06

Phase effects at stimulated Brillouin scattering

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In the study of stimulated Brillouin scattering (SBS) in the constant intensity approximation, the spatial behavior of the intensity of the Stokes scattering wave in a medium was investigated. It was found that as a result of nonlinear interaction of waves, the period of spatial beats changes. The intensity of the Stokes scattering component is considered as a function of the phase mismatch, pump and acoustic wave intensities. It is found that the efficiency of the scattering of the backward Stokes wave is affected by the total length of the nonlinear medium. According to the analytical expressions obtained in this work, the choice of the optimal parameters of the problem makes it possible to implement the regime of efficient generation of the Stokes scattering component in SBS. The process of generation and amplification of the Stokes scattering component is compared with experiment. By varying the pump intensity, one can control and manipulate the intensity of the output radiation of the Stokes component.

Keywords: SBS, backward Stokes scattering wave, constant intensity approximation.

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Introduction

Spectroscopic analysis of inelastic light scattering provides information about the processes occurring in the scattering medium, about the photoelastic and acoustic properties of substances. By determining the frequency shift, the intensity of the scattering component, it is possible to detect and judge what processes occur in solids, liquids and gases. The stimulated −Brillouin scattering (SBS) is successfully used for pulse compression [1], amplification and reversal of the wavefront of optical signals, plasma diagnostics [2]. SBS is characterized by a lower threshold than forced Raman scattering, which makes it attractive for applications. One of the promising areas of research with SBS is the study of the characteristics of oil wells [3,4]. Currently, SBS-amplifiers are actively used in communication systems as an optical filter due to the narrow gain band of the Stokes component $\sim 17\,\mathrm{MHz}$ (for comparison, a similar value with forced Raman scattering $\sim 5\,\mathrm{THz}$) [5,6].

The recent paper [7] is a review on SBS, in which an analysis of existing experimental studies and applications of this phenomenon is given. In [8], the SBS process is theoretically investigated in the special case when the change in the amplitude of an acoustic wave with a spatial coordinate occurs much slower compared to the magnitude of the acoustic wave itself. In the paper [9], threshold conditions for SBS in plasma were studied. The study of reverse Brillouin scattering in dense laser plasma at the picosecond duration of the pump pulse was carried out in [10], and in [11] the analysis of SBS in plasma under the action of ultrashort laser pump pulses was given. In [12], the SBS in violation of phase matched caused by phase modulation of interacting waves is considered in detail. The SBS theory is built, taking into account the change in the refractive index of the medium due to heating, which is in good agreement with the experiment. The following mechanism of this modulation is proposed: a part of the light wave propagating in the medium is absorbed, which leads to heating and to a change in the refractive index of the scattering medium.

The SBS analysis is carried out mainly in the inexhaustible pumping approximation, i.e. in the constant field approximation (CFA) or by numerical calculation of shortened equations [13–15]. However, already in the first experiments on SBS, a significant depletion of the pumping of [8,16] was observed. In addition, with the analytical solution of coupled wave equations, it is possible to understand the qualitative picture of the nonlinear interaction of waves. The analysis of nonlinear processes shows that the phase relations between interacting waves play a significant role [4,8,13,17–20], which is not taken into account in the CFA. We propose to use the following analytical research method - the constant intensity approximation (CIA), which takes into account the depletion of the pump wave and changes in the phases of interacting waves [21,22]. In this approximation, we have already considered the Raman scattering of Stokes and anti-Stokes components [23,24], parametric interaction with high-frequency pumping [25].

In this paper, the behavior of the intensity of the Stokes scattering component in a nonlinear medium as a function of the length of the nonlinear medium and the pumping intensity is analyzed in the process of SBS in the CIA, and the analytically obtained value of the gain of the Stokes component is compared with the existing experimentally measured values.

Theory

SBS — is a third-order nonlinear process resulting from the Bragg diffraction of laser radiation on a lattice of a nonlinear refractive index of the medium, which explains the parametric nature of the interaction when the refractive index modulation is caused by an acoustic wave. In this case, the scattered wave has the same polarization as the laser wave.

Thus, in the theory of coupled waves, the stimulated Brillouin scattering is considered as a parametric interaction in a nonlinear medium of two light waves (a laser pump wave and a scattered Stokes component) and an acoustic wave that occurs when this medium is excited [8]. Under the condition of phase matched for SBS, according to the dispersion relation obtained in this case, the resulting frequency shift of the Stokes component, in contrast to Raman scattering, depends on the angle between the incident and scattered beams. Thus, there is practically no inelastic scattering in the direction of the pump wave vector, and in the opposite direction (at 180° interaction), a maximum Brillouin frequency shift is observed, which is significant for applications. With such a geometry of the scattered wave propagation, the interaction is realized between the oncoming beams. And this means that the interference of the pump wave with the oncoming Stokes scattering component can lead to a more significant nonlinear interaction of waves due to the complete overlap of the oncoming wave beams compared to equally directed beams. The efficiency of this nonlinear process depends on the phase relationship between the interacting waves. We note that with the inverse scattering geometry, the Stokes component removes part of the energy of the pump wave from the nonlinear interaction region, which leads to an increase in the threshold of the scattering process.

We conduct the review assuming that a laser wave at the frequency ω_p (pump wave) is present at the entrance to the medium. Preliminary, the medium is excited, and, as a consequence, an acoustic wave propagates in it at the frequency ω_a. This can be, for example, a hypersonic wave having a thermal nature. The laser wave propagating in such a medium diffracts on the acoustic lattice, resulting in the inverse Stokes component of the simulated −Brillouin scattering at the frequency $\omega_s = \omega_p - \omega_a$.

The analytical analysis of the SBS will be carried out using a system of shortened equations, the solution of which we are looking for in a quasi-stationary approximation. The consideration is carried out in the case of a normal incidence of the pump wave from the left onto a nonlinear medium with cubic polarization $\chi^{(3)}$ along the positive direction of the z axis. To describe these processes in the quasi-stationary case with an arbitrary phase disorder, the system of wave equations has the form [8]:

$$\frac{dA_p}{dz} + \delta_p A_p = i\gamma_p A_s A_{ak} e^{-i\Delta z},$$

$$\frac{dA_s}{dz} + \delta_s A_s = -i\gamma_s A_p A_{ak}^* e^{i\Delta z},$$

$$\frac{dA_{ak}}{dz} + \delta_{ac} A_{ac} = i\gamma_{ac} A_p A_s^* e^{i\Delta z}. \quad (1)$$

Here $A_{p,s,ac}$ are complex amplitudes of the pump wave, Stokes and acoustic waves at frequencies $\omega_{p,s,ac}$, respectively, $\gamma_{p,s,ac}$ are coefficients of nonlinear coupling at SBS:

$$\gamma_p = \frac{\omega_p^2}{2k_p c^2}\left(\frac{\partial\varepsilon}{\partial\rho}\right)(\hat{e}_1 + \hat{e}_2),$$

$$\gamma_s = \frac{\omega_s^2}{2k_s c^2}\left(\frac{\partial\varepsilon}{\partial\rho}\right)(\hat{e}_1 + \hat{e}_2),$$

$$\gamma_{ac} = \frac{k}{4\pi\upsilon^2}\rho_0\left(\frac{\partial\varepsilon}{\partial\rho}\right)(\hat{e}_1 + \hat{e}_2),$$

$\delta_{p,s}$ and $\delta = \Gamma_B/\upsilon$ are corresponding linear losses, Γ_B is the half-width of the −Brillouin line in spontaneous scattering, υ is the velocity of the acoustic wave formed due to electrostriction in the dielectric, $\rho_0\left(\frac{\partial\varepsilon}{\partial\rho}\right)$ is the coefficient of electrostriction, ρ_0 is the density of the nonlinear medium, $\Delta = k_p + k_s - k_{ac}$ denotes the phase mismatching of the wave vectors of the pump wave k_p, Stokes k_s and acoustic k_{ac} waves, the cubic susceptibility of a nonlinear medium is determined by the expression

$$\chi_B^{(3)} = \frac{k_{ac}\rho_0}{(\Delta - i\Delta_{ac})}\left[\left(\frac{\partial\varepsilon}{\partial\rho}\right)(\hat{e}_1 + \hat{e}_2)4\pi\upsilon\right]^2.$$

In general, the cubic susceptibility contains resonant and non-resonant parts. From the above expression it can be seen that during phase synchronization, the resonant part $\chi_B^{(3)}$, which is purely imaginary and positive, is responsible for the SBS.

We solve the problem in the general case under the following boundary conditions:

$$A_p(z=0) = A_{po}, \quad A_s(z=l) = A_{sl}, \quad A_{ac}(z=0) = A_{aco}. \quad (2)$$

As is known, the acoustic wave losses in the scattering medium can exceed the losses of the Stokes wave. According to the experiment, in liquids the acoustic wave loss $\delta_{ac} \sim 10^4\,\mathrm{cm}^{-1}$ at the speed of the sound wave $\upsilon \sim 10^5$ cm/s [8,26,27], and in a quartz light guide $\delta_{ac} \geq 50\,\mathrm{cm}^{-1}$ at the speed of sound $\upsilon \sim 5.96$ km/s [14,26,27]. Therefore, solving the system (1) in AGI, i.e. $I_p(z=0) = I_{po}$, $I_{ac}(z=0) = I_{aco}$ taking into account (2), we neglect the losses of the Stokes wave.

In the case when phase synchronism is violated at SBS, which is studied in detail in [12], for the complex amplitude of the Stokes component we obtain ($\delta_{s,p,ac} = 0$)

$$A_s(z) = e^{-i\Delta\frac{z}{2}} \left[\frac{i\gamma_s A_{po} A^*_{aco} \frac{\sin\lambda l}{\lambda} + A_{sl} e^{-i\Delta\frac{l}{2}}}{\cos\lambda l \frac{i\Delta}{2} \frac{\sin\lambda l}{\lambda}} \times \left(\cos\lambda l - \frac{i\Delta}{2} \frac{\sin\lambda z}{\lambda} \right) - i\gamma_s A_{po} A^*_{aco} \frac{\sin\lambda z}{\lambda} \right], \quad (3)$$

where

$$\lambda = \sqrt{\Gamma_p^2 - \Gamma_{ac}^2 + \frac{\Delta^2}{4}}, \quad I_j = A_j A_j^*,$$
$$\Gamma_p^2 = \gamma_s \gamma_{ac} I_{po}, \quad \Gamma_{ac}^2 = \gamma_s \gamma_p I_{aco}.$$

If the root expression for λ is negative, then in (3) we proceed to hyperbolic sines and cosines. In the case of a given pumping field ($\delta_p = 0$, $\gamma_p = 0$), we get the result of the CFA.

The process of wave propagation in a nonlinear medium, as a result of nonlinear interaction, there is an energy exchange between counter wave packets of two types of waves: direct waves (pump waves and acoustic waves) and an inverse scattered Stokes wave. As a result, the energy of the pumping wave and the acoustic wave is pumped into the energy of the Stokes wave. As a consequence, the Stokes wave increases in the negative direction of the z axis, while the pump wave weakens as it propagates in the positive direction of the z axis.

Under the conditions of phase synchronism, $\Delta = 0$, from (3) for the intensity of the Stokes wave we obtain:

$$I_s(z) = \left[I_{sl} \frac{\cos^2\lambda' z}{\cos^2\lambda' l} + \frac{b^2}{\lambda'^2} \left(\cos\lambda' z \,\mathrm{tg}\,\lambda' l - \sin\lambda' z \right)^2 \right], \quad (4)$$

where $b = \gamma_s A_{po} A^*_{aco}$, $\lambda' = \sqrt{\Gamma_p^2 - \Gamma_{ac}^2}$.

In the general case (taking into account the attenuation of all interacting waves and phase detuning) from the system (1) at the left exit from the nonlinear medium, for the value of the complex amplitude of the Stokes component, we obtain

$$A_s^{\text{output}}(z) = A_s(z = 0) = \frac{i\gamma_s A_{po} A^*_{aco} \sin\lambda^* l/\lambda^* + A_{sl} \exp\left[(\delta_p + \delta_{ac} - \delta_s - i\Delta) \frac{l}{2} \right]}{\cos\lambda^* l + (\delta_p + \delta_{ac} + \delta_s - i\Delta) \sin\lambda^*/\lambda^*}, \quad (5)$$

where $\lambda^* = \sqrt{\Gamma_p^2 - \Gamma_{ac}^2 - \frac{(\delta_p + \delta_{ac} + \delta_s - i\Delta)^2}{4}}$.

It can be seen from (3) and (5) that the efficiency of the scattering process of the Stokes component is affected by the total length of the nonlinear medium.

As is known, in the presence of losses in the medium, the amplitude of propagating waves decreases in the direction of energy transfer (for the reverse wave, the attenuation occurs in the direction opposite to the forward waves). However, taking into account the nonlinearity of the medium, compensation of losses is possible, as can be seen from the exponential dependence in (5). And in this case, the wave will propagate with a constant amplitude and possibly even amplification of the reverse wave. Thus, in contrast to the interaction of passing waves in the studied case of counter interaction, the losses of the inverse Stokes wave can be compensated. This method of reducing losses is used in metamaterials, where wave attenuation is one of the significant problems existing for metamaterials [28,29]. In the case of the SBS process under consideration, the issue of compensation for reverse wave losses is relevant for media in which the losses of acoustic waves do not significantly exceed the losses of the pump wave and the Stokes scattering component, i.e. with small acoustic losses.

The signal wave gain at a length of z will be defined as $\eta_{\text{ampl}} = \frac{I_s(z)}{I_{sl}}$, and the efficiency of the frequency conversion of the Stokes wave in a medium of length z as $\eta_s = \frac{I_s(z)}{I_{po}}$. In the case of the absence of an acoustic wave at the entrance to the medium on the left, i.e. at $A_{aco} = 0$ from (4) for the signal wave gain at the length of z we get

$$\eta_{\text{ampl}}(z) = \frac{\cos^2\lambda' z}{\cos^2\lambda' l}. \quad (6)$$

Let us analyze the efficiency of converting $\eta_s = \frac{I_s(z)}{I_{po}}$ of the energy of two straight waves into the energy of a Stokes wave. It follows from (4) that there is an optimal value of the intensity of the main radiation at which the conversion efficiency is maximal. In the absence of a Stokes wave at the entrance to the medium, i.e. at $A_{sl} = 0$ at the exit of the medium ($z = 0$), the expression for efficiency taking into account the attenuation of all interacting waves, according to (5), takes the form ($\Delta = 0$)

$$\eta_s(z = 0) = \frac{\gamma_s}{\gamma_p} \times \frac{\Gamma_{ac}^2}{\left(\cos\lambda'' l + (\delta_p + \delta_{ac} + \delta_s) \frac{\sin\lambda'' l}{\lambda''} \right)^2} \left(\frac{\sin\lambda'' l}{\lambda''} \right)^2, \quad (7)$$

where

$$\lambda'' = \sqrt{\Gamma_p^2 - \Gamma_{ac}^2 - \frac{(\delta_p + \delta_{ac} + \delta_s)^2}{4}}.$$

Results and discussion

According to the expressions (3)-(6), graphically analyze the calculated dependencies obtained in the CIA. At the same time, the acoustic wave losses δ_{ac} will vary in a wide range of values, ranging from small values close to the pump wave losses and higher ($\delta_{ac} \sim 25\,\text{cm}^{-1}$ [14,28]) to large ($\sim 100\,\text{cm}^{-1}$ [14,30,31]) of practical interest. This will allow us to imagine how a nonlinear process depends on the loss level of δ_{ac}. The pumping intensity will be considered in the range of values from 10^{10} to $10^{12}\,\text{W/cm}^2$.

Fig. 1 below shows the dependences of the gain of the Stokes component $\eta_{\text{ampl}} = (I_s(z))/I_{sl}$ on the length of the nonlinear medium under the condition of phase synchronism, but at different pump wave intensities I_{po}

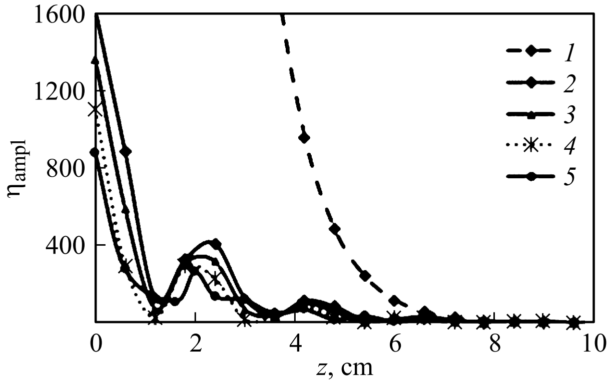

Figure 1. Dependence of the gain of the Stokes component $\eta_{\text{ampl}} = (I_s(z)/I_{sl}$ on the length of the nonlinear medium z at $\Delta = 0$, $\delta_s = \delta_p = 0.1\,\text{cm}^{-1}$, $\delta_{ak} = 0.5\,\text{cm}^{-1}$ for $I_{po} = 2 \cdot 10^{11}\,\text{W/cm}^2$ (*5*) and $2 \cdot 10^{12}\,\text{W/cm}^2$ (*2–4*) and $I_{ako} = 0$ (*4* and *5*), $2000\,\text{W/cm}^2$ (*3*), $4000\,\text{W/cm}^2$ (*2*). The curves *2–5* are calculated in CIA and the dashed curve *1* in CFA.

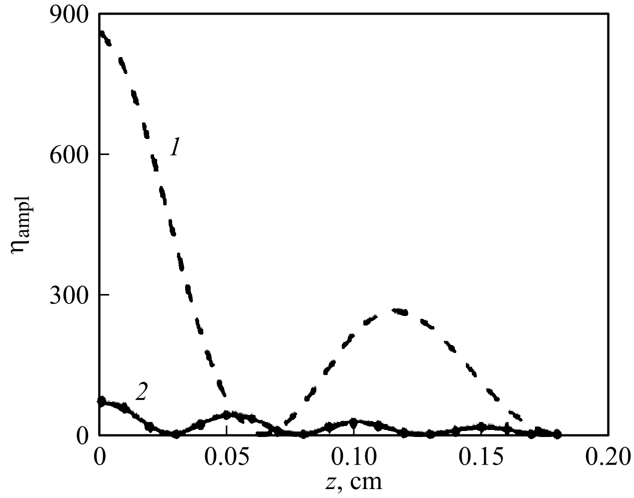

Figure 2. Dependence of the gain of the Stokes component $\eta_{\text{ampl}} = I_s(z)/I_{sl}$ on the length of the nonlinear medium z at $\Delta = 0$, $\delta_p = 0$, $\delta_{ac} = 10\,\text{cm}^{-1}$, $I_{aco} = 50\,\text{W/cm}^2$ for $I_{po} = 9 \cdot 10^{11}\,\text{W/cm}^2$ (*1*) and $2 \cdot 10^{12}\,\text{W/cm}^2$ (*2*).

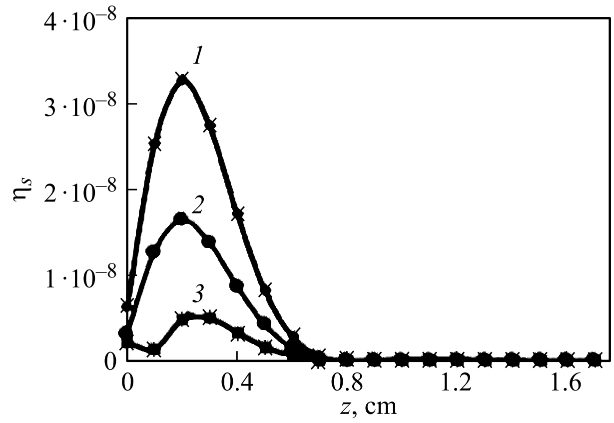

Figure 3. Dependence of the efficiency of the transformation of the Stokes component $\eta_s = \frac{I_s(z)}{I_{po}}$ on the length of the nonlinear medium z at $\Delta = 0$, $\delta_p = 0.5\,\text{cm}^{-1}$, $I_{po} = 5 \cdot 10^{10}\,\text{W/cm}^2$ for $\delta_{ac} = 5\,\text{cm}^{-1}$ (*1, 2*), $7\,\text{cm}^{-1}$ (*3*) and $I_{aco} = 4000\,\text{W/cm}^2$ (*1, 3*), $2000\,\text{W/cm}^2$ (*2*).

and acoustic wave I_{aco} for experimental parameters [5–6,8,14,16,26,27,32]. Spatial beats of the light wave are observed on the curves *2–5*. The maximum beats are reached at the coherent length of the nonlinear medium. An increase in the pumping intensity by an order of magnitude leads to an increase in the intensity of the Stokes component by about 1.3 times (compare the curves *4* and *5*). At the same pumping intensities, the behavior of the curves is affected by the initial value of the acoustic wave intensity. With the growth of I_{aco}, the gain factor increases, the influence of the acoustic wave on the process of amplification of the Stokes component increases (compare the curves *2–4*). Here is also the result of the CFA (dashed curve *1*), when Γ_{ac} and δ_p are assumed to be zero. As expected, the gain calculated without pumping depletion has higher values compared to the result in the CIA, which takes into account the inverse effect of the Stokes component on the pump wave (compare curves *1* and *2*). In addition, there is an increase in the period of spatial beats of the intensity of the Stokes component in the CFA ($\eta_s \sim I_s(z)$), what happens due to the exclusion in the CFA from the root expression for Γ_p from the root expression for λ', which is responsible for the inverse effect of the excited Stokes scattering component on the pump wave (compare curves *1* and *2*).

Fig. 2 shows how a change in the interaction pattern can be observed in the CIA compared to the CFA. With increasing pumping intensity (curves *1* and *2*), the frequency of oscillations increases, which follows from the root expression for λ'. This parameter (Γ_p) also affects the amplitude of the oscillations, which distinguishes the result in CIA from CFA. In CFA, with an increase in the pumping intensity, the gain definitely increases. In CIA, there is an optimal value of the pumping intensity. With these parameters of the problem, the optimal value of I_{po} is near the value of $I_{po} = 9 \cdot 10^{11}\,\text{W/cm}^2$, which explains the similar arrangement of the curves *1* and *2*.

In Figs. 3 and 4, the conversion efficiency is considered for small and large values of acoustic wave losses. The analysis has shown, that the conversion efficiency of η_s increases with an increase in the intensity of the acoustic wave I_{aco} (Fig. 3 and 4, curves *1* and *2*) and a decrease in losses as for the acoustic wave (Fig. 3, curves *1* and *3*; fig. 4, curves *2–4*), and for the pump wave. At high intensities of the acoustic wave at the entrance to the medium, energy is pumped into the Stokes wave from both direct waves much more efficiently. Conversion efficiency in the case of small acoustic wave losses $\delta_{ac} \sim (5–7)\,\text{cm}^{-1}$ takes the values $\eta_s \sim 10^{-8}$, and at higher losses $\delta_{ac} \sim (40–80)\,\text{cm}^{-1}$

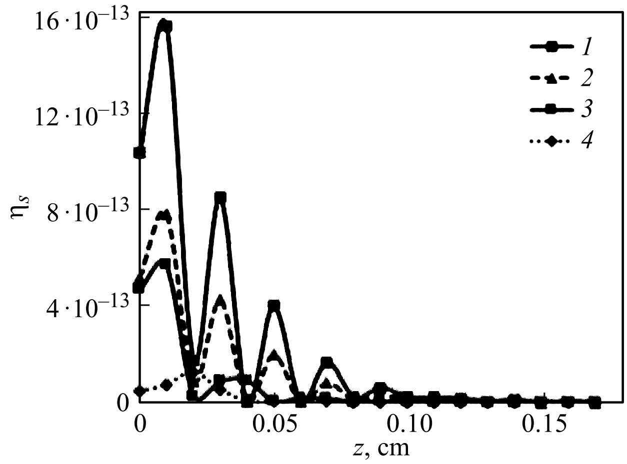

Figure 4. Dependence of the efficiency of the transformation of the Stokes component $\eta_s = \frac{I_s(z))}{I_{po}}$ of the length of the nonlinear medium z at $\Delta = 0$, $I_{po} = 9 \cdot 10^{12}$W/cm^2, $\delta_p = 0.5\,\text{cm}^{-1}$, $\delta_{ac} = 80\,\text{cm}^{-1}$ (*4*), $60\,\text{cm}^{-1}$ (*3*), $40\,\text{cm}^{-1}$ (*1, 2*) and $I_{aco} = 100\,\text{W/cm}^2$ (*2–4*) and $200\,\text{W/cm}^2$ (*1*).

the efficiency of the Stokes component is several orders of magnitude less and is equal to $\eta_s \sim 10^{-12}$. As further analysis showed, an increase in the attenuation of the acoustic wave from $\delta_{ac} = 7\,\text{cm}^{-1}$ (curve *3*, Fig. 3) to $40\,\text{cm}^{-1}$ leads to a nonlinear decrease in the conversion efficiency by five times with the following task parameters: $I_{aco} = 4000\,\text{W/cm}^2$, $\delta_p = 0.5\,\text{cm}^{-1}$, $I_{po} = 5 \cdot 10^{10}\,\text{W/cm}^2$.

We will perform a numerical calculation and compare the results obtained by the gain factor at SBS with the experimental and theoretical results given in [5–6,14,16,26–27,32]. In quartz ($\upsilon_{ac} = 5.97 \cdot 10^3$ m/s, $\gamma_e = 0.902$, $\rho_0 = 2.21\,\text{g/cm}^3$) the gain $\eta_{\text{ampl}}(z = 0)$, calculated in CIA and CFA, respectively is equal to $1.5 \cdot 10^3$ and $97 \cdot 10^4$. The large value of the gain in the CFA is explained by the neglect of pump depletion in this approximation. The corresponding experimental result in this case is (for $g_B = 4.5\,\text{cm/kW}$ [26–27] and $5 \cdot 10^{-11}$ m/W [5–6,32]). From here, we can estimate the efficiency of converting the energy of the pump wave into a Stokes component, in the CIA it is $3 \cdot 10^{-6}$, and in the CFA $-2 \cdot 10^{-4}$. At high pumping intensities, the contribution of the parameter $\Gamma_p(\sim \sqrt{I_{po}}$ increases in the CIA, which takes into account the inverse effect of the excited wave on the pumping wave, and the difference in results in both approximations increases.

Thus, the analysis shows that using the expression (4) under the condition of phase matched, it is possible to study the nonlinear scattering process at any stage of interaction of interest and obtain the optimal value of the pumping intensity for the required parameters of the problem (acoustic wave and pump wave losses, input intensity of Stokes and acoustic waves) at any stage of scattering for any value of z. By varying the values of losses and intensities of interacting waves, it is possible to control the rate of the SBS process by selecting the values of losses and intensities of interacting waves. In this case, the efficiency of the scattering process of the Stokes component also depends on the total length of the nonlinear medium.

Conclusion

Thus, the analysis shows that it is possible to investigate the process of the simulated −Brillouin scattering at any stage of interaction of interest and calculate the optimal value of the pumping intensity for the required parameters of the problem (acoustic wave and pump wave losses, input intensity of the Stokes and acoustic wave) for any value of the spatial coordinate z. The speed of the nonlinear SBS process can be controlled by selecting the values of losses and intensities of interacting waves. It was shown that the efficiency of the inverse Stokes wave scattering process is affected by the total length of the nonlinear medium.

Conflict of interest

The authors declare that they have no conflict of interest.

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