined using the following formulae:

$$t^{s,p} = \frac{2\exp(-ik_s d)}{Q_k^{s,p}(d) + F_k^{s,p}(d)}, \quad r^{s,p} = \frac{Q_{-k}^{s,p}(d) - F_{-k}^{s,p}(d)}{Q_k^{s,p}(d) + F_k^{s,p}(d)}, \qquad (3)$$

de $k_s = (2\pi/\lambda)\sqrt{\varepsilon_s}\cos\alpha$, α is angle of incidence. The functions $F^{s,p}$ and $Q^{s,p}$ are solutions of the following system of differential equations:

$$\frac{dF^{s,p}}{dz} = -iA^{s,p}Q^{s,p}, \qquad (4)$$

$$\frac{dQ^{s,p}}{dz} = -iB^{s,p}F^{s,p} \qquad (5)$$

with initial conditions on $z = 0$

$$F^{s,p}(0) = 1, \quad Q^{s,p}(0) = 1. \qquad (6)$$

For the electric field inside the PC layer, we have

$$E_{in}^{s,p}(z) = [F_{-k}^{s,p}(z) + r^{s,p}F_k^{s,p}(z)]E_i^{s,p}. \qquad (7)$$

Here

$$A^s = \frac{2\pi}{\lambda}\sqrt{\varepsilon_s}\cos\alpha, \quad B^s = \frac{2\pi}{\lambda}\frac{1}{\sqrt{\varepsilon_s}\cos\alpha}(\varepsilon(z) - \varepsilon_s\sin^2\alpha),$$

$$A^p = \frac{2\pi}{\lambda}\frac{\sqrt{\varepsilon_s}}{\varepsilon(z)\cos\alpha}(\varepsilon(z) - \varepsilon_s\sin^2\alpha),$$

$$B^p = \frac{2\pi}{\lambda}\frac{\varepsilon(z)}{\varepsilon_s}\cos\alpha.$$

Here and below, the indices k and $-k$ denote the functions computed before and after the inversion, $A^{s,p} \to -A^{s,p}$ and $B^{s,p}$ to $-B^{s,p}$ in the system of equations (4) and (5) [34]. The corresponding transmission, reflection and absorption coefficients are defined as

$$T^{s,p} = |t^{s,p}|^2, \quad R^{s,p} = |r^{s,p}|^2, \quad A^{s,p} = 1 - (T^{s,p} + R^{s,p}). \qquad (8)$$

Results and discussions

We consider two types of chirped PCs, namely the first type with

$$\Lambda(z) = \frac{\Lambda_{\max} - \Lambda_{\min}}{d} z + \Lambda_{\min} \qquad (9)$$

and the second type with

$$\Lambda(z) = \frac{\Lambda_{\min} - \Lambda_{\max}}{d} z + \Lambda_{\max}, \qquad (10)$$

and compare their properties. In the first case, along the direction of light propagation, the modulation period increases linearly from the value of $\Lambda_{\min}$ on the input surface to the value of $\Lambda_{\max}$ on the output surface, and in the second case, on the contrary, it linearly decreases from the value of $\Lambda_{\max}$ on the input surface to the value of $\Lambda_{\min}$ on the output surface. The calculation of the reflection (and transmission) coefficients in the absence of absorption for these two PCs shows that they differ from each other. But this does not mean the existence of nonreciprocity in these PCs, as stated in [35]. Indeed, by calculating the reflection (transmission), for example, for a PC with $\Lambda(z) = \frac{\Lambda_{\max} - \Lambda_{\min}}{d} z + \Lambda_{\min}$ for the direct direction of light propagation, and then the same for the reverse direction of propagation (to do this, in equations (1)−(8) we will replace z with $d - z$), and one can make sure that the quantity characterizing the nonreciprocity is $\Delta T = T_{\text{forward}} - T_{\text{backward}} = 0$, where T_{forward} and T_{backward} are coefficients transmission for forward and backward waves, respectively. The same takes place for the reflection coefficient $\Delta R = R_{\text{forward}} - R_{\text{backward}} = 0$ (in the case without absorption). In [35] the change in the modulation phase of the dielectric function for a wave with the opposite direction of propagation was not taken into account. Thus, according to (10), the PC with the law of variation for $\Lambda(z)$ is not a FK with the law (9) when the light is incident from the opposite side. Reflection/absorption asymmetry appears in these systems only in the presence of absorption in the system, while transmission is always symmetrical.
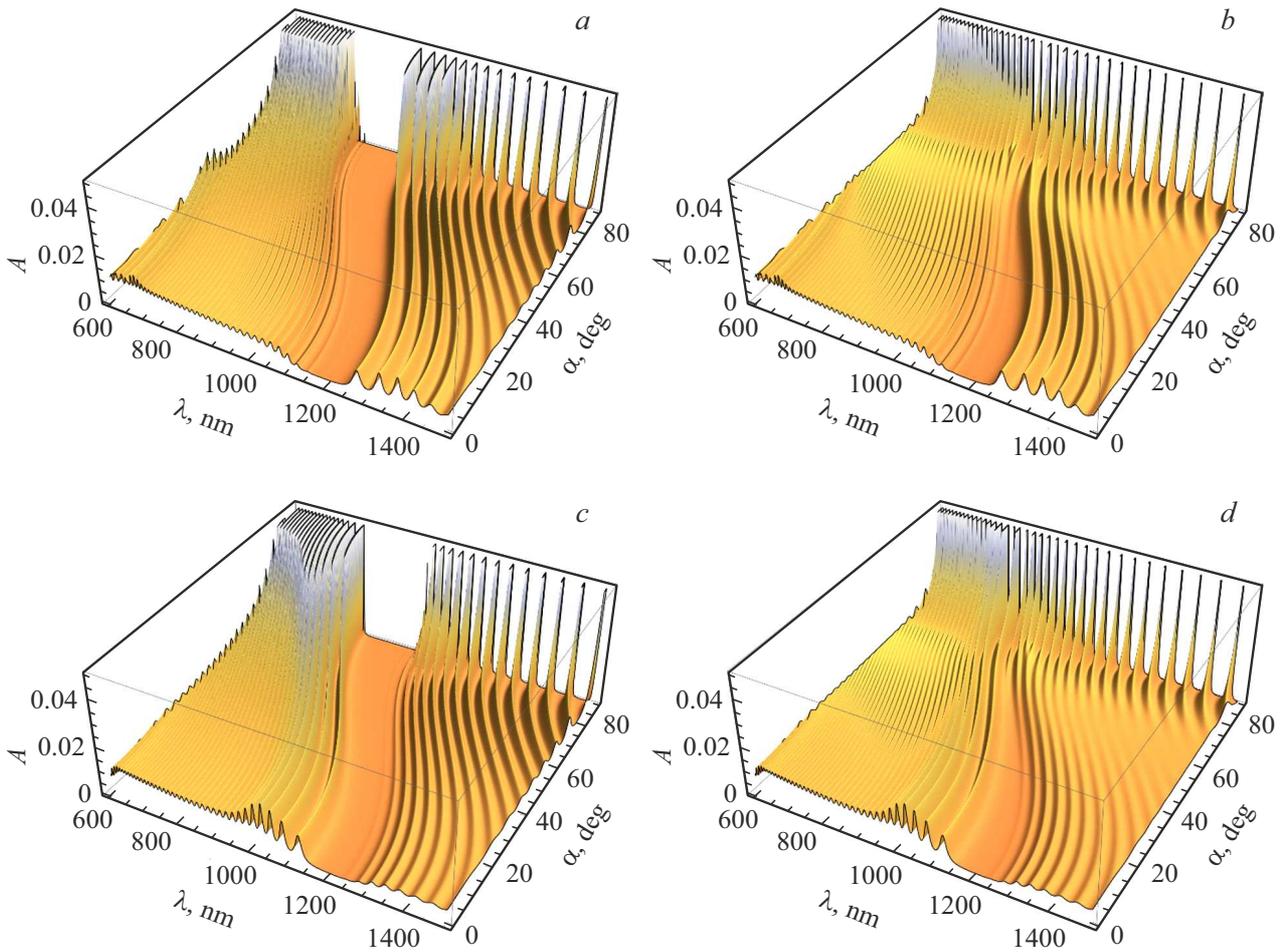
Figure 1. The angular dependence of the absorption spectrum of the chirped PC layer for the s polarization (a, c) and for the p polarization (b, d) in the case of the first (a, b) and second (c, d) types of chirpization. The parameters of the PC layer are as follows: $\varepsilon' = 2.25$; $\varepsilon'' = 0.0001$; $\Delta\varepsilon = 0.5$; $d = 16\,\mu m$; $\Lambda_{min} = 380\,nm$; $\Lambda_{max} = 420\,nm$; $n_s = 1$.

Next, we first consider the case when the imaginary part of the permittivity ε'' is constant and does not depend on z. Fig. 1 shows the angular dependence of the absorption spectra of the chirped PC layer for s-polarization (a, c) and p-polarization (b, d), for the first $((9); a, b)$ and second $((10); c, d)$ chirping types. Common denominator of all considered cases is the low absorption of radiation inside the PBG, not counting the coupled modes, which are characterized by a large penetration of radiation into the volume of the PC, due to which the absorption increases significantly. In addition, at angles of incidence near the Brewster angle, p-polarized light has less absorption than s-polarized light. This fact can be explained by the weakening of the influence of incident wave reflections inside the PC, which leads to a decrease in the localization of light energy that could be absorbed in the structure

Let's proceed to study the features of the light energy density w localized in the PC layer in order to compare their spectra with the absorption spectra A for a PC with a periodic structure and for chirped PCs of the first (9) and second (10) types. The light energy density in the PC layer

will be calculated using the following formula:

$$w = \frac{1}{d}\int_0^d |E_{in}(z)|^2 dz. \tag{11}$$

Fig. 2 shows the spectra of reflection R (curve 1), light energy density w (curve 2), and absorption spectra A (curve 3) for a PC with an ideal periodic structure (a), for a PC with the first (9) type of chirping (b), and for a PC with the second (10) type of chirping (c). As can be seen from Fig. 2, the positions of the minima and maxima of w and A coincide, and at the same time they coincide with the maxima and minima of the reflection coefficient R. Also note that as the wavelength decreases, the absorption A increases, oscillating, and it is reasonable, since the absorption is mainly determined by the imaginary part of the wave vector $\text{Im}\,k = \frac{2\pi}{\lambda}$, $\text{Im}(\varepsilon_m)$ increasing indefinitely as $\lambda \to 0$.

Then we calculated the total density of light energy F and absorption of radiation Q localized in the PC layer in a finite
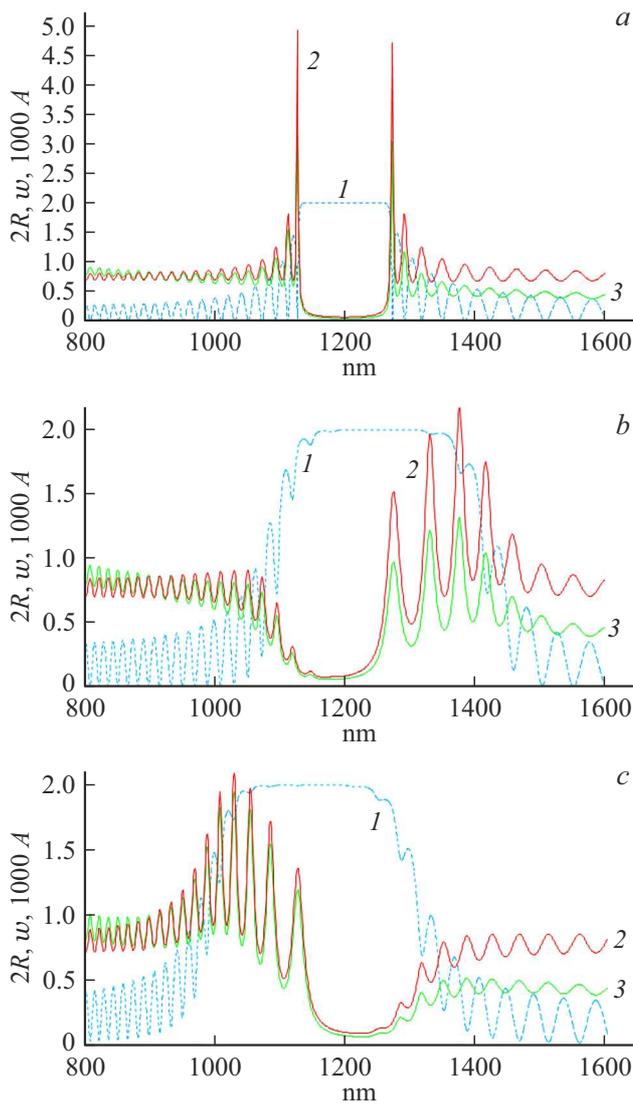
Figure 2. Spectra of reflection R (curve *1*), light energy density w (curve *2*), and absorption A (curve *3*) for a PC with ideal periodic structure (*a*), for a PC with the first (9) type of chirping (*b*) and for a PC with the second (10) type of chirping (*c*). Parameters are same as in Fig. 1.

spectral range from λ_1 to λ_2 using the following expression:

$$F = \int_{\lambda_1}^{\lambda_2} w(\lambda)d\lambda, \quad Q = \int_{\lambda_1}^{\lambda_2} A d\lambda. \quad (11)$$

As our calculations show, the value of F is practically the same for a PC with an ideal periodic structure and for a PC with the first and second types of chirping, i.e., violation of the ideal periodic structure (chirping) has practically no effective effect on the integral localization of light in a PC. For Q, we have the following pattern: the value of Q for the first type of chirping is less than Q for a PC with an ideal periodic structure, and Q for the second type of chirping is greater than Q for a PC with an ideal periodic structure . It follows from this that the violation of

the ideal periodic structure (chirping) can not only reduce the integral absorption, but also lead to an increase in the integral absorption in the PC. It depends on the form of chirping.

Now, denoting by x the volume $x = \Lambda_{max} - \Lambda_{min}$ (the chirping contrast), which characterizes the total change in the spatial period of the PC, we will study its effect on absorption. Fig. 3 shows the evolution of the $\ln(A)$ spectra with a change in x in the case of the first (9) type (*a*) and the second (10) type of chirping (*b*).

It follows from the presented results that strong absorption is observed in coupled modes and that mini-bands of transmission due to Zener tunneling appear in the PBG at certain values of the parameter x, and this happens differently for different chirping laws. This leads to the formation of bands with significant absorption.
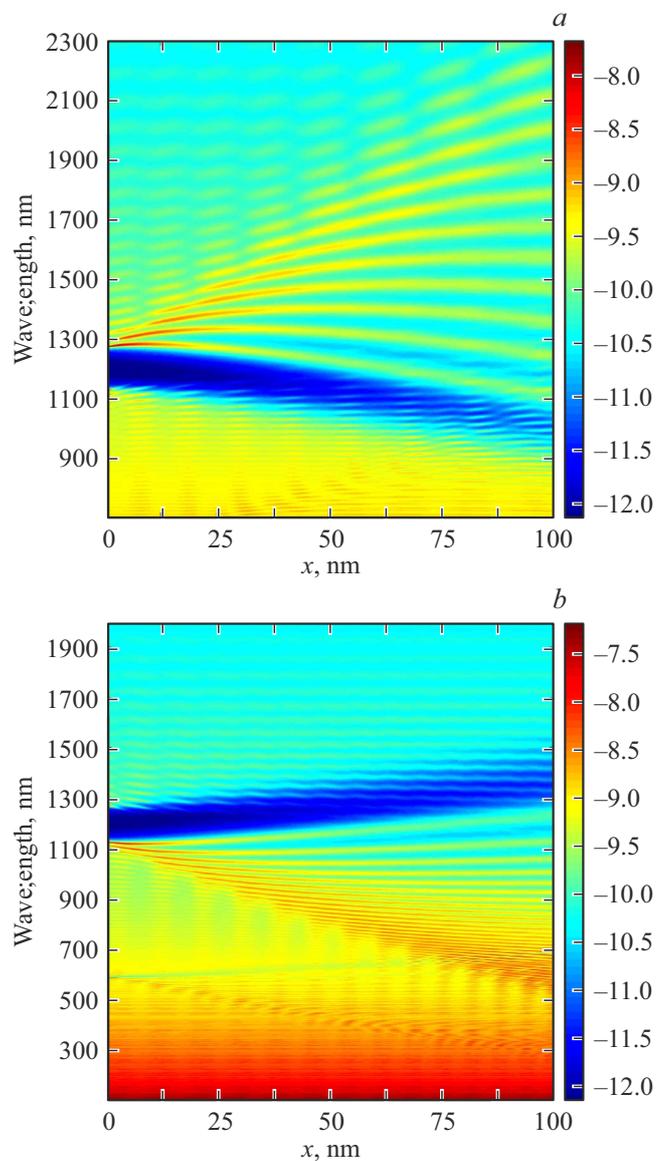

Figure 3. Evolution of $\ln(A)$ spectra with changing x in the case of the first (9) type of chirping (*a*) and the second (10) type of chirping (*b*). Parameters are same as in Fig. 1.

	$\varepsilon'' = \mathrm{const}$	$\varepsilon'' - 0.001 = 0.0005\sin(2\pi z/\Lambda(z))$	$\varepsilon'' - 0.001 = -0.0005\sin(2\pi z/\Lambda(z))$
$\Lambda(z) = \mathrm{const}$	43.4019	42.5860	44.1435
$\Lambda(z) = (\Lambda_{\max} - \Lambda_{\min})z/d + \Lambda_{\min}$	42.1913	43.0961	41.2520
$\Lambda(z) = \Lambda_{\max} - (\Lambda_{\max} - \Lambda_{\min})z/d$	44.9469	42.2461	47.5954

Let's proceed to the study of the effect of modulation of the imaginary part of the permittivity ε'' for the absorption A. Let ?s study and compare absorption A in the following three cases, namely: 1) $\varepsilon'' = \mathrm{const} = 0.001$, 2) $\varepsilon'' = 0.001 + 0.0005\sin\left(\frac{2\pi}{\Lambda(z)}z\right)$, i.e., absorption changes in phase with ε' and 3) $\varepsilon'' = 0.001 - 0.0005\sin\left(\frac{2\pi}{\Lambda(z)}z\right)$, t .e. absorption changes in antiphase with ε'. Fig. 4 shows the absorption spectra A for a PC with an ideal periodic structure (a), for a PC with the first (9) chirping type (b), and for a PC with the second (10) chirping type (c). As can be seen from Fig. 4, if the absorption changes in phase with ε', then the integral absorption A increases in the long-wave part of the spectrum and A decreases in the short-wave part of the spectrum (compared to the case of $\varepsilon'' = \mathrm{const}$). In the case when the absorption changes in antiphase with ε', we have the back-pattern, namely, the absorption A decreases in the long-wavelength part of the spectrum and A increases in the short-wave part of the spectrum. These features can be understood with this in mind it becomes that the localization maxima shift over the crystal with variation in the wavelength [33]. As follows from our calculations, in the short-wave part of the spectrum, the maxima of $|E_{\mathrm{in}}(z)|^2$ in the crystal coincide with (close) the minima of the refractive index, while in the long-wave part of the spectrum, the maxima of $|E_{\mathrm{in}}(z)|^2$ in the crystal coincide with (close to) the maxima of the refractive index. Therefore, it is reasonable that if the attenuation index changes in phase with the refractive index, then in the short-wave part of the spectrum there will be a decrease in absorption, and in the long-wave one will be a sharp increase, and vice versa, if the attenuation index changes in antiphase with the refractive index, then in the short-wavelength part of the spectrum there will be a sharp increase in absorption, and in the long-wave one will be a sharp decrease. Note also that, the imaginary part of the wave vector $\mathrm{Im}\,k = \frac{2\pi}{\lambda}\mathrm{Im}\left(\sqrt{\varepsilon}\right) = \frac{2\pi}{\lambda}\kappa$, where κ is the damping index. It should also be mindful that the κ exponent changes along the z axis even in the case of $\varepsilon'' = const$.

The table shows the values of the integral absorption in a finite spectral range (from $\lambda_1 = 800\,\mathrm{nm}$ to $\lambda_2 = 1600\,\mathrm{nm}$) for a PC with an ideal periodic structure, for a PC with the first (9) chirping type, and for FC with the second chirping type in the three cases considered above, namely: 1) at $\varepsilon'' = \mathrm{const} = 0.001$, 2) at $\varepsilon'' = 0.001 + 0.0005\sin\left(\frac{2\pi}{\Lambda(z)}z\right)$ and 3) at $\varepsilon'' = 0.001 - 0.0005\sin\left(\frac{2\pi}{\Lambda(z)}z\right)$.

From the presented results, it follows that by variation the character of the modulation for the spatial period of the PC and the character of the modulation for the absorption, it
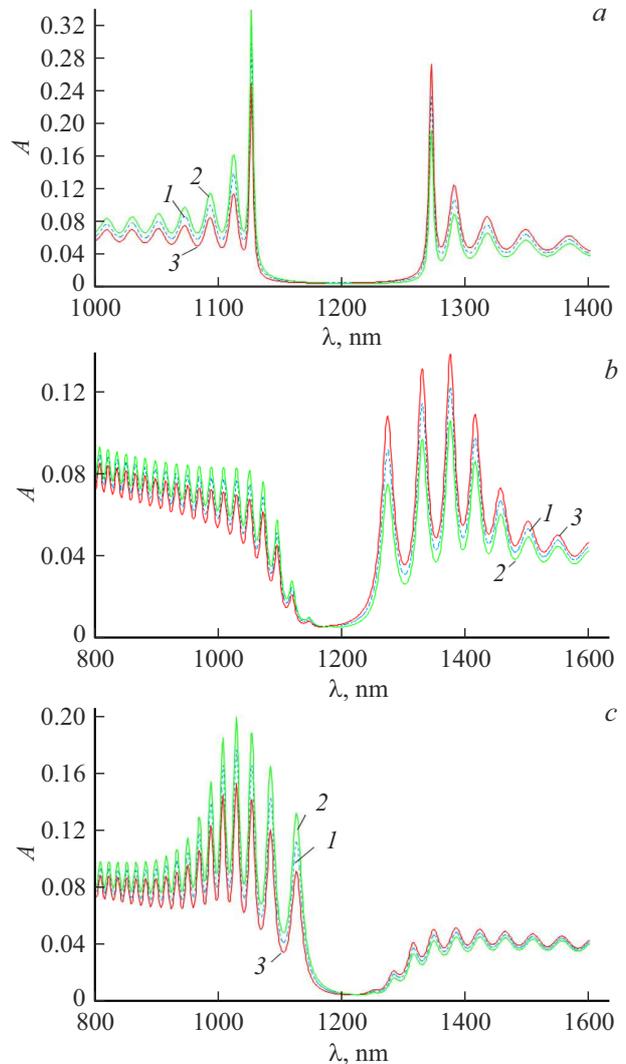
Figure 4. Absorption spectra A for a PC with an ideal periodic structure (a), for a PC with the first (9) chirping type (b), and for a PC with the second (10) chirping type (c). Curve *1*: $\varepsilon'' = \mathrm{const} = 0.00$; curve *2*: $\varepsilon'' = 0.001 + 0.0005\sin\left(\frac{2\pi}{\Lambda(z)}z\right)$; curve *3*: $\varepsilon'' = 0.001 - 0.0005\sin\left(\frac{2\pi}{\Lambda(z)}z\right)$. Parameters are same as in Fig. 1.

is possible to change the value of the integral absorption in the PC in the finite spectral range within 15%.

Conclusions

In present work, the features of light absorption in one-dimensional chirped harmonic PCs are studied. Two

cases of chirping are studied: in the first case, along the direction of light propagation, the modulation period increases linearly from Λ_{min} on the input surface to Λ_{max} on the output surface; in the second case, it linearly decreases from the value of Λ_{max} on the input surface to the value of Λ_{min} on the output surface. Study of the evolution of the absorption spectra with a change in the angle of incidence for these two cases showed that although chirping leads to violation of the ideal periodicity, the integral over a wide spectral region of the light energy density and absorption can not only decrease, but also increase, and this depends on the form of chirping and the shape modulation of the imaginary part of the permittivity. Then we studied the evolution of the absorption spectra with a change in the chirping contrast and showed that a change in the nature of the modulation of the spatial period of the PC and the nature of the modulation of the absorption can lead to a change in the value of the total absorption in the PC in the final spectral range within 15%.

Funding

Conflict of interest

The author declares that he has no conflict of interest.

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