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Localization of electromagnetic waves in a zigzag lattice of waveguides with competing cubic and quintic nonlinear responses

© E.V. Kazantseva, A.I. Maimistov

National Research Nuclear University „MEPhI", Moscow, Russia

e-mail: elena.kazantseva@gmail.com, aimaimistov@gmail.com

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The zigzag waveguide array is considered, where even waveguides are composed of optically linear material, and their nearest neighbors are characterized by positive cubic nonlinearity and negative fifth-order nonlinearity. In the continuum (long-wave) approximation the solutions of the system of coupled wave equations are found, which describe waves localized in the lattice and harmonic propagating waves.

Keywords: nonlinear waves, discrete photonics, waveguides, solitons.

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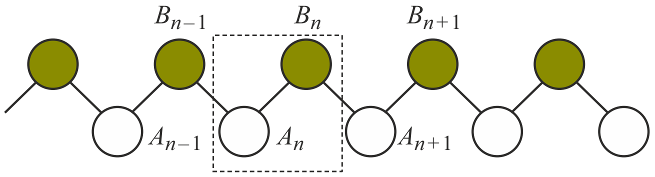
1. Introduction

The localization of electromagnetic radiation is a fundamental problem that has attracted attention for many years. In linear optics, this purpose is achieved with the help of resonators (micro- and nanoresonators i.e.photon dots), waveguides, and excitation of surface waves. Nonlinear optics provides another way to localize radiation, which is known as the formation of solitons, or in general, stable solitary waves. In 1967 the term „photonics" [1] entered science and later, approximately from the 70s of the last century, a branch of optics began to form, where phenomena were studied in which photons play the main role, similar to that which electrons play in electronics. Discrete media formed by a periodic distribution of the permittivity in space have become important in photonics. Due to the peculiarities of the spectrum of electromagnetic radiation, which contains bandgaps and allowed bands, such media are called photonic crystals [2]. Another example of discrete media is metamaterials [3–5], which are formed by metallic or dielectric elements of a complex shape periodically located in a dielectric matrix, having dimensions comparable to the radiation wavelength. In a broad sense, metamaterials include arrays of quantum dots, microresonators, Josephson contacts (SQUIDs), layers of thin films of metal or graphene, and waveguides. If these „meta-atoms" are located periodically in space, then one speaks of photonic lattices of meta-atoms. The results concerning the nonlinear localization of light in discrete structures are presented in a large number of reviews and books, among which it is enough to mention [4,6].

In recent years, publications have appeared on the results of studying the optical properties of photonic lattices formed from one-dimensional and two-dimensional arrays of waveguides, and the lattice cell contains more than two waveguides. It is assumed that radiation can penetrate into neighboring waveguides only due to interruption of total internal reflection. This situation is analogous to an electronic crystal under strong coupling conditions. The radiation frequency in a photonic lattice depends on the propagation constant along the waveguide and on a discrete set of transverse wave numbers. Thus, the dispersion curves form surfaces above the Brillouin zone, similar to the allowed bands of electrons in a crystal. The curvature of the zones characterizes the diffraction of light on a photonic lattice, which is referred to as discrete diffraction [6,7].

In addition to photonic lattices with two or more atoms in the lattice cell, zigzag lattices [11], binary lattices [8–10] and zigzag binary lattices [12,13] were studied. In the linear case, radiation is not localized in such photonic lattices. The Kerr-type nonlinearity effect can lead to the formation of a discrete soliton [6,14,15]. At present, interest in binary photonic lattices has again arisen in connection with research in the field of non-Hermitian photonics [16,17]. In a lattice cell, one waveguide is an absorber, and the second waveguide is an amplifier.

In most cases, lattices of dielectric waveguides were considered, the nonlinear properties of which are described by the Kerr nonlinearity. But a weaker quadratic nonlinearity can lead to the formation of discrete solitons [6,18–20]. The effect of nonlinearity saturation on the formation of discrete solitons has been studied for simple lattices (i.e., with one „atom" in the lattice cell) [21–23]. Here one will consider a model of discrete photonics, which is a generalization of a binary zigzag lattice (Figure) for the case of competing nonlinearities of the third and fifth orders. The system of equations of coupled waves can be solved in the continuum (or long-wavelength) approximation, which leads to localized distributions of electromagnetic wave intensities along the lattice waveguides.



Waveguide configuration corresponding to a one-dimensional zigzag lattice. The dashed line marks the lattice cell. Integers n enumerate lattice cells.

2. Main equations of the model

The propagation of electromagnetic radiation in a line of waveguides is usually described on the basis of the coupled wave theory [24]. An electromagnetic wave is represented by a linear superposition of quasi-harmonic waves localized in the n-th waveguide. Using the approximation of slowly varying amplitudes, a differential–difference equation is derived from the wave equation for the amplitudes of coupled waves in the n-th waveguide [25–29].

For a zigzag lattice shown schematically in the Figure, the slowly varying amplitudes of the electric fields A_n and B_n are determined by a system of equations of the following form

$$i\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\xi}\right)A_n+e^{i\delta_b\xi}(B_n+B_{n-1})$$
$$+c_2(A_{n+1}+A_{n-1})+G_1[A_n]A_n=0, \tag{1}$$

$$i\left(\frac{\partial}{\partial\tau}+\sigma\frac{\partial}{\partial\xi}\right)B_n+e^{-i\delta_a\xi}(A_{n+1}+A_n)$$
$$+c_2(B_{n+1}+B_{n-1})+G_2[B_n]B_n=0, \tag{2}$$

where ξ is spatial coordinate, measured in units of coupling length L_c, τ is time, measured in units of $t_c=L_c/v_g$, v_g is group velocity of electromagnetic wave in a waveguide. In the model under consideration, it is assumed that the group velocities are the same for all waveguides. The parameter $\delta_a=(\beta_b-\beta_a)L_c$ is the difference between the propagation constants β_b and β_a of waves localized in neighboring type B and A waveguides, respectively. The terms $G_1[A_n]$ and $G_2[B_n]$ describe the local nonlinear properties of the waveguides. Parameter c_2 is the ratio of coupling constants between waveguides following nearest neighbors and nearest neighbors. The symbol $\sigma=\pm1$ takes into account the fact that type B waveguides can be made of a material with positive ($\sigma=1$) or negative ($\sigma=-1$) refraction.

Here, a model will be considered in which only type A waveguides are nonlinear, and their nonlinear properties are described by terms in equations (1) and (2) of the following form:

$$G_1[A_n]=\mu(|A_n|^2-\varrho|A_n|^4),\quad G_2[B_n]=0.$$

In the work [13] the case of Kerr nonlinearity was considered: $G_1[A_n]=\mu\,|A_n|^2$. It is considered that the phase-matching condition is fulfilled: $\delta_a=0$.

If we restrict ourselves to the case of continuous radiation and make the replacement of variables $A_n=(-1)^n\tilde{A}_n$ and $B_n=(-1)^n\tilde{B}_n$, then equations (1) and (2) take the following form:

$$i\frac{\partial}{\partial\xi}\tilde{A}_n+(\tilde{B}_n-\tilde{B}_{n-1})-c_2(\tilde{A}_{n+1}+\tilde{A}_{n-1})$$
$$+\mu(|A_n|^2-\varrho|A_n|^4)\tilde{A}_n=0, \tag{3}$$

$$i\sigma\frac{\partial}{\partial\xi}\tilde{B}_n+(\tilde{A}_n-\tilde{A}_{n+1})-c_2(\tilde{B}_{n+1}+\tilde{B}_{n-1})=0. \tag{4}$$

3. Nonlinear waves in the continuum approximation

Let the radiation propagates along the waveguides as a harmonic wave with the wave number $k_\xi=\beta$. Then one can make the replacement $\tilde{A}_n(\xi)=e^{i\beta\xi}a_n$ and $\tilde{B}_n(\xi)=e^{i\beta\xi}b_n$, which reduces the system of differential-difference equations to the system of difference equations:

$$-\beta a_n+(b_n-b_{n-1})-c_2(a_{n+1}+a_{n-1})$$
$$+\mu\left(|a_n|^2-\varrho|a_n|^4\right)a_n=0, \tag{5}$$

$$-\sigma\beta b_n+(a_n-a_{n+1})-c_2(b_{n+1}+b_{n-1})=0. \tag{6}$$

All the amplitudes in these equations can be considered as real quantities, since there are no complex numbers in (5) and (6).

In the continuum approximation, this system of equations reduces to the following system:

$$\frac{\partial b}{\partial\xi}-(2c_2+\beta)a+\mu\left(a^3-\varrho a^5\right)=0, \tag{7}$$

$$\frac{\partial a}{\partial\xi}+(2c_2+\sigma\beta)b=0. \tag{8}$$

The system of equations (7) and (8) can be reduced to one equation for the field $a(\xi)$:

$$\frac{\partial^2 a}{\partial\xi^2}+(2c_2+\sigma\beta)(2c_2+\beta)a$$
$$-\mu(2c_2+\sigma\beta)\left(a^3-\varrho a^5\right)=0.$$

Let $\sigma=-1$. In this case, the resulting equation can be written in the following form:

$$\frac{\partial^2 a}{\partial\xi^2}=p^2a-\kappa(a^3-\varrho a^5), \tag{9}$$

where $p^2=\beta^2-4c_2^2$ and $\kappa=\mu(\beta-2c_2)$. In the case when $\beta^2<4c_2^2$, equation (9) has no bounded solutions with zero asymptotics at infinity. However, if $\beta^2>4c_2^2$, then bounded solutions exist. Thus, the interaction with the next after nearest neighbors leads to the appearance of a gap (bandgap) for propagation constants β.

Equation (9) can be understood as the equation of motion of mass particle in a given potential. This equation has a first integral (which is obtained in the standard way):

$$\left(\frac{\partial a}{\partial \xi}\right)^2 - p^2a^2 + \frac{\kappa}{2}\left(a^4 - \frac{2\varrho}{3}a^5\right) = I_1.$$

For the boundary conditions corresponding to localized waves, the I_1 integral is equal to zero[1].

If replace $a = a_0u^{-1/2}$, then for the new dependent variable $u(\xi)$, one can get the equation

$$\left(\frac{1}{2}\frac{\partial u}{\partial \xi}\right)^2 = p^2u^2 - \frac{\kappa a_0^2}{2}u + \frac{\kappa\varrho a_0^4}{3}.$$

Having defined a new independent variable $\tau = 2p\xi$ and a normalization parameter $a_0^2 = 4p^2/\mid\kappa\mid$, one can write the equation obtained above in a simple form:

$$\left(\frac{\partial u}{\partial \tau}\right)^2 = \nu u^2 - 2u + \theta = (u-1)^2 - (1-\theta),$$

where $\theta = (16/3)\varrho p^2/\kappa$, $\nu = \kappa/\mid\kappa\mid$. In what follows, it will be assumed that $\kappa > 0$ is a third-order focusing nonlinearity.

The obvious replacement is $u - 1 = \Delta w$, where the parameter Δ is arbitrary, leads to the equation

$$\Delta^2\left(\frac{dw}{d\tau}\right)^2 = \Delta^2w^2 - (1-\theta). \qquad (10)$$

In the value range θ: $0 < \theta < 1$, one can define $\Delta^2 = 1 - \theta$. In this case, the solution of equation (10) is written as $w(\tau) = \cosh(\tau - \tau_0)$, where τ_0 is the integration constant. Therefore, $u = 1 + \Delta\cosh(\tau - \tau_0)$, and in the original variables this solution has the form

$$a^2(\tau) = \frac{a_0^2}{1 + \Delta\cosh(\tau - \tau_0)}. \qquad (11)$$

In the region $\theta > 1$, the parameter Δ^2 is chosen as $\Delta^2 = \theta - 1 > 0$, and the equation for w takes the following form:

$$\left(\frac{dw}{d\tau}\right)^2 = w^2 + 1.$$

Its solution is $w(\tau) = \sinh(\tau - \tau_0)$. Therefore,

$$a^2(\tau) = \frac{a_0^2}{1 + \Delta\sinh(\tau - \tau_0)}. \qquad (12)$$

The right side of this expression vanishes at some point on the τ axis and changes the sign, therefore, this solution has no physical meaning.

[1] It is enough that the amplitude a and its derivative would turn to zero at one of the lattice boundaries

At the point $\theta = 1$, the parameter Δ turns to zero. The function satisfies the equation

$$\left(\frac{dw}{d\tau}\right)^2 = w^2, \qquad (13)$$

or $dw/d\tau = \pm w$. This implies two solutions: $w^{(\pm)} = \exp[\pm(\tau - \tau_0)]$. Therefore,

$$a^{(\pm)2}(\tau) = \frac{a_0^2}{1 + \exp[\pm(\tau - \tau_0)]}. \qquad (14)$$

Using this result, one can write it in another form

$$a^{(\pm)2}(\tau) = \frac{a_0^2\exp[\mp(\tau - \tau_0)/2]}{2\cosh[(\tau - \tau_0)/2]}. \qquad (15)$$

These solutions describe domain walls similar to those found in the case of a rhombic lattice [30]. But such a wave is not localized in type A waveguides.

When there is no competing nonlinearity, the medium has only third-order nonlinearity, and $\theta = 0$. In this limiting case, it follows from (10) that

$$a^2(\tau) = \frac{a_0^2}{1 + \cosh(\tau - \tau_0)} = \frac{2a_0^2}{\cosh^2[(\tau - \tau_0)/2]}.$$

This yields the solution for the case of the Kerr nonlinearity:

$$a(\tau) = \frac{\sqrt{2}a_0}{\cosh[(\tau - \tau_0)/2]}.$$

To obtain the distribution of fields in type B waveguides, one must refer to equation (8), from which it follows that

$$(2c_2 - \beta)b(\xi) = -\frac{da}{d\xi} = -2p\frac{da}{d\tau}.$$

Because

$$a(\tau) = \frac{a_0}{(1 + \Delta\cosh\tau)^{1/2}},$$

the derivative with respect to τ of the field amplitude a is

$$\frac{da}{d\tau} = \frac{(-1/2)a_0}{(1 + \Delta\cosh\tau)^{3/2}}\Delta\sinh\tau.$$

Remembering that $p = \sqrt{\beta^2 - 4c_2^2}$, one can get the final formula for the amplitude b:

$$b(\xi) = -\sqrt{\frac{\beta + 2c_2}{\beta - 2c_2}}\frac{a_0\Delta\sinh\tau}{(1 + \Delta\cosh\tau)^{3/2}}, \quad \tau = 2p\xi. \quad (16)$$

Comparing the obtained expressions for $a(\xi)$ and $b(\xi)$, one can see that the distribution of fields $a(\xi)$ is symmetric, and distribution of fields $b(\xi)$ is antisymmetric. This is reflected in equation (8) and is its consequence. Accounting for higher order derivatives can change this result.

The solution in the form of a domain wall is found similarly. By setting (where $s = \pm1$)

$$a^{(s)}(\tau) = \frac{a_0\exp[-s\tau/4]}{\sqrt{2}\cosh^{1/2}(\tau/2)},$$

the derivative can be calculated

$$\frac{da^{(s)}}{d\tau} = \frac{a_0 \exp[-s\tau/4]}{\sqrt{2}\cosh^{1/2}(\tau/2)} \left(-\frac{s}{4}\right)$$

$$+ \frac{a_0 \exp[-s\tau/4]}{\sqrt{2}\cosh^{3/2}(\tau/2)} \left(-\frac{1}{4}\sinh(\tau/2)\right) =$$

$$= -\frac{a_0 \exp[-s\tau/4]}{4\sqrt{2}\cosh^{1/2}(\tau/2)} \left[s + \tanh(\tau/2)\right].$$

The final formula for the field $b(\xi)$ will have the following form:

$$b^{(\pm)}(\xi) = -\sqrt{\frac{\beta + 2c_2}{\beta - 2c_2}} \frac{a_0 e^{\mp\tau/4}}{2\sqrt{2}\cosh^{1/2}(\tau/2)} \left[\tanh(\tau/2) \pm 1\right],$$

$$\tau = 2p\xi. \tag{17}$$

4. Conclusion

In this paper we consider one of the models of discrete photonics, namely a model in which two waveguide lines are shifted relative to each other, which allows interaction, in addition to nearest neighbors, also with waveguides following the nearest neighbors. Such a zigzag lattice was considered earlier in [11,12]. If the lattice cell contains two types of waveguides, one waveguide is linear, and the other is characterized by Kerr nonlinearity, then radiation can be localized in such a lattice if a certain threshold condition is met [13]. But this localization takes place in the direction of the axis of the waveguides. Localization in the transverse direction, when the radiation is concentrated in only a few waveguides, has not been investigated. If assume that the field strength changes little from site to site of the lattice, one can use the continuum approximation and pass from difference equations to differential ones. The solution of the equations obtained in this way demonstrates the existence of localized distributions of the amplitudes (or intensities) of the electromagnetic field along the waveguides. The parameters of the localized intensity distributions depend on the ratio of the interaction constants between the nearest waveguides and the waveguides following the nearest ones. This parameter can change when the geometric characteristics of the zigzag lattice change.

In addition to localized distributions of amplitudes over waveguides, there are distributions in the form of domain walls. It should be noted that the presence of competing nonlinearities ensured the existence of a solution for the domain wall type. For a rhombic lattice, the same solution took place, taking into account the competing cubic and quintic nonlinearities [30]. The same solution was obtained when considering the problem on the propagation of extremely short pulses in a medium described by the Duffing model [31]. It can be concluded that the formation of a distribution of field strengths (or intensities) of electromagnetic waves in the form of domain walls is inherent in waveguide lattices with non-Kerr nonlinearity.

The model considered here can be generalized in order to study the possibility of the existence of completely localized distributions in waveguides, both in the transverse and in the longitudinal directions.

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Conflict of interest

The authors declare that they have no conflict of interest.

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