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## Anisotropy of the Paramagnetic Susceptibility in the 3D Dirac Semimetal $\text{Cd}_3\text{As}_2$ Caused by Chromium Impurity: the EPR on $\text{Cr}^{3+}$ Ions

© Yu.V. Goryunov<sup>1</sup>, A.N. Nateprov<sup>2</sup>

<sup>1</sup>FRC Kazan Scientific Center RAS,  
Kazan, Russia

<sup>2</sup>Institute of Applied Physics, Moldova State University,  
Chişinău MD2028, Republic of Moldova

E-mail: gorjunov@kfti.knc.ru

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In the literature devoted to magnetic impurities in Dirac semimetals, the appearance of anisotropic and long-range non-oscillating contributions to the Ruderman–Kittel–Kasuya–Yosida interaction is predicted. These contributions are due to the hybridization of the wave functions of magnetic impurities with the wave functions of Dirac electrons, which have a linear dispersion law. We have studied electron paramagnetic resonance (EPR) in powder samples of topological 3D Dirac semimetal  $\text{Cd}_3\text{As}_2$  with an admixture of chromium ions. Anisotropy of the magnetic susceptibility in the paramagnetic state was found from the behavior of the resonance field. The appearance of magnetic anisotropy in the paramagnetic state is associated with an increase in the contribution of the orbital moments of chromium ions  $\text{Cr}^{2+}$  and  $\text{Cr}^{3+}$  due to their interaction with the orbital moments of donor Dirac electrons, which have anomalously large values of  $g$  factors.

**Keywords:** magnetic resonance, topological materials, magnetic impurities.

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### 1. Introduction

A characteristic feature of topological materials is the strong coupling of the spin and orbital momentum of Dirac electrons, which leads, in particular, to the fact that the directions of the spins of such electrons turn out to be rigidly tied to their direction of motion. This primarily applies to the parent electrons of the topological material. However, since the modification of the electronic properties of semiconductors or semimetals is carried out by introducing donor electrons or specific scattering centers into the material, the study of the behavior of donor electrons becomes almost of greater importance. The complexity of such a problem is exacerbated by the presence of orbital moments of the donor impurity. Therefore, initially [1,2] we undertook a study of the behavior of donor magnetic impurities with a purely spin state in a three-dimensional (3D) topological Dirac semimetal  $\text{Cd}_3\text{As}_2$ . It was noticed that there is a significant relationship between the spins of conduction electrons and localized purely spin states, leading, in particular, to an increase in the  $g$ -factor of the localized state and the appearance of a non-oscillating contribution to their interaction of Rudermann–Kittel–Kasui–Yosida (RKKY) [3–6]. From this naturally arises the question of the mutual influence of Dirac electrons and localized electronic states, which, along with the spin angular momentum, also have an orbital one. Theoretical calculations point to the opportunity of anisotropy in the RKKY interaction [7]. In the sphere of such interest, the study of the effect of chromium impurity on the properties of the 3D topological

Dirac semimetal  $\text{Cd}_3\text{As}_2$  was undertaken. The  $d$ -transition metal ions (Fe and Mn) with the  $3d^5$  configuration are in the  $S$ -state, and their total orbital angular momentum is zero. Therefore, the magnetic state of these ions is traditionally considered to be purely spin with a  $g$ -factor equal to that of a free electron (2.0023). In case of a chromium impurity, which can be represented by a  $\text{Cr}^{2+}$  ion with a  $3d^4$  configuration, and a  $\text{Cr}^{3+}$  ion with a  $3d^3$  configuration, the magnetic states of the ions are not purely spin, and there is a significant contribution of the orbital angular momentum [8]. In addition, since the isovalent substitution of a  $\text{Cd}^{2+}$  ion by a  $\text{Cr}^{2+}$  ion does not directly produce donor Dirac electrons [9], the latter is associated with the formation of  $\text{Cr}^{3+}$ , mixed valence effects [10] and charge compensation violation. Thus, manifestations of the effects of the influence of the freedom orbital degree and various valence states hybridization of chromium ions with the wave functions of free electrons of the  $\text{Cd}_3\text{As}_2$  matrix on its magnetic properties.

### 2. Experiment

$\text{Cd}_3\text{As}_2$  ingots doped with 0.45 at% Cr were synthesized at 1023 K as a direct interaction of the components: Cd (purity 99.999%), As (99.9999%) and Cr (99.99%), and through Cd–Cr alloys in glassy graphite crucibles placed in vacuum sealed quartz ampoules. The crystal structure and stoichiometry were confirmed by energy-dispersive analysis (EDX) and powder X-ray diffraction (PXRD), and repeat

the results reported in our works [1,2]. Electron paramagnetic resonance (EPR) was measured on a standard modulating Bruker EPR spectrometer in the X-band (9.3 GHz). The samples were paraffin-suspended powders of particles approximately  $4\ \mu\text{m}$  across. The cylindrical ampoule was rotated around its axis. Before measurements, the ampoule was held in a magnetic field 1.3 T at a temperature  $\sim 370\ \text{K}$  (above the melting point of paraffin) for 15 min and cooled to room temperature (until solidification of paraffin) in the same field.

### 3. Results and discussion

#### 3.1. Crystal structure

As mentioned above, the PXRD data confirm the identity of the synthesized pure and doped compounds of the low-temperature tetragonal modification  $\alpha\text{-Cd}_3\text{As}_2$  [11] with symmetry space group  $I4_1cd$  and parameters lattices  $a = b = 12.653\ \text{\AA}$ ,  $c = 25.456\ \text{\AA}$ . The lattice parameters of pure  $\alpha\text{-Cd}_3\text{As}_2$  obtained in the same way were  $a = b = 12.6539\ \text{\AA}$ ,  $c = 25.4586\ \text{\AA}$ . The experimental and calculated data as per PXRD, as well as in [1,2], practically coincide. It should be noted that each  $\text{cm}^3$   $\alpha\text{-Cd}_3\text{As}_2$  contains  $2.564 \cdot 10^{20}$  crystal cells. The concentration of conduction electrons is only  $6 \cdot 10^{17}\ \text{cm}^{-3}$ . And this is despite the fact that each cell contains 160 atoms, in particular, 96  $\text{Cd}^{2+}$  ions and 64  $\text{As}^{3-}$  ions in stoichiometric positions. In addition, the tetrahedral vacancies of the crystal cell, the main compositional motif of which is an antiferroite [12] type structure, can contain up to 0.16 at.%  $\text{Cd}^{2+}$  ions or, in the presence of impurities, ions  $\text{Cr}^{2+}$ . I.e. about 1 „extra“ atom of cadmium per six cells. However, the electrons from these cadmium ions do not enter the conduction band, since the possible concentration of such ions is 2 orders of magnitude higher than the electron concentration in a pure sample. The detachment energies of the second electrons Cd and Cr are very close. But for detachment of the third electron, significantly different energies are required: in the case of  $\text{Cr}^{2+}$  — 30.1 eV versus 37.5 eV for  $\text{Cd}^{2+}$  [13]. Divalent ion sizes  $D_{\text{Cd}} \approx 1.98\ \text{\AA}$  and  $D_{\text{Cr}} \approx 1.66\ \text{\AA}$  [13] also differ significantly and cause „subsidence“ of the lattice at the location of impurity chromium ions and, accordingly, contraction of bonds in the near coordination spheres.

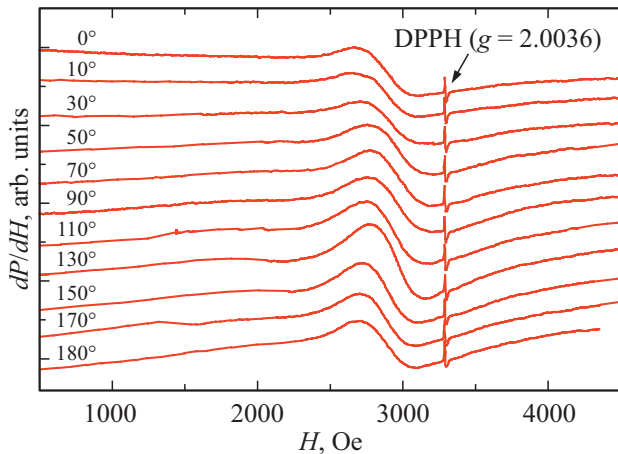
#### 3.2. Electron paramagnetic resonance

Based on general considerations on the opportunity of observing EPR on chromium ions [14], we can talk about the observation in our system of EPR on chromium ions  $\text{Cr}^{2+}$ ,  $\text{Cr}^{3+}$ ,  $\text{Cr}^{4+}$ . The chromium ion  $\text{Cr}^{2+}$  has the configuration  $3d^4$  with spin  $S = 2$  and orbital momentum  $L = 2$ , and its valence state completely corresponds to the substituted ion  $\text{Cd}^{2+}$ , as, for example, in the case of EPR in fluorite  $\text{CaF}_2$  [15] and  $\text{CdGa}_2\text{S}_4$  [16]. The chromium ion  $\text{Cr}^{4+}$  has the configuration  $3d^2$  with  $S = 1$ . However, the formation of a  $\text{Cr}^{4+}$  ion due to the high detachment

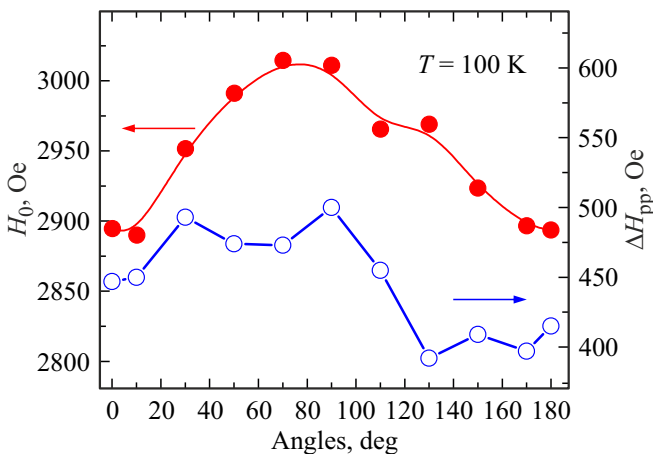
energy of the 4th electron in this system is incredible, as is its detection in the EPR measurements of the X-frequency range. Also, in this frequency range, it is impossible to observe EPR on the  $\text{Cr}^{2+}$  ion, which is obviously the main impurity when  $\text{Cd}_3\text{As}_2$  is doped with chromium. Observation is impossible due to the strong splitting of the energy levels of this ion in the crystal field [14], which is larger than the RF field quantum. Thus, in the case of EPR measurements at frequencies in the X-range, only EPR on  $\text{Cr}^{3+}$  ions can be considered.

The EPR spectra of  $\text{Cr}^{3+}$  ions should have components corresponding to  $S = 3/2$  and  $L = 3$ , which, in the case of low symmetry of the ion environment, decay into three components of the fine structure, a c in the case of the influence of the spin of the nucleus — also on the components of the hyperfine structure. The observed EPR spectrum is dominated by the resonance line, which has a distorted, almost symmetrical shape, in which traces of a fine or hyperfine structure can be seen (see Fig. 1). Note that although a natural mixture of chromium isotopes contains only 9% of an isotope with a non-zero nuclear spin (and this greatly reduces the intensity and the possibility of resolving the hyperfine structure), the manifestation of a super-hyperfine structure is also possible [17] caused by the interconfigurational interaction [18] with the nuclei of the Cd and As ligands, which in a natural mixture of isotopes contain nuclei with magnetic moments. Thus, the spectrum of the sample has a very complex structure, which allows us to state with a sufficient degree of confidence that the observed spectrum of the powder sample consists of minimum two groups of allowed EPR lines with a width of 300 Oe. Based on the results of experiments on doping  $\alpha\text{-Cd}_3\text{As}_2$  with other impurities, we associate these groups with  $\text{Cr}^{3+}$  ions located in the tetrahedral environment in the substitutional positions of the ions  $\text{Cd}^{2+}$  and in the positions of tetrahedral vacancies in the antiferroite structure [12]. However, most of these positions, in accordance with the level of doping, are occupied by  $\text{Cr}^{2+}$  ions, which, of course, affect the widths and positions of the EPR lines originating from  $\text{Cr}^{3+}$  ions.

In the temperature range of about 80–120 K, a plateau was observed in the EPR spectrum, which is characteristic of powder samples with an anisotropic  $g$ -factor. After a transient in the resonance field  $H_0$  at  $T \approx 120\ \text{K}$ , a rapid decrease  $H_0$  was observed with decreasing temperature. This shift below 50 K was accompanied by a sharp broadening of the main resonance line. The temperature behavior of the EPR spectrum in our experiment requires a more painstaking presentation. The present paper, however, aims to report on a very important and interesting moment in this study — the observation of the angular dependence of the position and width of the EPR signal (see Fig. 1 and 2). Deal is that the sample is a powder with a particle size of approximately  $4\ \mu\text{m}$ , placed in paraffin and a cylindrical ampoule. The sample was rotated in a magnetic field around the ampoule axis. The  $\text{Cr}^{3+}$  ion is characterized by a slight anisotropy of the  $g$ -factor (typical values are



**Figure 1.** Variations in the EPR spectra during the rotation of the ampoule with powder around its axis. The vertical line refers to the standard marker diphenyl–picryl–hydrazyl (DPPH).



**Figure 2.** Dependences of the position  $H_0$  and the peak-to-peak linewidth of EPR  $\Delta H_{pp}$  on the angle of rotation of the ampoule around its axis.

$g_x, g_y, g_z \approx 1.95-1.98$ ). In our case (see Fig. 1) the  $g$ -factor ranges from 2.13 to 2.18. In case of a powder, where all crystallographic directions are equally probable, the ampoule rotation cannot lead to a change in the position of the EPR signal. In our experiment, the 2 angular dependences shown in Fig. 1 were obtained after paraffin melting and holding the sample for 15 min at a temperature of about  $\sim 380$  K in a 13 kOe field and cooling in the same field until the paraffin solidifies. Angular dependencies indicate the presence of the 2nd axis, as well as in the work [19]. The same type of angular dependence is repeated in the line width. However, this does not directly indicate the nature of the anisotropy symmetry, since the particles are oriented only along the „direction of the easy axis“ (most likely, the  $c$  axis) when the orientation of other crystallographic axes is random. The fact that there is „an easy axis of magnetization“ in the paramagnetic state

means that at a given temperature ( $\sim 400$  K) the value of the induced magnetization of chromium ions depends on the crystallographic direction. Let us emphasize that pure  $\alpha\text{-Cd}_3\text{As}_2$  is a strong diamagnet. The appearance of texture in a powder sample can be associated with only two factors: the magnetic anisotropy of the shape of the powder particles and the magnetic crystalline anisotropy. In the first case, the shape of the particles should again be tied to the crystal structure. For example, when preparing a powder, due to the pronounced cleavage of the crystal in the direction perpendicular to the  $c$  axis, disk-shaped particles (flakes) are formed. These particles in a magnetic field will line up with planes parallel to the field, even in the case of diamagnetism. The  $c$  axis will turn out to be perpendicular to the external magnetic field, and there will be disorder in the arrangement of the  $a$  and  $b$  axes. When preparing the powder, no pronounced manifestation of the cleavage plane was observed; although the powder particles were not spherical, they were characterized by similar sizes in three directions. Therefore, only the second factor remains significant — the presence of magnetic crystalline anisotropy in the paramagnetic state.

Thus, the orbital degrees of freedom significantly affect the magnetic state of the  $\text{Cr}^{2+}$  and  $\text{Cr}^{3+}$  ions, and in the case of Dirac quasiparticles, this effect can be expected to be enhanced due to the exchange interaction between localized magnetic moments and free Dirac electrons, the spins of which are rigidly related to their orbital moments. This results in an increase in the  $g$ -factor of localized magnetic moments and the appearance of an anisotropy of the induced magnetization in the paramagnetic state. Anisotropy manifests itself in the orientation of powder particles in a magnetic field even at temperatures  $\sim 400$  K, which are noticeably higher than room temperature. At these temperatures, the positive magnetoresistance is still significant — both in pure and chromium-doped cadmium arsenide. In connection with this, it can be expected that, as in the case of doping with  $\text{Mn}^{2+}$  ions, magnetic polarons are formed on  $\text{Cr}^{2+}$  ions, which, however, will have anisotropic polarizability. Meanwhile, the source of donor electrons are the same  $\text{Cr}^{2+}$  ions, which donate one electron to the conduction band and turn into  $\text{Cr}^{3+}$  ions. Ions  $\text{Cr}^{3+}$ , on which EPR is observed, may have a stabilized valence, or may participate in a double exchange [10], the carriers of which in this case are donor Dirac electrons, the state of motion which are topologically protected. In this case, the lifetime of the  $\text{Cr}^{3+}$  and  $\text{Cr}^{2+}$  ions, or the exchange frequency, will affect the EPR linewidth and the phase of the Dirac electron wave function, which determines the modes of weak localization or antilocalizations [20–23]. This circumstance indicates the opportunity that the conditions for the implementation of the weak antilocalization regime will depend on the direction of motion of the Dirac electrons and, depending on the direction of the current, a positive [24] or a negative [25] magnetoresistance. Reducing the lifetime of the  $\text{Cr}^{3+}$  ion and its interaction with the conduction electron leads to a partial reduction in the

magnetic moment of the ion. And in case of a large  $g$ -factor of an electron, this causes a significant change in the  $g$ -factor of the ion.

Assuming that the value of the  $g$ -factor of the ion is the result of interaction with the magnetic moments of the current carriers, which reduce the magnetic moment, and using the results [26] for two interacting electronic subsystems with the corresponding  $g$ -factors  $g_1$ ,  $g_2$  and spins  $S_1$ ,  $S_2$ ,

$$g_{\text{eff}} = (g_1 S_1 + g_2 S_2) / (S_1 + S_2),$$

$$g_{\text{eff}} = g_{\text{Cr}} [1 - \mu(g_e/g_{\text{Cr}})] / (1 - \mu^*),$$

where  $\mu^* = (\mu_{\text{Cr}} - \mu_0) / \mu_{\text{Cr}}$ . For  $g_e = 16$ ,  $g_{\text{Cr}} = 2.0$ , depending on the degree of reduction  $\mu_{\text{Cr}}/\mu_0$  of the magnetic moment of the ion  $\text{Cr}^{3+}$ , we get the following effective  $g$ -factors: for  $\mu_{\text{Cr}}/\mu_0 = 1.00$ ,  $g_{\text{eff}} = 2.00$ ; for  $\mu_{\text{Cr}}/\mu_0 = 0.99$ ,  $g_{\text{eff}} = 2.154$ .

It should also be taken into account that the  $\text{Cr}^{3+}$  ion is a Jan-Teller ion [27–30], and local symmetry breaking of the tetrahedral environment of the ion in this topological material can be associated with it. This can also pin the magnetic moments of impurities in the crystal lattice and enhance the anisotropy in the paramagnetic state.

In addition, a few remarks about the opportunity of the presence in the sample of foreign phases or compounds from the Cr–As system, which, in principle, could give EPR signals and not be detected by other measurement methods. According to [31], in the Cr–As system there are 7 compounds with a melting temperature above or close to the synthesis temperature of  $\text{Cd}_3\text{As}_2$  samples, which could remove chromium from investigated compound  $\text{Cd}_3\text{As}_2$  and substitute its magnetic properties. Pure chromium is antiferromagnetic with a Neel temperature slightly above room temperature. However, for the compound with the highest chromium content,  $\text{Cr}_3\text{As}$ , an anomaly is observed in the temperature behavior of the magnetic susceptibility at 973 K. As the chromium content decreases, compounds with a complex magnetic structure are observed; however, naturally, the temperatures of magnetic transitions decrease. The Neel temperature for the sufficiently studied compounds  $\text{Cr}_2\text{As}$  and  $\text{CrAs}$  [32–38] is approximately 430 and 280 K, respectively. Thus, the magnetic transition temperature for the compound with the highest arsenic content  $\text{CrAs}_2$  can be expected to be close to 200 K. In this range, there are no sharp features in the behavior of the EPR spectrum, which usually accompany phase transitions. This once again confirms the data on the composition and crystal structure of the samples: chromium dissolves well in  $\alpha$ - $\text{Cd}_3\text{As}_2$  and does not form separate phases with each of the components during synthesis.

## 4. Conclusion

Thus, we believe that in EPR on  $\text{Cr}^{3+}$  ions in  $\alpha$ - $\text{Cd}_3\text{As}_2$  there is an increase in the anisotropy of the paramagnetic susceptibility of localized spins ensemble of impurity

chromium ions due to the orbital angular momentum and anisotropy of the  $g$ -factor of impurity ions interacting with the spin and anomalously large orbital momenta of donor Dirac electrons produced by the same impurity ions.

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## Conflict of interest

The authors declare that they have no conflict of interest.

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