09;01.1;01.5

Optical excitation and control of extended orbits in quadrupole traps

© S.S. Rudyi, V.V. Rybin, D.P. Shcherbinin, Y.V. Rozhdestvensky, I.A. Kosternoi

ITMO University, St. Petersburg, Russia
E-mail: semyonrudiy@gmail.com

Received April 25, 2022
Revised June 22, 2022
Accepted June 22, 2022

In the present work a formation of extended orbits of single charged particles in a linear radiofrequency trap under the action of the light pressure force of laser radiation is considered. The conditions for the formation of extended orbits depending on the characteristics of the laser radiation and the object of localization are determined.

Keywords: ion traps, quadrupole traps, light pressure, extended orbits, period-doubling bifurcation, nonlinear dynamics.

DOI: 10.21883/TPL.2022.08.55059.19233

At present, RF microparticle traps are widely used for the localization and characterization of single micro- and nanosized charged particles. The specificity of the microparticles localization allows the use of radio frequency traps under atmospheric pressure conditions [1,2]. In this case, nonlinear and chaotic effects can appear in the dynamics of a charged particle.

One of such effects is the formation of the so-called extended orbits — two-dimensional quasi-periodic oscillations, the spectrum of which is characterized by harmonics with frequencies that are multiples of half of the frequency of the alternating confining field of the trap [2,3]. The formation of expanded orbits is associated with the transition from laminar to turbulent motion in the event of a nonlinear dissipation of energy on the buffer gas [4]. Being a typical artifact of non-linear dynamics, expanded orbits can be used to characterize localized charged particles: non-destructive determination of charge, mass and size [3]. The described NDI method (nonlinear damping identification) is promising, but it requires monitoring of buffer gas parameters, including temperature, pressure, and partial composition [5]. Maintaining stationary conditions of the localization environment is possible, but in practice it is associated with significant technological difficulties. Besides, the mechanisms for the formation of expanded orbits presented in the literature are based on a hydrodynamic description of the nonlinear dissipation of energy. Such a description imposes restrictions on the minimum characteristic size of the localization object (on the order of $1\,\mu$m). Thus, the search for new mechanisms for the formation of expanded orbits for submicron particles is especially important. Thus, the excitation of expanded orbits for charged particles can be carried out with the help of an additional external optical effect.

The purpose of this paper is to determine the conditions for optical excitation of expanded orbits in a linear quadrupole ion trap depending on the physical characteristics of localized particles and laser radiation parameters.

Let us consider the case of localization of a single particle in a linear quadrupole trap, the spatial distribution of the potential in which has the form

$$\Phi = \left[V\cos(\omega t)\right]\frac{x^2 - y^2}{2r_0^2}, \tag{1}$$

where V — variable component of the voltage on the electrodes, ω — frequency of the alternating voltage on the electrodes, r_0 — radius of the working area of the trap (Fig. 1).

In the case of microparticles localization in a buffer gas, damping effects must be taken into account. For small values of the Reynolds number [3,6], nonlinear effects can be neglected and the particle motion can be considered laminar. The resulting friction force can be described by the Stokes friction force as

$$F_D = -6\pi\mu r\mathbf{v} = -\gamma\mathbf{v},$$

where $\mathbf{v}$ — particle velocity vector, r — particle radius, μ — buffer gas dynamic viscosity coefficient, γ — coefficient linear friction.

The light pressure force of a laser beam directed along the straight line $y = x$ (Fig. 1) can be described in the framework of Mie scattering theory as [7]:

$$F_r = \frac{E}{c}A_d Q_{pr},$$

where $E = 4P(x, y, z)/(\pi D^2)$, P — spatial distribution of optical radiation power, A_d — projection illuminated surface area, Q_{pr} — light pressure efficiency. The radiation pressure efficiency Q_{pr} determines the efficiency of pulse transfer from the radiation to the particle. Mie scattering theory provides an accurate means of calculating the efficiency of radiation pressure for a given particle size and laser wavelength [7,8]. Taking into account the transverse distribution of the laser radiation intensity within the framework of the Gaussian profile collimated to the beam width D, the light

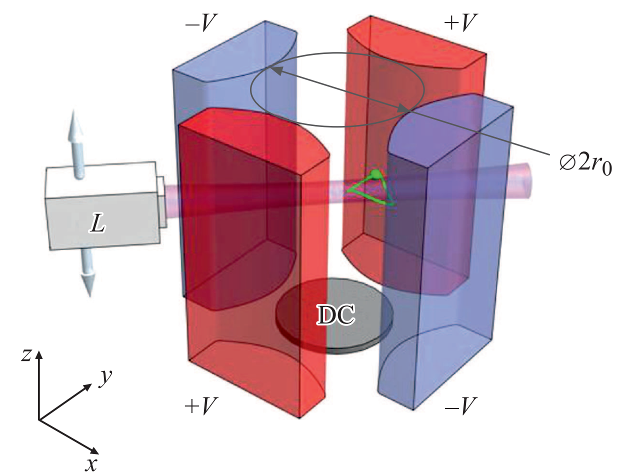

Figure 1. Process of localization in a linear quadrupole trap with a blocking electrode. L — source of laser radiation, $\pm V$ — amplitude of high-frequency voltage on electrodes connected in phase $(+)$ and antiphase $(-)$, respectively, DC — blocking electrode, r_0 — trap radius.

pressure force takes the form

$$F_r(x, y, z) = \exp\left[-\frac{(x-y)^2 + 2z^2}{D^2}\right]\frac{4P_0}{c}\frac{r^2}{D^2}Q_{pr}, \quad (2)$$

where P_0 — nominal power of the laser emitter. It is important to note that the vertical coordinate of the beam is determined by the height of the charged particle localization above the surface of the blocking electrode (denoted DC in Fig. 1). Combining the position of the localization plane of the charged particle XY and the height of the laser emitter is possible by varying the voltage on the blocking electrode.

For $r \ll D$, taking into account the replacements

$$q = \frac{2eV}{m\omega^2 r_0^2}, \ \beta = \frac{2\gamma}{m\omega}, \ \eta = \sqrt{2}\frac{8P}{mc\omega^2}\frac{r^2}{D^3}Q_{pr}, \ \tau = \frac{\omega t}{2}$$

the equations of motion in the XY plane will take the final form

$$\ddot{X} + (2q\cos[2\tau])X + \beta\dot{X} + \eta\exp[-(X-Y)^2] = 0, \quad (3a)$$

$$\ddot{Y} - (2q\cos[2\tau])Y + \beta\dot{Y} + \eta\exp[-(X-Y)^2] = 0, \quad (3b)$$

where the normalization $X = x/D$, $Y = y/D$ is accepted.

In general, in system (3) it is possible to implement a wide range of nonlinear dynamic modes, including chaotic motion. However, the transition between oscillations at frequencies multiples of ω (T_1 mode) and at frequencies multiples of $\omega/2$ (T_2 mode) is of practical importance. This transition can be provided by controlling the light pressure force parameter η through changing the laser radiation power (2).

To identify the dynamic mode depending on the values of the parameters q, η and β, we calculated the senior Lyapunov exponent λ, which characterizes the divergence of trajectories [9] at $q \in [0.65, 0.9]$, $\eta \in [0.1, 0.9]$, $\beta = 0.1$ (Fig. 2, a). The range of the parameter q is determined in accordance with the values of the first stability zone of the Ines–Straet diagram [1]; the initial conditions for the coordinate of each pair of parameters (q, η) are random within a normal distribution with a dispersion of $\sigma_{xy} = 0.05$D.

The stability of localization in the XY plane satisfies the conditions imposed on the parameters (q, β) corresponding to the stability zones of the Ines–Straet diagram in the absence of constant voltage component on the electrodes, regardless of the values of the light pressure parameter η. In this case, as noted above, when η changes, a transition between different dynamic modes is observed. For any small $\eta > 0$, the trajectory is stretched with a shift of the equilibrium position relatively to the geometric center of the trap in the direction of the laser beam (Fig. 3, a, $\eta = 0.1$). The spectral composition of the oscillations is characterized by harmonics that are multiples of the oscillation period of the alternating field ω (mode T_1). The spectrogram of the motion sweep over $0X$ and $0Y$, obtained as a result of the Fourier transform, is shown in the lower part of Fig. 3. In Fig. 2, a the range of values of the parameters (q, η), at which the described motion is formed, is highlighted by a rectangle.

The η parameter increasing leads to the formation of one of the varieties of expanded orbits (Fig. 3, b, $\eta \geqslant 0.198$, T_2 mode). Note that there is no rigorous formulation of the expanded orbit effect; this term is understood as any manifestation of the closed orbital motion of charged particle during localization in the electrodynamic trap under conditions of energy dissipation. Usually, the formation of $\omega/2$-multiple quasi-periodic trajectories is associated with the appearance of nonlinear friction effects during the localization of charged particles under atmospheric pressure [5,6]. However, in this case, expanded orbits mean the mode of oscillations, when the spectral composition of oscillations corresponds to $k\omega/2^{n-1}$, $k = 1, 2, 3, \ldots$, $n \geqslant 1$, where n corresponds to the oscillatory mode T_n. Thus, as a result of external forces action, the expanded orbits are also formed. Optical monitoring in this case makes it possible to excite the transition $T_1 \leftrightarrow T_2$ only for particles in the region of interaction with optical radiation (Fig. 3).

Considering the value of the senior Lyapunov exponent, there is a clear boundary between the modes $T_1 \leftrightarrow T_2$ at the boundary of the first stability region in q ($\eta \leqslant 0.25$, $q > 0.83$, it is marked with a dashed line in Fig. 2, a). The subsequent growth of η leads to a bifurcation with successive doubling of the period and enrichment of the spectral composition of oscillations by the corresponding $k\omega/2^{n-1}$ frequencies (Fig. 3, b, c).

Note that the calculation of the exponents $\lambda(q, \eta)$ was carried out for a fixed value β. At that, change in the reduced friction coefficient affects only the position of the boundary $T_1 \leftrightarrow T_2$ in the space of parameters (q, η), while the linear dependence of the boundary $\eta(q)$ is preserved (Fig. 2, b). When approximating the results of numerical

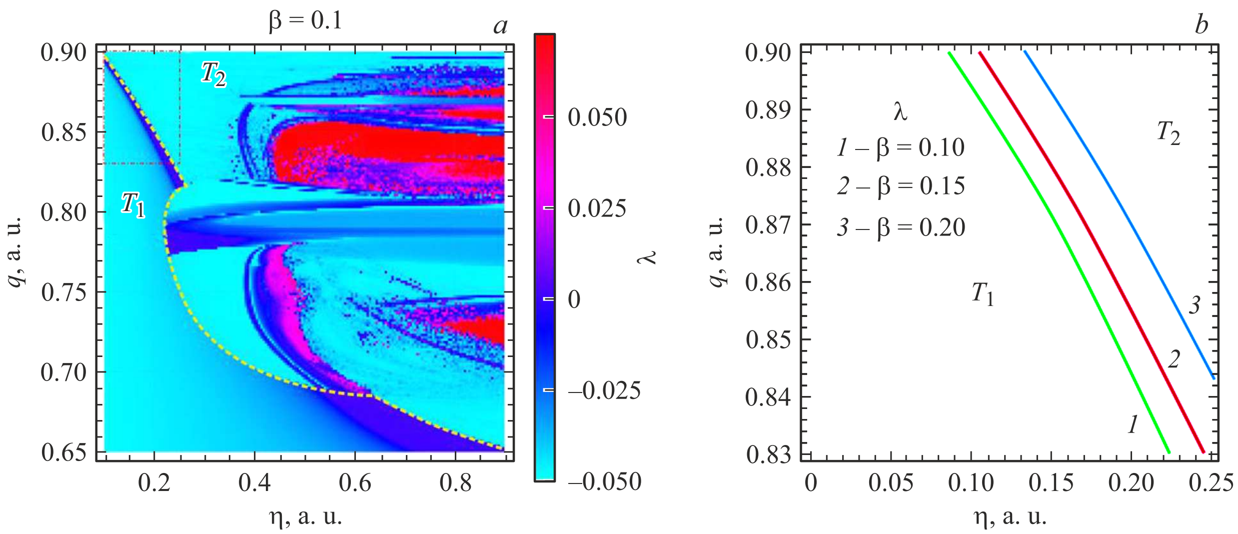

Figure 2. Calculation of the highest Lyapunov index λ for model (3). The dashed line shows the transition $T_1 \leftrightarrow T_2$. a — for $\beta = 0.1$, $q \in [0.65, 0.9]$, $\eta \in [0.1, 0.9]$. b — enlarged fragment for $\beta = 0.1$, 0.15, 0.2. The lines show the boundary $T_1 \leftrightarrow T_2$.

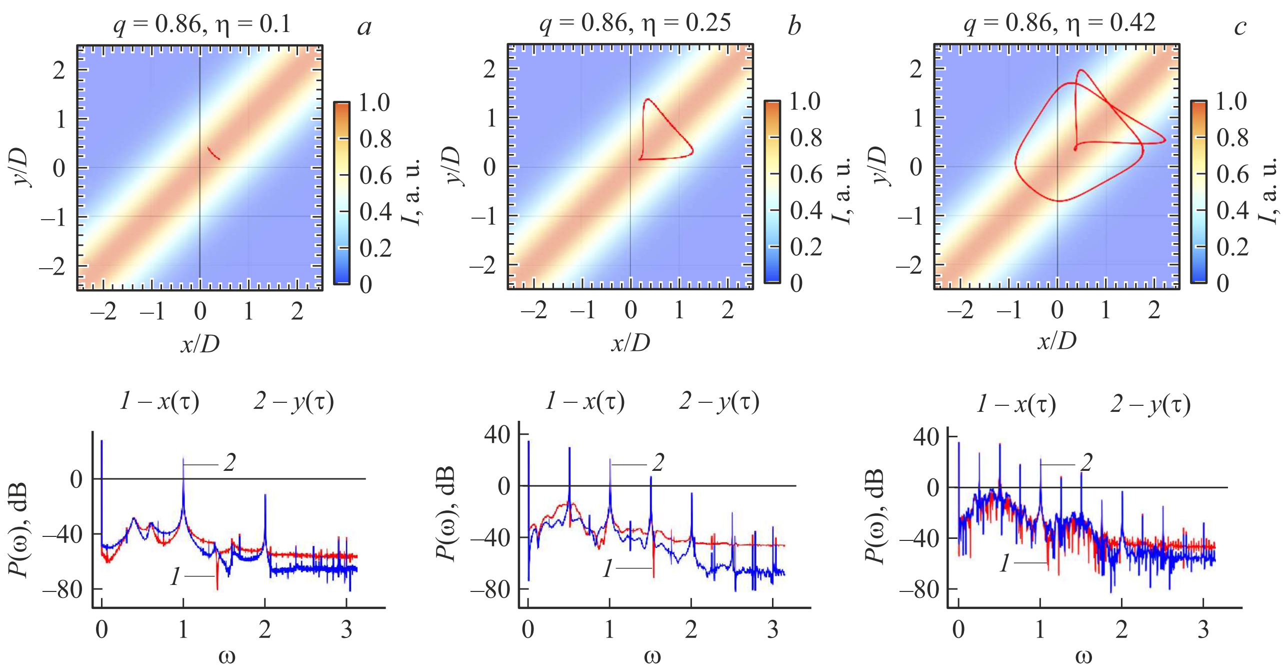

Figure 3. Trajectories and vibrational spectrum of motion at $q = 0.86$. a — $\eta = 0.1$ (T_1), b — $\eta = 0.25$ (T_2), c — $\eta = 0.42$.

calculation, the equation of boundary $\eta(q, \beta)$ for the transition $T_1 \leftrightarrow T_2$ at $0.83 \leqslant q \leqslant 0.903$, $0.05 \leqslant \beta \leqslant 0.25$ becomes

$$\eta = A(\beta) + qB(\beta),$$

where A, B — functions of the reduced coefficient of friction of the form

$$A(\beta) = 2.118 - 0.962\beta + 5.763\beta^2 - 11.405\beta^3 - 68.253\beta^4,$$

$$B(\beta) = -2.556 + 7.431\beta - 49.555\beta^2 + 83.515\beta^3 + 411.228\beta^4 - 955.659\beta^5.$$

Thus, the formation of expanded orbits can be ensured not only by nonlinear energy dissipation, but also by an external optical effect. By focusing and positioning laser radiation, it is possible to control the formation of expanded orbits of individual charged particles.

The optical monitoring of expanded orbits can be used for non-destructive determination of the parameters of a single charged particle by analogy with the NDI method [4]. This fact seems to be especially important in the case of studying nanosized particles, where the implementation of nonlinear energy dissipation is limited [3]. The ability to monitor the formation of expanded orbits and the subsequent determination of the boundaries of transitions between dynamic modes $T_1 \leftrightarrow T_2$ allows not only to find the characteristic size of particle (through the coefficient β), but also to separately determine the mass and charge. Optical monitoring in this case requires additional refinement, taking into account Rayleigh scattering [10].

Funding

The research was carried out as part of project № 22-42-05002 of the Russian Science Foundation.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] B.M. Mihalcea, S. Lynch, Appl. Sci., **11** (7), 2938 (2021). DOI: 10.3390/app11072938
[2] R. Syrovatka, V. Filinov, L. Vasilyak, V. Pecherkin, L. Deputatova, V. Vladimirov, **19** (3), 564 (2021). DOI: 10.5937/jaes0-28342
[3] E.A. Vinitsky, E.D. Black, K.G. Libbrecht, Am. J. Phys., **83** (4), 313 (2015). DOI: 10.1119/1.4902185
[4] V.V. Rybin, S.S. Rudyi, O.O. Kokorina, Vibroeng. Procedia, **32**, 156 (2020). DOI: 10.21595/vp.2020.21537
[5] T. Al-Hababi, M. Cao, B. Saleh, N.F. Alkayem, H. Xu, Sensors, **20** (24), 7303 (2020). DOI: 10.3390/s20247303
[6] B.P. Le Clair, A.E. Hamielec, H.R. Pruppacher, J. Atmospheric Sci., **27** (2), 308 (1970). https://journals.ametsoc.org/view/journals/atsc/27/2/1520-0469_1970_027_0308_ansotd_2_0_co_2.xml
[7] M.M. Abbas, P.D. Craven, J.F. Spann, W.K. Witherow, E.A. West, D.L. Gallagher, M.L. Adrian, G.J. Fishman, D. Tankosic, A. LeClair, R. Sheldon, E. Thomas, Jr., J. Geophys. Res.: Space Phys., **108** (A6), 1229 (2003). DOI: 10.1029/2002JA009744
[8] W.J. Wiscombe, *Mie scattering calculations: advances in technique and fast, vector-speed computer codes* (National Technical Information Service, Boulder, Colorado, 1979). DOI: 10.5065/D6ZP4414
[9] N.N. Rosanov, N.V. Vysotina, JETP, **128** (6), 840 (2019). DOI: 10.1134/S1063776119060074.
[10] M.Y. Amusia, A.S. Baltenkov, JETP Lett., **111** (8), 472 (2020). DOI: 10.1134/S0021364020080019.