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Diagnostics of changes in the dynamics of complex systems from transient processes based on multiresolution wavelet analysis

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With the use of multiresolution wavelet analysis, the possibilities of diagnosing changes in the dynamical regimes of complex systems from transient processes depending on the rate of variations of control parameters are studied. Estimates are made of the minimum sample size that allows diagnosing a change in the dynamical regime of systems with self-sustained oscillations on the example of transient processes during the formation or destruction of synchronous chaotic oscillations.

Keywords: multiresolution analysis, random process, nonstationarity, wavelet.

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The analysis of dynamics of complex systems based on experimental data is usually performed for the stationary regime with no regard for transient processes. This allows one to utilize a wide range of standard digital signal processing techniques that include probabilistic and spectral methods and other approaches presupposing the ergodicity of examined processes and providing for estimation of statistical characteristics via time averaging [1]. However, the non-stationary dynamics of systems often provides a crucial insight into the specifics of their operation (e.g., in the examination of momentary responses to external inputs), and the inclusion of transient processes then becomes essential to the study of dynamics of such systems. A number of methods for examination of systems with time-dependent characteristics, which include approaches involving wavelet transform [2–7], empirical modes [8], fluctuation analysis [9–13], etc., are being used at present. However, their capabilities in terms of estimation of dynamic characteristics based on short data fragments (significantly shorter than the transient process duration) generally differ, and the issue of determination of the minimum sample size for reliable diagnostics of the system state remains open.

In the present study, we investigate the potential for diagnosing changes in the dynamics of complex systems by transient processes using the example of transitions between synchronous and asynchronous chaotic oscillations in the model of interacting Rössler systems and estimate the minimum sample size for reliable diagnostics of regimes of chaotic self-sustained oscillations with the use of multiresolution wavelet analysis. The chosen model is characterized by the following system of six ordinary first-order differential equations:

$$\frac{dx_{1,2}}{dt} = -\omega_{1,2}y_{1,2} - z_{1,2} + \gamma(x_{2,1} - x_{1,2}),$$

$$\frac{dy_{1,2}}{dt} = \omega_{1,2}x_{1,2} + ay_{1,2}, \quad \frac{dz_{1,2}}{dt} = b + z_{1,2}(x_{1,2} - c), \tag{1}$$

where parameters $a = 0.15$, $b = 0.2$, and $c = 6.8$ govern the dynamics of individual systems, $\gamma = 0.02$ is the coupling parameter, and $\omega_{1,2} = 1.0 \pm \Delta$ are oscillation frequencies with mismatch Δ. As Δ varies near 0.0097, the synchronization region boundary is crossed [14]; this transition is probed in the present study by examining sequences of return times to Poincaré secant $x_1 = 0$.

These sequences are analyzed using multiresolution wavelet analysis [1] that involves signal decomposition over a basis, which is formed by conjugate mirror filters (scaling function $\varphi(t)$ (low-pass filter) and wavelet $\psi(t)$ (high-pass filter)), via their dilations and translations:

$$\varphi_{j,k} = 2^{j/2}\varphi(2^j t - k), \qquad \psi_{j,k} = 2^{j/2}\psi(2^j t - k). \tag{2}$$

Signal $f(t)$ is expanded into a series at the chosen resolution level m

$$f(t) = \sum_k s_{m,k}\varphi_{m,k} + \sum_{j\geqslant m}\sum_k d_{j,k}\psi_{j,k}(t), \tag{3}$$

where $s_{m,k}$ and $d_{j,k}$ are approximation and detail coefficients. Daubechies wavelet D^8 is used as the base one, and standard deviations of wavelet coefficients at different resolution levels serve as quantitative characteristics of dynamics:

$$\sigma(j) = \sqrt{\frac{1}{J}\sum_{k=1}^{J}\left[d_{j,k} - \langle d_{j,k}\rangle\right]^2}, \tag{4}$$

where J is the number of detail coefficients at level j. This approach was used successfully in solving a wide variety of problems in diagnostics of complex processes and systems [2,15,16], although a more thorough analysis of

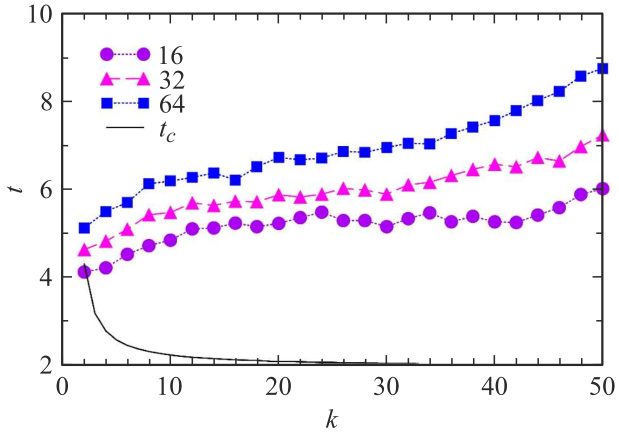

Figure 1. Dependence of the Student's t-test value, which characterizes the differences between synchronous and asynchronous chaotic oscillations in model (1), on number k of data segments, which are used for averaging of the results, for three segment sizes. The critical values for confidence level $p < 0.05$ are denoted as t_c.

coefficients may also be performed [17]. Note that the problem of diagnostics of dynamic regimes of non-autonomous systems by transient processes has been discussed earlier in [18,19].

Prior to analyzing transient processes directly, we estimate the minimum sample size needed to differentiate between synchronous and asynchronous oscillation regimes in model (1) in the case of stationary processes. Let us use the Student's t-test for independent samples with confidence level $p < 0.05$ for this purpose. Figure 1 shows how the value of t varies with the size of a data segment chosen for analysis and with number k of these segments used for averaging of the results. It can be seen that data sets containing approximately 64 return times allow one to differentiate reliably between synchronous and asynchronous chaotic oscillation regimes. It often makes no substantial difference whether one averages $\sigma(j)$ values over two segments with a length of 32 samples or uses a single segment 64 samples in length; however, according to our estimates, the latter method is preferable. Note that $\sigma(1)$ estimates yield higher t values than $\sigma(2)$; i.e., it is reasonable to perform an analysis at the first resolution level in this case.

Let us now turn to transient processes between synchronous and asynchronous chaotic oscillation regimes, which are modeled by either a step-wise or a smooth (linear growth of frequency mismatch with crossing of the synchronization region boundary) Δ change. In the former case, the transient process evolves faster, and the moment of crossing is identified accurately by $\sigma(1)$ values for a sample of 128 return times (Fig. 2, a). Smaller samples may also be used, but ambiguities may arise (if, e.g., transitions between regimes are identified by introducing a threshold level for $\sigma(1)$).

In the latter case (the case of a slower parameter variation), the needed sample size increases (to 256 samples in the discussed example) (Fig. 2, b). On the one hand, this is to be expected of a longer transient process; on the other hand, it still remains possible to identify a regime change by examining a relatively small data set. This provides an opportunity to use multiresolution analysis as a diagnostic tool for specific features of transient processes, including their duration and projected completion time estimated based on a short data fragment. The latter option is viable if the examined data fragment covers the onset of a transient process, which corresponds to a significant change in variance of wavelet coefficients. Note that the obtained estimates of the minimum sample size are also valid for other transient processes in model (1) (specifically, transitions between synchronous chaotic and hyperchaotic oscillations or between asynchronous chaotic and hyperchaotic oscillations). The discussed approach to analysis of transient processes may be applied in the study of non-stationary dynamics of systems of various nature, and other characteristics of distributions of detail coefficients may be used alongside with standard deviations.

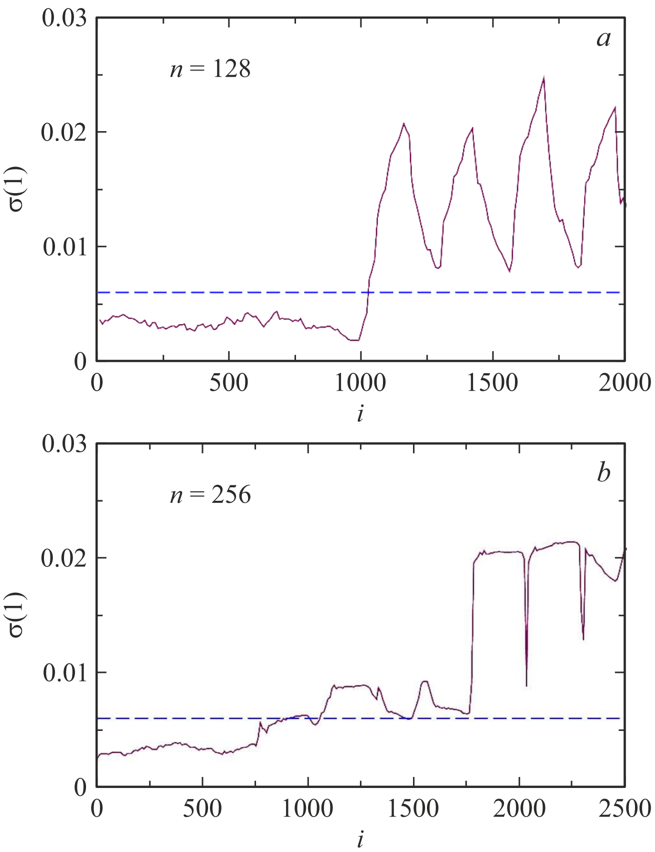

Figure 2. Variations of standard deviations of detailing coefficients at the first resolution level under fast (a) and relatively slow (b) changes of the frequency mismatch parameter in the case of a transition from synchronous to asynchronous chaotic oscillations. The threshold level, which serves as a boundary between dynamic regimes, is represented by the horizontal dashed line.

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Conflict of interest

The authors declare that they have no conflict of interest.

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