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Evaluation of the thresholds of stimulated nonlinear scattered CW laser radiation in high-power fiber amplifiers

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The paper describes a new approach for determining the threshold for stimulated nonlinear scattering in active and passive optical fibers. It is proposed to fix the power of the Stokes component at a certain level, which will make it possible to determine the threshold values of laser radiation powers, which characterize the manifestation of stimulated nonlinear scattering at the expected power level, in a convenient way from the applied point of view. Verification of the obtained expressions is given.

Keywords: stimulated nonlinear scattering, fiber amplifier, stimulated nonlinear scattering threshold.

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Introduction

With increasing power and narrowing the width of the spectral line of the radiation of fiber laser systems, the inevitable manifestation of such physical phenomena as stimulated scattering occurs. The main types of stimulated scattering are stimulated Raman (SRS) [1,2] and Mandelshtam–Brillouin (SMBS) scattering [3–5]. These scatterings are inelastic and are characterized by experimental thresholds of manifestation. The thresholds are determined by the value of the pumping radiation power for stimulated scattering, at which the Stokes radiation (index „$\mathbb{S}$“) will be reliably registered in the experiment [5]. To eliminate during further analysis of the confusion in the use of the term „pumping“the pumping radiation for stimulated nonlinear scattering will be called excitation laser radiation according to the terminology in [4] (index „$\mathbb{P}$“).

In the case of fiber optic media the issue of developing an analytical method that allows estimation of the experimental thresholds for the manifestation of stimulated nonlinear scattering in passive optical fibers was first considered by Smith in his paper [6]. The essence of this method lies in the fact that a sufficient (from the experimental point of view) value of the integral gain of the Stokes component (SRS or SMBS) is determined from the condition that the power of this component reaches a value equal to the power of the exciting laser radiation at the observation point (for SRS — co-directional with the exciting radiation at the output of the optical fiber and opposite to it, for SMBS — at the point of input of the exciting radiation). This choice was fully justified, since the goal pursued by Smith in his analysis is to determine the threshold power of exciting laser radiation for stimulated nonlinear scattering, upon reaching which it becomes possible to detect Stokes radiation in extended low-power ($\sim$mW) fiber-optical communication lines. The corresponding analytical expressions were obtained:

$$P_{R+}^{th} \approx 16\,\frac{A_{\text{eff}}}{g_R L_{\text{eff}}}, \tag{1a}$$

$$P_{R-}^{th} \approx 20\,\frac{A_{\text{eff}}}{g_R L_{\text{eff}}}, \tag{1b}$$

$$P_{B}^{th} \approx 16\,\frac{A_{\text{eff}}}{g_R L_{\text{eff}}}, \tag{2}$$

where $P_{R,B}^{th} = P_{\mathbb{P}}(0)$ — threshold values of the excitation power at the point $z = 0$ of the optical fiber for SRS (index „R“) and SMBS (index „B“), the indices „+“ and „–“ determine the scattering direction; A_{eff} — effective fiber cross-section area; $L_{\text{eff}} = 1/\alpha_{\mathbb{P}}\big(1 - \exp(-\alpha_{\mathbb{P}} L)\big)$ — effective interaction length (L — optical fiber length; $\alpha_{\mathbb{P}}$ — linear loss coefficient at the exciting laser wavelength); g_R and g_B — gains of SRS and SMBS, respectively. The value of the threshold power of the exciting radiation was determined based on the conditions: $P_{\mathbb{P}}(L) = P_{\mathbb{S}}(L)$ — for scattering codirectional with the exciting radiation, and $P_{\mathbb{P}}(0) = P_{\mathbb{S}}(0)$ — for scattering towards the exciting radiation.

At laser radiation power levels significantly exceeding the level considered in [6], and the transition to fiber-optic amplifiers, the Smith's condition becomes less relevant. However, note that expressions (1) and (2) are actively used in the theoretical analysis of the limit power characteristics of fiber laser systems [7–11], despite the fact that they were obtained for passive optical fibers with fixed parameters [6].

In later papers [12–15] Smith's criterion about the power equality of the Stokes component of the exciting radiation power at the point of observation was replaced by a more flexible one — the Stokes component power is equal to the fraction (generally arbitrary) of the exciting radiation power at the point of observation. This transition was due to nominal increase of the optical radiation power under consideration and convenience from the point of view of experimental registration, in particular, the SRS effect codirectional with the exciting laser radiation (the manifestation of this scattering is determined from the spectrum of the output radiation, in which the Stokes component is displayed in relative units at the level of the its exciting laser radiation). With a further increase in the power level of the exciting laser radiation to ∼kW, this approach also loses its relevance, especially with regard to the SMBS effect.

In the present paper we propose a new approach for determining the value of the integral gain of stimulated nonlinear scattering in high-power fiber amplifiers, which corresponds to their experimental manifestation threshold. It is proposed, when obtaining analytical expressions that make it possible to estimate the value of the threshold power of exciting laser radiation, corresponding to the manifestation of stimulated nonlinear scattering in optical fibers, to base on the empirical nature of this threshold, namely, to record the value of the Stokes radiation power at a specific level, i. e. the power value reliably observed in the experiment.

1. Threshold formulas determining SRS and SMBS occurrence in high-power fiber amplifiers

In the course of our discussions we will assume that the effects of SRS and SMBS for laser radiation amplified in fiber amplifier prevail over similar processes for pumping radiation. Thus, the amplified laser radiation (or the radiation of the main laser signal) will act as the exciting laser radiation, therefore below, instead of the index „$\mathbb{P}$“ we will write „$\mathbb{S}$“. We consider the case when the signal and Stokes radiation propagate inside the core of an active optical fiber. Let us describe the physical model of an active optical fiber, which we will use further.

We consider that the optical fiber is a cylinder, the length of which significantly exceeds its linear dimensions in the cross-section ($a \ll L$, where a — the radius of the active region (core) of the fiber, L — fiber length) surrounded by a medium with a lower refractive index. The active impurity (ions of rare earth elements) is uniformly distributed in the volume of the cylinder, the structure of the energy levels is known, i. e. their location and the distance between them are known, the probabilities of transitions between levels are also considered to be known.

We will consider the stationary mode of amplification of cw laser radiation, which is implemented according to the quasi-two-level scheme [16,17]. The propagation of optical radiation inside the active region is described in terms of the energy balance [17]. Analytical solutions for the distribution of pumping radiation and signal in the optical fiber amplifier were obtained not considering the amplification of spontaneous radiation and nonlinear scattering, in the Hardy-Kelson strong pumping approximation [18], and also taking into account the fact that the probability of a spontaneous transition from excited state, which determines its lifetime (τ), is negligible. In this case it is assumed that at a sufficiently high pumping power the signal radiation power in the amplifier becomes high enough to excite the active medium and suppress spontaneous emission:

$$P_p(z)\lambda_p\Gamma_p\sigma_{12}(\lambda_p) > P_s^{sat}(z)\lambda_s\Gamma\sigma_{12}(\lambda_s),$$

$$P_s(z) \gg P_s^{sat}(z), \quad P_s^{sat} = \frac{hcA_{co}}{\lambda_s\Gamma_s\left(\sigma_{12}(\lambda_s)+\sigma_{21}(\lambda_s)\right)}, \quad (3)$$

where λ_s and λ_p — signal and pump wavelengths; h — Planck's constant; c — speed of light in vacuum; $\sigma_{12}(\lambda)$ and $\sigma_{21}(\lambda_s)$ — cross sections of stimulated absorption and emission, respectively; $P_k(z)$ — power of optical radiation ($k = p$ — pumping, $k = s$ — signal); A_{co} — active fiber core area ($A_{co} = \pi a^2$, a — core radius); Γ_s — effective overlap integral for signal radiation [19]; $\Gamma_p = A_{co}/A_{cl}$ — effective overlap coefficient for pump radiation (A_{cl} — area of pump radiation input region).

As Smith's paper [6] we assume that at the birth of the Stokes component, its power is much less than the signal power. This assumption makes it possible to use a fixed distribution function (the result of the analytical solution of the corresponding differential equation) to distribute the signal radiation power along the fiber length. We also assume that the amplification of the Stokes component mainly occurs due to stimulated scattering, i. e. the amplification due to the activity of the laser medium is not taken into account.

1.1. Analytical solutions for distribution of optical radiation powers

We write differential equations for changing the signal powers and Stokes components scattered in the forward „+“ and backward „–“ directions in the framework of the above approximations:

$$\frac{dP_s(z)}{dz} = \Big(g\big(P_s(z), z\big) - \alpha_s\Big)P_s(z) \quad (4a)$$

$$\frac{dP_{\mathbb{S}}^{\pm}}{dz} = \left(\frac{g_{\mathbb{S}}}{A_{\text{eff}}^{s,\mathbb{S}}}P_s(z) - \alpha_{\mathbb{S}}\right)P_{\mathbb{S}}^{\pm}(z), \quad (4b)$$

and the concentrations of active impurity ions in different energy states, according to the accepted physical model, are determined by the relations:

$$N_2(z) \approx \frac{R_{12}+W_{12}}{W_{12}+W_{21}}N,$$

$$N_1(z) = N - N_2(z), \quad (5)$$

here

$$R_{12} = \frac{\sigma_{12}(\lambda_p)\lambda_p}{hc}\frac{P_p(z)\Gamma_p}{A_{co}},$$

$$W_{12} = \frac{\sigma_{12}(\lambda_s)\lambda_s}{hc}\frac{P_s(z)\Gamma_s}{A_{co}}, \quad W_{21} = \frac{\sigma_{21}(\lambda_s)\lambda_s}{hc}\frac{P_s(z)\Gamma_s}{A_{co}},$$

$g\left(P_s(z), z\right) = \Gamma_s\left(\sigma_{21}(\lambda_s)N_2(z) - \sigma_{12}(\lambda_s)N_1(z)\right)$ — signal radiation gain; N — active impurity concentration in fiber core; $P_k(z)$ — optical power ($k = p$ — pump, $k = s$ — signal, $k = \mathbb{s}$ — Stokes radiation); $g_{\mathbb{s}}$ — gain of the Stokes component during stimulated scattering, the value of this gain depends on the frequency shift of signal radiation, and in the case of SMBS, also on the width of the spectral line of the signal; α_s and $\alpha_{\mathbb{s}}$ — linear loss coefficients for signal radiation and Stokes component;

$$A_{\text{eff}}^{s\mathbb{s}} = \frac{\int_0^{2\pi} d\phi \int_0^{\infty} i_{\upsilon_s,V_s}(r,\phi)rdr \int_0^{2\pi} d\phi \int_0^{\infty} i_{\upsilon_s,V_s}(r,\phi)rdr}{\int_0^{2\pi} d\phi \int_0^{\infty} i_{\upsilon_s,V_s}(r,\phi)rdr},$$

where $i_{\upsilon_s,V_s}(r,\phi)$ — normalized distribution function optical radiation intensity at frequency υ_j $(j = \mathbb{s}, s)$ in the optical fiber section.

After substituting (5) into (4a), this differential equation is transformed into

$$\frac{dP_s(z)}{dz} = \Gamma_p\sigma_{12}(\lambda_p)N\frac{\lambda_p}{\lambda_s}P_p(z) - \alpha_s P_s(z),$$

its solution has the form

$$P_s(z) = \left(\Gamma_p\sigma_{12}(\lambda_p)N\frac{\lambda_p}{\lambda_s}\int_0^z P_p(\xi)e^{\alpha_s\xi}d\xi + P_s(0)\right)e^{-\alpha_s z}, \quad (6)$$

where $P_s(0)$ is value of the signal power at the amplifier input. We write the solutions of equations (4b) in the form

$$P_s^+(z) = P_s^+(0)\exp\left(g_{\mathbb{s}}\frac{1}{A_{\text{eff}}^{s\mathbb{s}}}\int_0^z P_s(\xi)d\xi - \alpha_{\mathbb{s}}z\right). \quad (7)$$

$$P_s^-(z) = P_s^-(L)\exp\left(g_{\mathbb{s}}\frac{1}{A_{\text{eff}}^{s\mathbb{s}}}\int_L^z P_s(\xi)d\xi - \alpha_{\mathbb{s}}(L - z)\right). \quad (8)$$

It is easy to see that we left the distribution function of the pumping radiation along the length of the fiber in function (6) in its general form. This was done to carry out a primary analysis without focusing on the type of active fiber within the accepted physical model, or rather, on the method of introducing pumping radiation into the active cylindrical core.

1.2. Threshold formulas

When analyzing the manifestations of stimulated nonlinear scattering, it is proposed to assume the empirical nature of this phenomenon threshold, namely, to fix the value of the Stokes radiation power at a specific level, i.e. the power value reliably observed in the experiment or admissible in a particular setting of the experiment. The threshold condition for the manifestation of stimulated nonlinear scattering then takes the form

$$P_{\mathbb{s}}^+(L) = \mathbb{F}, \quad (9)$$

$$P_{\mathbb{s}}^-(0) = \mathbb{B}. \quad (10)$$

Here $\mathbb{F}$ and $\mathbb{B}$ are the Stokes radiation powers in forward and backward scattering, respectively. In contrast to the approach described in the papers [6,14,15], when the Stokes component power is fixed at the level of a fraction of the exciting radiation power at the observation point, the proposed approach makes it possible during threshold expressions determination to ignore the function that determines the distribution of the excitation radiation power along the length of the fiber, and thereby simplify the algebraic equations for finding the corresponding integral gain. Note that this method makes it possible to carry out a preliminary analysis of high-power fiber amplifiers based on the requirements imposed on them in terms of the power level of nonlinear scattering, which is undoubtedly more convenient than the traditional approach.

1.2.1. Forward scattering

In this case, scattering occurs only due to the SRS effect. According to (7), the expression that determines the Stokes radiation power during forward scattering at the point $z = L$ can be written in general form as follows (hereinafter, index „$\mathbb{s}$“is replaced with „R“):

$$P_R^+(L) = P_R^+(0)\exp\left(g_R\frac{P_s(0)L_{\text{eff}}^s}{A_{\text{eff}}^{sR}} - \alpha_R L\right), \quad (11)$$

where L_{eff}^s is the effective interaction length, which is given by the expression [6,14]

$$L_{\text{eff}}^s = \frac{\int_0^L P_s(z)dz}{P_s(0)}.$$

Let us take into account that the distribution function of the signal radiation power along the length of the active fiber is determined by expression (6), then

$$L_{\text{eff}}^s = \frac{\int_0^L P_s(z)dz}{P_s(0)} = \mathbb{L}_{\text{eff}}^s + \frac{\Upsilon_p}{P_s(0)},$$

$$\mathbb{L}_{\text{eff}}^s = \frac{1}{\alpha_s}(1 - e^{-\alpha_s L}),$$

$$\Upsilon_p = \Gamma_p\sigma_{12}(\lambda_p)N\frac{\lambda_p}{\lambda_s}\left(\int_0^z P_p(\xi)e^{\alpha_s\xi}d\xi\right)e^{-\alpha_s z}dz.$$

If we introduce the designation

$$\mathbb{P}(0) = P_s(0) + \frac{\Upsilon_p}{\mathbb{L}_{\text{eff}}^s},$$

then the expression (11) is transformed into

$$P_R^+(L) = P_R^+(0)e^{g_R \frac{\mathbb{P}_s(0)\mathbb{L}_{\text{eff}}^s}{A_{\text{eff}}^{sR}} - \alpha_R L}. \quad (12)$$

Value $P_R^+(0)$ represents the actually absent seed Stokes radiation in the system. According to [3,6] this value can be replaced by an equivalent value determined in terms of the effective width of the Stokes radiation spectrum (B_{eff}^R):

$$P_R^+(0) = h\nu_R B_{\text{eff}}^R, \quad (13a)$$

where ν_R is frequency of Stokes radiation during SRS, and B_{eff}^R for the Lorentz contour of the spectral line [6,20]:

$$B_{\text{eff}}^R = \frac{\Delta\nu_R}{2}\sqrt{\frac{\pi}{G_R}}, \quad G_R = g_R \frac{P_S(0)L_{\text{eff}}^s}{A_{\text{eff}}^{sR}} = g_R \frac{\mathbb{P}_S(0)\mathbb{L}_{\text{eff}}^s}{A_{\text{eff}}^{sR}}, \quad (13b)$$

where $\Delta\nu_R$ is the linewidth of the Stokes radiation spectrum during SRS.

In accordance with expressions (12) and (13) the equation for finding the integral gain G_R^{th} based on threshold condition (9) will be written in the form

$$h\nu_R \frac{\Delta\nu_R}{2}\sqrt{\frac{\pi}{G_R^{th}}}\, e^{G_R^{th} - \alpha_R L} = \mathbb{F}$$

or

$$(G_R^{th})^{1/2} e^{-G_R^{th}} = \frac{1}{\mathbb{F}} h\nu_R \frac{\Delta\nu_R \pi}{2} e^{-\alpha_R L}, \quad (14)$$

and the threshold value of the initial signal power ($z = 0$), according to (13b):

$$P_s^{th} = \frac{1}{\mathbb{L}_{\text{eff}}^s}\left(\frac{G_R^{th} A_{\text{eff}}^{sR}}{g_R} - \Upsilon_p\right). \quad (15)$$

The transition to the standard expression for passive optical fibers is carried out at $\Upsilon_p = 0$, but, undoubtedly, in contrast to [6], another criterion is used when calculating G_R^{th}. It is easy to see that, according to (14), with the approach, we have chosen, the coefficient G_R^{th} depends only on the length of the considered fiber and the linear loss coefficient at the frequency of the scattered radiation. For modern fiber amplifiers $\exp(-\alpha_R L) \approx 1$, in this case the integral gain G_R^{th} in (15) no longer depends on the optical fiber parameters, except for the matrix of the glass from which it is made (in terms of ν_R and $\Delta\nu_R$).

1.2.2. Backscattering

Backscattering can occur both due to the SRS effect and due to the SMBS effect. According to (8), the Stokes radiation power during backscattering at the point $z = 0$ can be generally represented by the function:

$$P_{\mathbb{S}}^-(0) = P_{\mathbb{S}}^-(L)\exp\left(g_{\mathbb{S}} \frac{P_s(0)L_{\text{eff}}^s}{A_{\text{eff}}^{s\mathbb{S}}} - \alpha_{\mathbb{S}} L\right). \quad (16)$$

Due to the absence of real seed radiation at the Stokes frequency for the value $P_{\mathbb{S}}^-(L)$ during backscattering, as well as for the case of forward scattering, the equivalent value is used. We will adhere to the results presented in the papers [6,20] and define it as follows (for the case of SRS we replace the index „$\mathbb{S}$“ by „R“, and for SMBS by index „B“ respectively)

$$P_R^-(L) = h\nu_R B_{\text{eff}}^R \frac{1}{G_R}, \quad B_{\text{eff}}^R = \frac{\Delta\nu_R}{2}\sqrt{\frac{\pi}{G_R}},$$

$$G_R = g_R \frac{P_s(0)L_{\text{eff}}^s}{A_{\text{eff}}^{sR}} = g_R \frac{\mathbb{P}_s(0)\mathbb{L}_{\text{eff}}^s}{A_{\text{eff}}^{sR}}, \quad (17)$$

$$P_B^-(L) = h\nu_B B_{\text{eff}}^R \frac{1}{G_B}, \quad B_{\text{eff}}^B = \frac{\Delta\nu_R}{2}\sqrt{\frac{\pi}{G_B}},$$

$$G_B = g_B \frac{P_s(0)L_{\text{eff}}^s}{A_{\text{eff}}^{sB}} = g_B \frac{\mathbb{P}_s(0)\mathbb{L}_{\text{eff}}^s}{A_{\text{eff}}^{sB}}, \quad (18)$$

where ν_B and $\Delta\nu_B$ are frequency and linewidth of the Stokes radiation spectrum in SMBS, respectively; N_{ph} is average number of acoustic phonons

$$N_{ph} = \frac{1}{\exp\left(\frac{h\nu_{ph}}{kT}\right) - 1} \approx \frac{kT}{h\nu_{ph}},$$

where ν_{ph} — phonon frequency; k — Boltzmann's constant; T — medium temperature; the quantities $\mathbb{P}_s(0)$ and $\mathbb{L}_{\text{eff}}^s$ are defined by expressions identical to the case of forward scattering.

Substitute (17) and (18) into (16), and, according to the new condition for fixing the threshold for the manifestation of nonlinear scattering in the opposite direction (10), we obtain algebraic equations for finding integral gains G_R^{th} and G_B^{th}:

$$h\nu_R \frac{\Delta\nu_R}{2}\sqrt{\frac{\pi}{G_R^{th}}}\, G_R^{th} e^{G_R^{th} - \alpha_R L} = \mathbb{B}_R,$$

$$\frac{\nu_B}{\nu_{ph}} \frac{\Delta\nu_B}{2}(kT)\sqrt{\frac{\pi}{G_B^{th}}}\, G_B^{th} e^{G_B^{th} - \alpha_B L} = \mathbb{B}_B,$$

or

$$(G_R^{th})^{3/2} e^{-G_R^{th}} = \frac{1}{\mathbb{B}_R} h\nu_R \frac{\Delta\nu_R\sqrt{\pi}}{2} e^{-\alpha_R L}, \quad (19)$$

$$(G_B^{th})^{3/2} e^{-G_B^{th}} = \frac{1}{\mathbb{B}_B} \frac{\nu_B}{\nu_{ph}} \frac{\Delta\nu_B\sqrt{\pi}}{2}(kT) e^{-\alpha_B L}. \quad (20)$$

The threshold value of the initial signal power ($z = 0$), according to (17) and (18), is determined by the expression

$$P_s^{th} = \frac{1}{\mathbb{L}_{\text{eff}}^s}\left(\frac{G_{R,G}^{th} A_{\text{eff}}^{s(R,B)}}{g_{R,B}} - \Upsilon_p\right). \quad (21)$$

The transition to the standard expression for passive optical fibers, as well as for forward scattering, occurs at $\Upsilon_p = 0$. We see that, according to equations (19) and (20), in the case of backscattering, as in the case of forward scattering, the values $G_{R,B}^{th}$ depend on the length of the considered fiber and loss factor at the frequency of the scattered radiation. For modern fiber amplifiers, the integral gains $G_{R,B}^{th}$ in (21) also further do not depend on the parameters of the optical fiber, except for the matrix of glass from which it is made (through the parameters ν_R, $\Delta\nu_R$ and ν_B, $\Delta\nu_B$, ν_{ph} for SRS and SMBS, respectively).

2. Boundary width of spectral line

The threshold values for nonlinear scattering of signal radiation power (in the general case, we will talk about exciting radiation) at the input to the optical fiber (at the point $z = 0$) are determined by expressions (15) and (21), while the SMBS gains depend on the width of the spectral line of the signal, and for the Lorentz gain contour can be written as [7]:

$$g_B(\Delta\nu_s) = g_B^0 \frac{\Delta\nu_B}{\Delta\nu_B + \Delta\nu_s}, \tag{22}$$

where g_B^0 — maximum SMBS gain; $\Delta\nu_s$ — linewidth of the signal emission spectrum. Thus, if we accept (22), then it becomes possible to determine the width of the spectral line of the signal, which „delimits" the effects of SRS (forward scattering) and SMBS: when the width of the spectral line exceeds this boundary value, the threshold value of the signal power for the SBS effect will be lower than for the SMBS effect, in other words, SRS will be observed earlier than SMBS upon increase in the signal radiation power at the input to the fiber amplifier, all other things being equal. Equate (15) and (21), and taking into account (22) we get

$$\Delta\nu_s^{th} = \Delta\nu_B \left(\frac{g_B^0}{g_R} \frac{G_R^{th}}{G_B^{th}} \frac{A_{\text{eff}}^{sR}}{A_{\text{eff}}^{sB}} - 1 \right). \tag{23}$$

This expression is valid for both active and passive optical fibers. In practical application the determination of $\Delta\nu_s^{th}$ value makes it possible to make a preliminary estimate of the expected effects when fixing the width of the spectral line of the signal (in the general case, exciting) radiation.

3. Verification of threshold formulas

Next, we compare the results of estimating the threshold values of the signal radiation power in high-power fiber amplifiers, determined by expressions (15) and (21), with experimental results available in the open press and sufficient in terms of the amount of data presented.

In the course of the comparison, we will consider active double-clad fibers (DCFs), in which the analytical function that determines the distribution of the pumping radiation power along the length of the active fiber can be represented as follows [17]:

$$P_p(z) = P_p^0 \exp\{-(\Upsilon_p + \alpha_p)z\} + P_p^L \exp\{-(\Upsilon_p + \alpha_p)(L - z)\}, \tag{24}$$

where P_p^0 and P_p^L are the pumping powers introduced into the amplifier at the points $z = 0$ and $z = L$, respectively; α_p is linear loss coefficient at the pumping wavelength; $\Upsilon_p = \Gamma_p\sigma_{12}(\lambda_p)N$ is absorption coefficient of pumping radiation by the active impurity in the fiber core (absorption coefficient of the active region). The transition to consideration of other types of active fibers can be made by choosing the appropriate distribution function of the pumping radiation along the length of the active fiber: GTWave(1+1) [21], GTWave(2+1) [22] (the effective coupling coefficients in GTWave type fibers can be determined by the calculation-experimental method [23, 24]).

3.1. SCR manifestation threshold

To verify the expression (15), which determines the value of the threshold power of the signal for SRS in the forward direction, we will use the results of the paper [25]. This paper relates to experimental analysis of the influence of the introduction direction of pumping radiation into fiber amplifier on the SRS manifestation threshold. In the paper [25] the spectrum of radiation going towards the signal one is not given, and no information is given about its presence or absence. The ytterbium DCF 20/400(0.06/0.46) was chosen as the active fiber (this notation is typical for DCF description and corresponds to the following sequence of parameters $d_{co}/d_{cl}(NA_{co}/NA_{cl})$, where d_{co} and NA_{co} — diameter and numerical aperture of the fiber core, d_{cl} and NA_{cl} — diameter and numerical aperture of fiber inner sheath). The distribution function of the pumping radiation power along the length of the active optical fiber is determined by expression (24). The absorption coefficient of the active region at the pumping wavelength 976 nm for the selected sample was $\Upsilon_p = 1.44$ dB/m. The length of the amplifier was chosen to be 19 m. The master oscillator power was 65 W at a wavelength of 1080 nm with a spectral linewidth of 4 nm (according to the data given in Fig. 2 of the paper [25]). These parameter values were used in the numerical analysis, and the linear loss coefficients at all considered wavelengths of optical radiation were taken equal to 10^{-3} 1/m.

3.1.1. Pumping co-directional with signal radiation (direct pumping)

In the paper [25], when describing the results of experiments with direct pumping ($P_p^L = 0$), the first mentioning of the SRS manifestation corresponds to the case when the Stokes radiation was at the level of 24 dB of the main signal, while the signal power was 1.52 kW, and the pumping power was — 1.8 kW. Thus, the Stokes component power was 6 kW. The wavelength of the Stokes radiation was $\sim$ 1135 nm (Fig. 2 [25]). The value of this parameter allows us to assume that a quartz active fiber [26] was used as an amplifier, therefore, in the course of numerical analysis, here and below, when using the results of the paper [25] for the SRS gain the value $g_R = 1.86 \cdot 10^{-13}$ W/m [26] was accepted.

Let us determine the value of the integral gain G_R^{th} by substituting $\mathbb{F} = 6$ W into equation (14), taking into account the values of the spectral characteristics given in the paper [{]25 }: $\lambda_R = 1135$ nm, $\Delta\lambda_R = 5$ nm (these values were used in the numerical analysis for all options

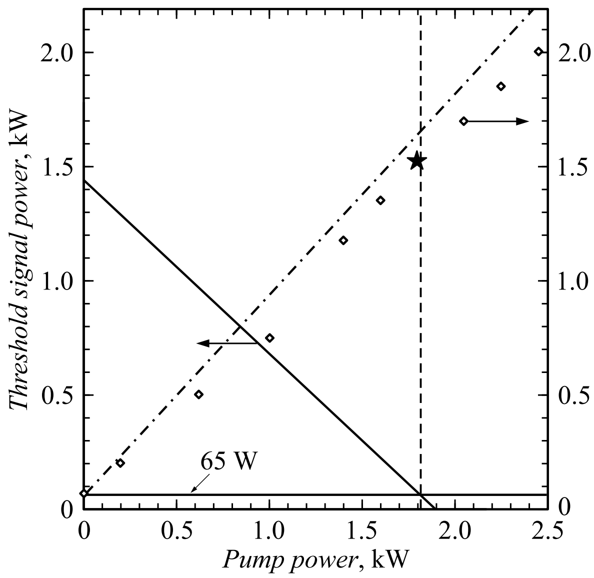

Figure 1. Threshold signal radiation power vs. the pumping power and experimental data on the dependence of the signal radiation output power vs. pumping power according to [25] (the dashed-dotted line indicates the analytical dependence represented by the function (6); dotted line — predicted boundary of SRS manifestation at the 6 W level; asterisk — experimental value of SRS manifestation at the 6 W level).

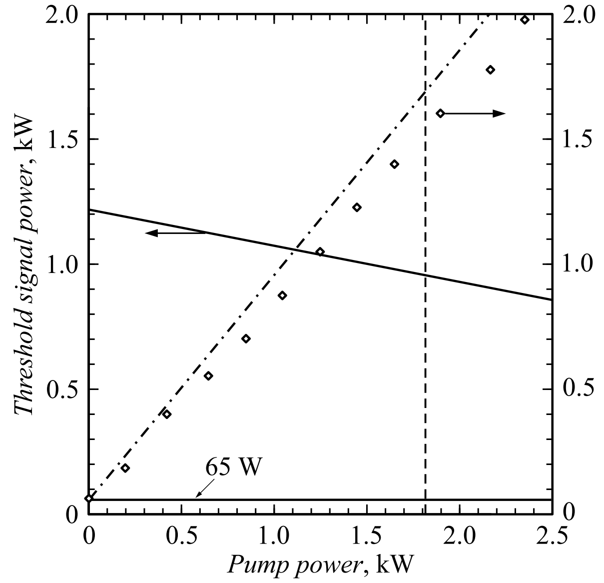

Figure 2. Threshold signal radiation power vs. the pumping power and experimental data on the dependence of the signal radiation output power vs. pumping power according to [25] (the dashed-dotted line indicates the analytical dependence represented by the function (6)).

of pumping radiation introduction). As a result, we get $G_R^{th} \approx 19$.

Figure 1 shows the threshold signal power given by expression (15) vs. the pumping radiation power introduced into the active fibre; experimental dependence of the signal output power on the pumping power (dots with an asterisk for the value corresponding to the SRS manifestation at the level 6 W) [25]; analytical dependence of the signal radiation power at the amplifier output on the pumping power, determined by expression (6) for $P_s(0) = 65\,\mathrm{W}$ (corresponds to the power of the master oscillator used in the experiment).

It is easy to see that the estimate of the threshold power of the signal radiation, obtained using expression (15), agrees well with experiment, which allows us to state the expediency of its use in the analysis of fiber amplifiers with direct input of pumping radiation for the manifestation of the SRS effect.

3.1.2. Pumping towards the signal radiation (reverse pumping)

Let us consider the experiment with pumping radiation introduction opposite to the signal radiation ($P_p^0 = 0$). In the paper [25] to objectively estimate the effect of changing the introduction direction of pumping radiation on the manifestation of the SRS effect in the forward direction the parameters of the amplifier and master oscillator were retained. As a result, when reverse pumping is used, in contrast to direct pumping, SRS was not observed in the experiment. In this regard, to determine the integral gain G_{th}^R from equation (14) we take $\mathbb{F} = 630\,\mathrm{mW}$ (at the noise level in the experiment) in order to deliberately underestimate the possible level of the predicted threshold for the SRS manifestation. As a result, we get $G_R^{th} \approx 16$.

Figure 2 shows the threshold power of signal radiation given by expression (15) vs. the pumping radiation power introduced into the active fibre; experimental dependence of the signal radiation on the pumping power [25] (dots); analytical dependence of the signal radiation power at the amplifier output on the pumping power, determined by expression (0) for $P_s(65) = 65\,\mathrm{W}$ (corresponds to the power of the master oscillator).

We see that the results of the numerical analysis are in good agreement with the experimental data, and for the case when the pumping radiation is introduced opposite the signal radiation. According to the estimates presented, SRS will not manifest itself in the case of reverse pumping, which was observed in the paper [25] in the course of experiments.

3.1.3. Double-sided pumping

For the case of double-sided pumping, in the paper [25] the first mentioning of the SRS manifestation corresponds to the case when the Stokes radiation was at a level of 33 dB of the main signal with power of 2.48 kW at pumping power of 3 kW (P_p^{in}), thus, for the Stokes component power we have the value 1 W. The pumping radiation was introduced uniformly into both ends of the fiber amplifier $P_p^0 = P_p^L = P_p^{in}/2$. Let us determine the value G_R^{th} by

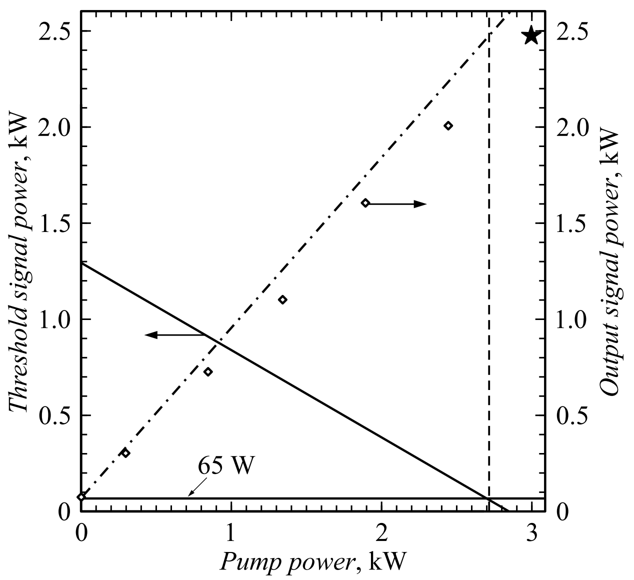

Figure 3. Threshold signal radiation power vs. the pumping power and experimental data on the dependence of the signal radiation output power vs. pumping power according to [25] (the dashed-dotted line indicates the analytical dependence represented by the function (6); dotted line — predicted boundary of SRS manifestation at the 1 W level; asterisk — experimental value of SRS manifestation at the 1 W level).

substituting $\mathbb{F} = 1$ W into equation (14). As a result, we get $G_R^{th} \approx 17$.

Figure 3 shows the threshold power of signal radiation given by expression (15) vs. the pumping radiation power introduced into the active fibre; experimental dependence of the signal radiation output power on the pumping power (dots with an asterisk for the value corresponding to the SRS manifestation at the level 1 W) [25]; analytical dependence of the signal radiation power at the amplifier output on the pumping power, determined by expression (6) for $P_s(0) = 65$ W (corresponds to the power of the master oscillator). It is easy to see that in this case also there is good agreement between the results of numerical analysis and experimental data.

3.1.4. Output

According to the above comparison of the numerical results obtained using expression (15) together with equation (14), with the experimental data found in the open press, we can conclude whether it is expedient to apply the approach presented in this paper to analyze the SRS manifestation in high-power fiber amplifiers operating in a continuous mode, the optical pumping radiation power and the signal power of which satisfy the condition (3). In this case, the option of pumping radiation introduction can be any: forward, reverse, and two-sided pumping. Note also that the values of the signal power at the amplifier output, obtained from the analytical solution (6), are in good agreement with the experimental data in the case of high-power fiber amplifiers.

3.2. SMBS manifestation threshold

Unfortunately, in open press we did not find a comprehensive experimental analysis of the Stokes component power during SMBS vs. direction of pumping radiation introduction in high-power fiber amplifiers with a set of data sufficient for comparison. Therefore, we will verify expressions $(21) + (20)$ and the proposed approach for determining the threshold signal power for SMBS based on the experimental data presented in [27]. In this paper we assume that when reverse pumping is used, the SMBS manifestation threshold due to signal radiation in the high-power fiber amplifier is higher than during forward pumping. Based on this, the results of experiments with forward pumping are not presented in [27].

As a remark, note that in the paper [27] the question of the influence of the direction of pumping radiation introduction into the fiber amplifier on the broadening of the line of the master oscillator radiation spectrum is experimentally studied in detail. It is shown that when the pumping radiation is introduced opposite to the signal radiation, the broadening of the radiation spectrum of the master oscillator is by 3 times less than when the input is codirectional. Regardless of [27], similar results were presented in papers [28,29], where it was theoretically and experimentally shown that the level of noise occurred due to amplification of spontaneous radiation depends on the direction of pumping radiation introduction and leads to spectrum broadening in narrow-band fiber lasers.

Let us proceed to the experimental results [27] comparison with the results obtained according to $(21) + (20)$. In the paper [27], when describing the results of experiments with reverse pumping, SMBS was not observed, and the noise level in the spectrum of the output radiation of the signal was by 44 dB behind the main peak of the signal radiation with power of 2.19 kW during pumping 2.7 kW (noise at the level of 100 mW). The ytterbium DSF 25/400(0.06/0.46) was chosen as an amplifier, the distribution function of the pumping radiation power along the length of which is determined by expression (24), and the absorption coefficient of the active region at pumping wavelength of 975 nm is $\Upsilon_p = 1.7$ dB/m. The length of the amplifier was chosen to be 14.5 m. The master oscillator power was 80 W at a wavelength of 1070 nm with a spectral linewidth of 36.6 pm (9.6 GHz). SRS was not observed during the experiments. The presented values of the amplifier parameters were used in the course of the numerical analysis. According to the frequency shift expected in [27] for SMBS, it can be assumed that a quartz active fiber [30] was used. During the numerical analysis, for the SMBS maximum gain the value $g_B^0 = 5.8 \cdot 10^{-11}$ W/m was taken, and for the linewidth of the Stokes radiation spectrum $\Delta\nu_B = 50$ MHz [13,30].

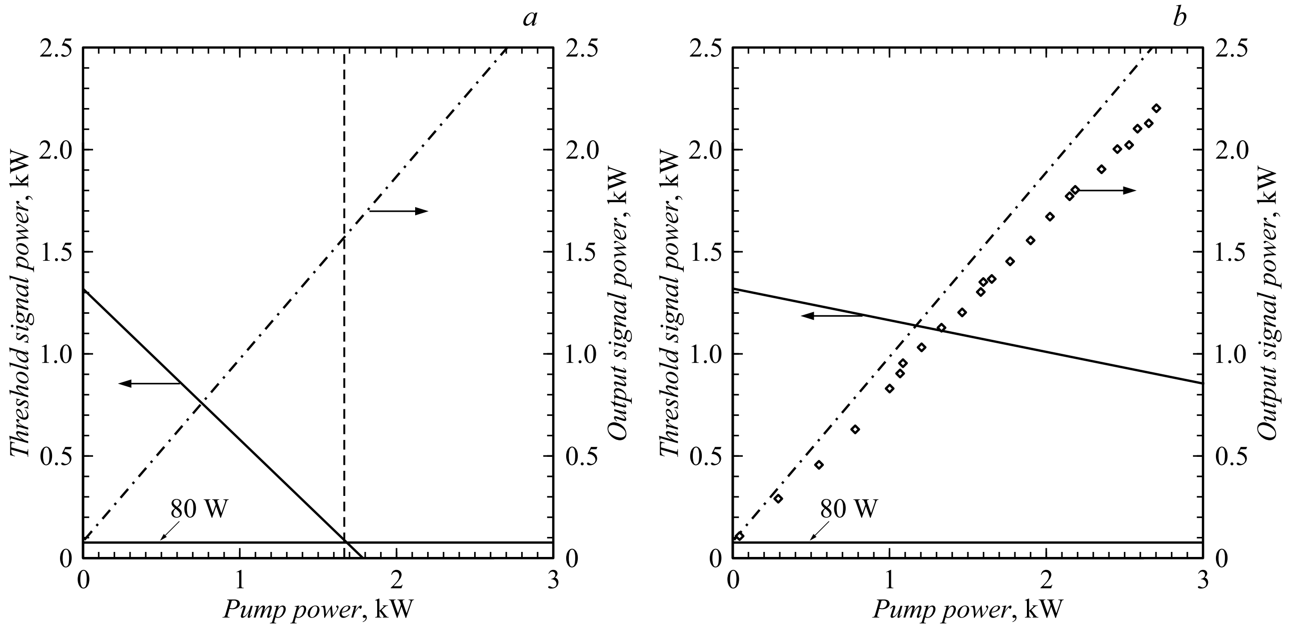

Figure 4. Threshold and output powers of the signal radiation vs. value of the pumping power for two options of its introduction: *a* — forward pumping; *b* — reverse pumping. Experimental data [27] are shown with dots

First of all, it is necessary to determine the value of the boundary width of the spectral line of the signal radiation $\Delta\nu_s^{th}$ according to expression (23). To do this, we calculate the values of the integral gains G_R^{th} and G_B^{th} by substituting $\mathbb{F} = 100\,\text{mW}$ (the noise level in the output radiation spectrum) into equation (14) and $\mathbb{B} = 1\,\text{mW}$ into equation (20). For the spectral parameters of Stokes radiation during SRS the values $\lambda_R = 1125\,\text{nm}$ and $\Delta\lambda_R = 5\,\text{nm}$ were taken. As a result, we have $G_R^{th} \approx 15$ and $G_B^{th} \approx 17$, and $\Delta\nu_s^{th} \approx 14\,\text{GHz}$ $(\Delta\nu_s^{th} > \Delta\nu_s)$. As mentioned above, no SRS was observed in the course of the experiments; therefore, we can conclude that expression (23) together with equations (14) and (20) really makes it possible to estimate the boundary of manifestation of SRS and SMBS effects from the width of the spectral line of signal (in the general case, exciting) radiation.

Figure 4 shows the threshold power of signal radiation given by expression (21) vs. the pumping radiation power introduced into the active fibre (solid line); experimental dependence of the signal radiation on the pumping power [27] (dots); analytical dependence of the signal radiation power at the amplifier output on the pumping power, determined by expression (6) (dash-dotted line) for $P_s(0) = 80\,\text{W}$ (corresponds to the power of the master oscillator).

We see that the numerical results shown in Fig. 4 are in good agreement with the experimental data (during reverse pumping), and the experimental values of the output signal power with change in the pumping radiation power agree with the result obtained using function (6). The paper [27] is presented as an experimental illustration of the fact that the reverse pumping use makes it possible to increase the threshold for SMBS manifestation in fiber amplifiers, and, according to the numerical analysis presented, this is justified. Thus, we can conclude whether it is expedient to use expressions (21) and (23) when analyzing the manifestation of the SMBS effect in high-power fiber amplifiers (pumping power and signal power satisfy the condition (3)).

Conclusion

In the paper we propose a new approach for determining the value of the integral gain of stimulated nonlinear scattering in high-power fiber amplifiers, which corresponds to their experimental manifestation threshold. This approach is based on the physical determination of the manifestation threshold of the phenomena under consideration: the values of the Stokes radiation power are fixed for stimulated scattering in the forward and backward directions, which can be reliably observed in the experiment. The corresponding expressions for determining the values of the threshold power of the exciting radiation are obtained.

The comparison results of estimating the threshold power of the signal radiation in high-power fiber amplifiers, determined by expressions obtained in paper, with experimental results available in the open press. Good agreement between the results of numerical analysis and experimental data is obtained.

Conflict of interest

The authors declare that they have no conflict of interest.

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