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Cooling of a Vapor-Gas Mixture by Evaporating Water Drops: Dependence of the Cooling Time on the Initial Temperature of the Vapor-Gas Mixture

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Numerical modeling of the cooling of a vapor-gas mixture by evaporating water drops has been carried out. An expression is proposed using the characteristic temperature, which determines the dependence of the cooling time on the initial temperature of the mixture. A technique for determining the characteristic temperature based on the results of numerical simulation is presented. An estimate of the error in the results of using the obtained expressions is made.

Keywords: vapor-gas mixture, thermal relaxation, numerical simulation, heating and evaporation of droplets, vapor-gas mixture cooling time, characteristic temperature.

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Out of various use cases of evaporation of droplets [1], the following two are the most relevant to the present study. The first of them is the use of evaporation for rapid cooling of droplets (including cooling in the process of nanoparticle synthesis by LPSP (low-pressure spray pyrolysis) [2,3]). The second scenario is the use of evaporation of droplets for rapid cooling of the carrier vapor–gas flow. This is the method that is proposed for cooling of methane conversion products and is expected to inhibit the formation of soot [4]. Another use scenario is stabilization of temperature conditions in thermal processing of wood biomass (torrefaction), which features an exothermic stage [5,6]. Fine-droplet spraying of cooling water within the section of cooling of black pellets is one of the possible solutions here. The endothermic process of droplet evaporation should then balance out the heat release in the exothermic reaction. Estimates of the time of cooling of gas by evaporating water droplets are needed to assess the feasibility of this solution. A considerable number of papers focused on solving the problems of heat and mass exchange in gas flows with evaporating droplets have already been published (see, e.g., [2,4,7,8]). However, the obtained results have not been generalized into relations that allow one to estimate the time of cooling of a vapor–gas mixture by evaporating water droplets as a function of the initial temperature of this mixture and the temperature interval of cooling. The aim of the present study is to derive such relations.

The formulation of the problem corresponds to the one used in [9,10]: a flow of a hot vapor–gas mixture and cool water droplets was considered. The flow is steady, laminar, and single-velocity. The problem was solved by simulating the process of thermal relaxation in a gas–droplet flow numerically and analyzing and generalizing the obtained results.

Just as in [10], the mathematical model of the process of thermal equilibration (thermal relaxation) of the flow included the equations of continuity and energy for gas, vapor, and droplets:

— the mass fraction of gas is constant, and the variation of mass fractions of vapor and droplets is governed by the balance condition

$$\frac{dg_g}{dt} = 0, \qquad \frac{dg_v}{dt} = -\frac{dg_d}{dt}; \tag{1}$$

— the number of droplets remains unchanged in the process of gas cooling, and the variation of the mass fraction of droplets for a monodisperse distribution is specified by the mass and radius variation of a single droplet

$$\frac{dg_d}{dt} = \frac{3g_d}{r_d}\frac{dr_d}{dt}; \tag{2}$$

— the temperature of droplets is specified by the intensity of their evaporation and heating in the process of heat exchange with vapor and gas

$$g_d c_{pd}\frac{dT_d}{dt} = \frac{dg_d}{dt}\big(L(T_d) + c_{pv}(T_{vg} - T_d)\big) + Q_{vg\text{-}d}, \tag{3}$$

$$\big(g_v c_{pv} + g_g c_{pg}\big)\frac{dT_{vg}}{dt} = -Q_{vg\text{-}d}. \tag{4}$$

It was taken into account here that $u\frac{d}{dx} = \frac{d}{dt}$ for a steady flow, where x and u are the coordinate and the velocity along the flow axis; g_i, c_{pi}, and T_i are the mass fraction, the heat capacity at constant pressure, and the temperature of component i of the vapor–gas–droplet mixture, respectively; r_d is the droplet radius; $L(T_d)$ is the heat of evaporation at the droplet temperature; and

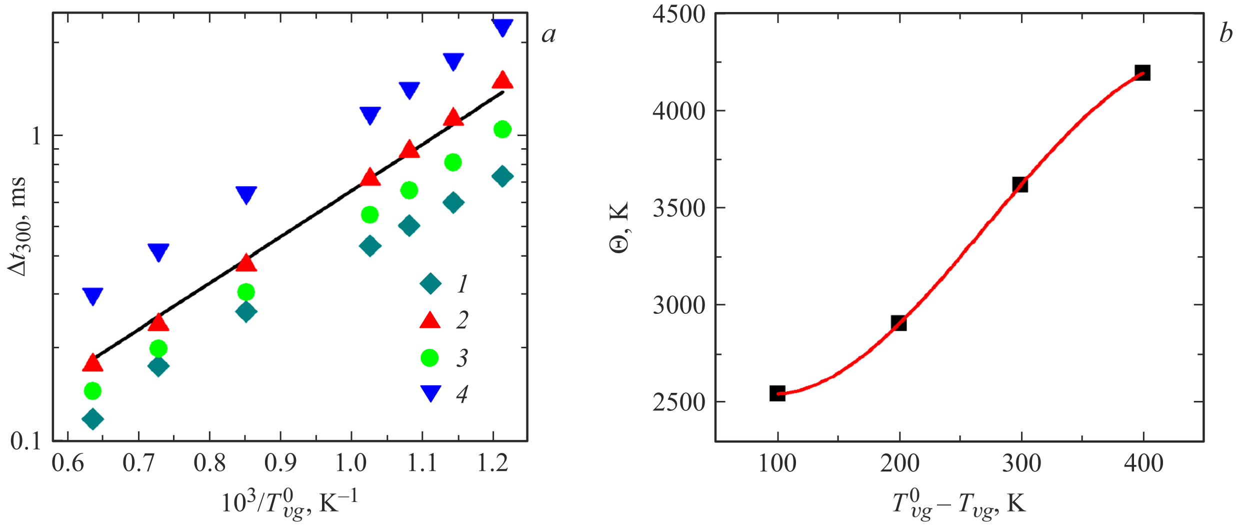

Figure 1. *a* — Cooling time of the vapor–gas mixture at $\Delta T = 300$ K as a function of the initial temperature of this mixture. Symbols denote the results of calculation for different values of the initial mass fraction of droplets and different initial compositions of the vapor–gas mixture (*1* — $g_d^0 = 0.5$, $g_{vgf}^0 = 0$; *2* — $g_d^0 = 0.5$, $g_{vgf}^0 = 0.999$; *3* — $g_d^0 = 0.5$, $g_{vgf}^0 = 0.5$; *4* — $g_d^0 = 0.333$, $g_{vgf}^0 = 0.5$), and the solid line is the result of approximation performed using (8) in order to determine the characteristic temperature. *b* — Dependence of the characteristic temperature on the length of the temperature interval. Symbols and the solid line represent the results of processing of calculated data and approximation performed in accordance with (10), respectively.

$Q_{vg\text{-}d}$ is the amount of heat transferred from vapor and gas to droplets in a unit mass of the mixture in a unit time,

$$Q_{vg\text{-}d} = \left(\frac{3g_d}{\rho_l r_d^2}\right)\frac{\mathrm{Nu}}{2}\lambda_{vg}(T_{vg} - T_d). \quad (5)$$

Here, λ_{vg} is the coefficient of heat conduction of the vapor–gas mixture and Nu is the Nusselt number. This number is $\mathrm{Nu} = 2$ in the single-velocity approximation used here (droplets are embedded into the flow).

The following form of the expression for the rate of evaporation of droplets (with an initial radius of $\sim 10\,\mu$m) in the diffusion regime with account for the Stefan flow, which emerges due to the presence of noncondensable gas in the mixture, was used [11]:

$$\frac{dr_d}{dt} = -\frac{\rho_{vg}D_{vg}}{\rho_l r_d}\left(\ln\frac{1-\alpha_1}{1-\alpha_2}\right), \quad (6)$$

where

$$\alpha_1 = \frac{\rho_\Sigma g_v}{\rho_{vg}}, \quad \alpha_2 = \frac{\rho_v^s}{\rho_{vg}}, \quad \rho_{vg} = \rho_\Sigma\left(g_v + g_g\frac{\mu_v}{\mu_g}\right), \quad (7)$$

ρ_i and μ_i are the density and the molar mass of component i of the mixture, ρ_v^s is the vapor density at saturation, and D_{vg} is the coefficient of diffusion of vapor molecules in the vapor–gas mixture. Indices v, g, d, l, and Σ correspond to vapor, gas, droplets, the droplet material, and the mixture as a whole, respectively. A similar model of evaporation of heated droplets was used in [12]. This study differs from the present one in that droplets were heated ([12]) by laser radiation and the temperature of ambient gas (atmospheric air) was assumed to be constant.

The assumptions made in the thermal relaxation model (rapid equalization of vapor and gas temperatures in intermolecular collisions that makes it possible to use a single energy equation for the gas phase; uniformity of the temperature profile within evaporating droplets; quasi-steady nature of the process of droplet evaporation) and the corresponding time constraints have been discussed in or previous study [10]. Their validity was verified.

The Kutta–Merson method with automatic step selection and proprietary software (COND-KINET-1 [13]) were used to solve the system of ordinary differential equations (1)–(4), (6). The TOL parameter, which governs the choice of smaller steps, was set to a level at which the obtained results ceased to change with further reduction of the TOL value. Numerical simulations of cooling were performed for a water vapor–nitrogen vapor–gas mixture with the initial mass content of vapor varying from 0 to 100%. The initial temperature of water droplets was 293 K. The needed reference data on the properties of nitrogen, water vapor, and water were taken from [14]. The initial values of vapor–gas mixture temperature T_{vg}^0 and mass fractions of droplets (g_d^0) and water vapor (g_v^0), which specified the initial concentration of water vapor in the gas phase ($g_{vgf}^0 = g_v^0/(1 - g_d^0)$), were varied. Temporal dependences (along the flow axis) of the vapor–gas mixture temperature and the mass and temperature of droplets were determined. Cooling times for the vapor–gas mixture corresponding to specific temperature intervals were derived from the calculated data. The parameters varied within the following intervals in calculations: g_d^0 (0; 0.5; 1), g_{vgf}^0 (0; 0.5; 1), T_{vg}^0 (823–1573 K), and $\Delta T = T_{vg}^0 - T_{vg}$ (100–400 K). It

Polynomial expansion coefficients in expression (10)

Θ_0, K	Θ_1, K,	Θ_2, K	Θ_3, K
3.0149	−10.4825	65.72	−80.45

was found that the dependence of the obtained data on the initial vapor–gas mixture temperature is given by

$$\Delta t = \Delta t_{973}\exp\left(\Theta\left(\frac{1}{T_{vg}^{0}}-\frac{1}{973}\right)\right), \tag{8}$$

where Θ is the characteristic temperature and Δt_{973} is the cooling time at $T_{vg}^0 = 973\,\mathrm{K}$, which is determined in accordance with the following expression:

$$\Delta t_T = A\left(r_d^0/10\right)^2\left(g_d^0/\left(1-g_d^0\right)\right)^{-n}. \tag{9}$$

The relation between Δt_{973} and the initial composition of the vapor–gas mixture and the temperature interval of cooling was defined in [10] in terms of the coefficients of polynomial expansions for A and n in parameter $Y = \Delta T/1000$, which depend on g_{vgf}^0 and were determined at $T_{vg}^0 = 973\,\mathrm{K}$. In general, characteristic temperature Θ also depends on the values of g_d^0, g_{vgf}^0, and ΔT. In order to simplify the practical application of expression (8), we tried to determine Θ for each ΔT value by analyzing data for the entire interval of variation of g_d^0, g_{vgf}^0, and T_{vg}^0; i.e., we checked whether the dependence of Θ on the initial composition of the vapor–gas mixture may be neglected. With this end in view, we plotted cooling times for each ΔT value and the entire spectrum of initial mixture compositions in $\lg(\Delta t)-1/T_{vg}^0$ coordinates and performed a linear approximation of these data. The coefficient at the linear term specified the sought-for Θ value. Figure 1, *a* illustrates this approach. The obtained Θ values (Fig. 1, *b*) were approximated by a polynomial in parameter $Y = \Delta T/1000$ of degree 3

$$\Theta = 10^3\left(\sum_{i=0}^{i=3}\Theta_i Y^i\right). \tag{10}$$

The approximation coefficients are listed in the table.

The validity of the proposed approach was verified by comparing two values of $\Phi = \Delta t(T_{vg}^0)/\Delta t_{973}$: Φ_{appr} obtained using expressions (8), (10) and Φ_{calc} derived directly from calculated data. The values of ratio Φ_{appr}/Φ_{calc} are presented in Figs. 2. The relative error of calculation of the vapor–gas mixture cooling time as a function of the initial mixture temperature performed using the proposed approach and expressions (8), (10) was determined as

$$\delta = \frac{|\Phi_{appr}-\Phi_{calc}|}{\Phi_{calc}}\cdot 100\%. \tag{11}$$

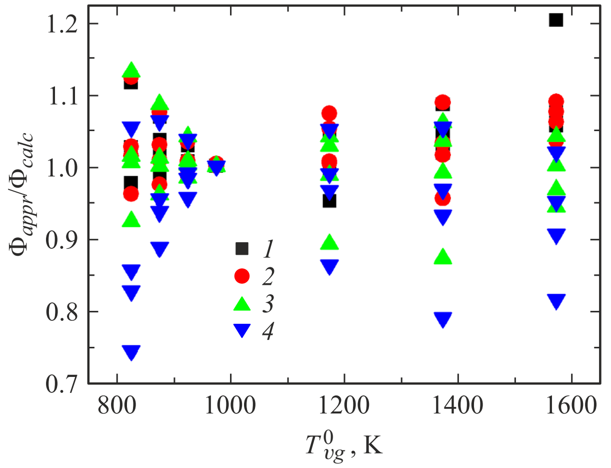

Figure 2. Results of application of expressions (8), (10) corresponding to different initial vapor–gas mixture temperatures and temperature intervals $\Delta T = 100$ (*1*), 200 (*2*), 300 (*3*), and 400 K (*4*).

Having processed the results presented in Fig. 2 (96 values), we determined the dependence of the error of data obtained using expressions (8), (10) on the initial vapor–gas mixture temperature (averaging over ΔT, squares) and the temperature interval length (averaging over T_{vg}^0, triangles) (Fig. 3, *a*). The distribution of the results of application of expressions (8), (10) over the error value is shown in Fig. 3, *b*, where N_Σ is the total number of calculated values (96) and ΔN is the number of values calculated in accordance with (11) and having an error that does not exceed the one corresponding to a specified δ interval. It can be seen that the approach utilizing expression (8), where characteristic temperature Θ, which specifies the temperature dependence of the cooling time, depends only on temperature interval ΔT of cooling, yields results with an average error within 10% and a maximum error of 25%. It may be assumed that approximating expression (8) yields results with a greater error when applied in conditions differing from the ones in which it was derived ($T_{vg}^0 = 823-1573\,\mathrm{K}$, $\Delta T = 100-400\,\mathrm{K}$, $r_d^0 \leqslant 10\,\mu\mathrm{m}$). Calculated data from [2] on cooling of gas by evaporating droplets were used to verify this assumption. According to [2], the time of cooling of gas by $\Delta T = 20\,\mathrm{K}$ in a gas–droplet flow with initial temperature $T_{vg}^0 = 296\,\mathrm{K}$ and mass content $g_d^0/(1-g_d^0) = 0.03$ of micrometer-sized droplets (the authors of [2] did not provide more accurate size data) is 10 ms. The same time determined using expressions (8)−(10) and tabulated data for $r_d^0 = 1-2\,\mu\mathrm{m}$ is 3.5−14 ms. If we take into account the difference in conditions (T_{vg}^0 and ΔT values) between [2] and the present study and the uncertainty of r_d^0, the result of application of expressions (8)−(10) appears to be satisfactory.

Expressions (8), (10) and data listed in the table amount to fulfillment of the declared objective of the present study; i.e., they allow one to determine the dependence of the cooling time of a vapor–gas mixture in a specified temperature interval on its initial temperature. Future studies will

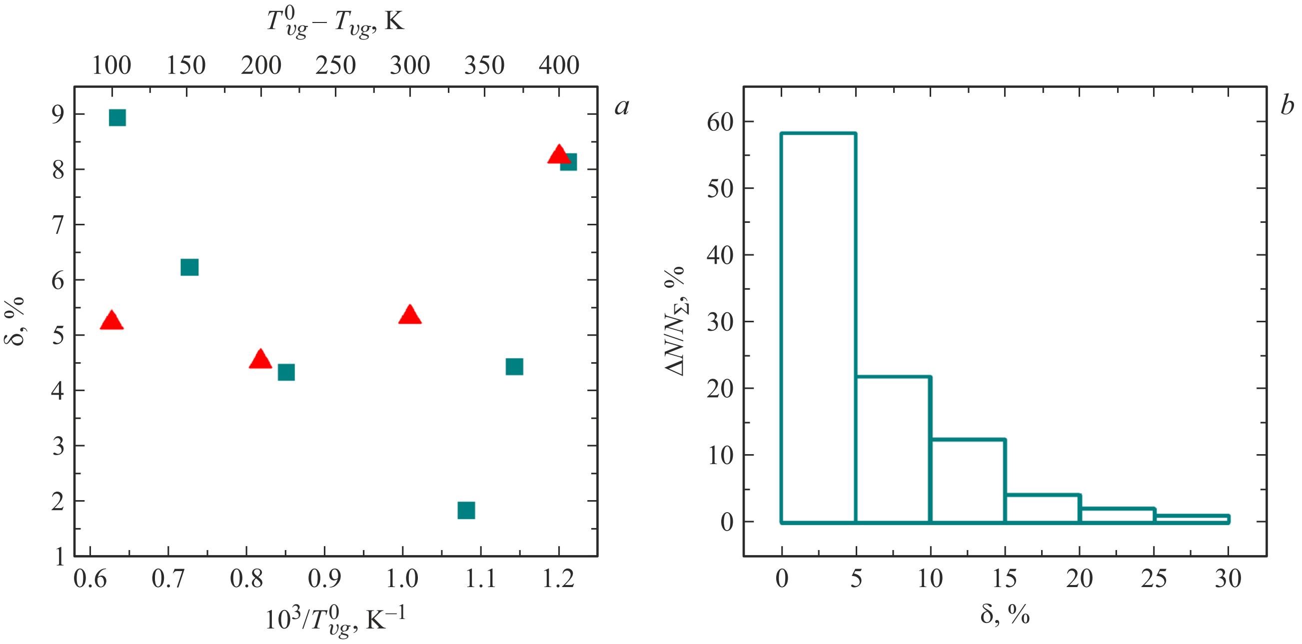

Figure 3. *a* — Variation of the relative error of data obtained using expressions (8), (10) with initial temperature of the vapor–gas mixture (squares) and length of the temperature interval (triangles). *b* — Distribution of the results of application of expressions (8), (10) over the error value.

be focused on applying the proposed approach with the characteristic temperature, which specifies the dependence of the cooling time on the initial mixture temperature, to turbulent flows in the double-velocity approximation.

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Conflict of interest

The author declares that he has no conflict of interest.

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