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On a „paradox“ in the scattering theory

© L.A. Apresyan

Prokhorov Institute of General Physics, Russian Academy of Sciences,
Moscow, Russia
e-mail: leon_apresyan@mail.ru

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The solution of the „paradox“in scattering theory is considered, according to which the extinction cross section is expressed in terms of the forward scattering amplitude (the so-called „optical theorem“), whereas for a point source, and as a consequence, for any emitter located at a finite distance from the scatterer, a similar ratio is often written through a scattered field near the emitter, i.e. determined by „backscattering“. A clear picture of the formation of radiation losses during the transition of energy from the source to the scatterer is presented. It is shown that although the field backscattered to the source determines the change in its radiation characteristics (the Purcell effect), the optical theorem includes an extinction factor which is generally related to the work of the incident wave on the currents induced in the scatterer. This factor passes into the forward scattering amplitude in the limiting case of a plane incident wave.

Keywords: optical theorem, energy conservation, radiation losses, Purcell effect, point source of radiation.

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Introduction

Most physical measurements are ultimately associated with the energy transition from one form to another. In the theory of wave or particle scattering in describing the energy balance the so-called optical theorem plays a fundamental role (see, for example, [1–6], and also the history of the problem in [7]; in foreign literature the names „extinction theorem“, „forward scattering theorem“, „optical cross-section theorem“ are also used). According to this theorem, the extinction cross-section, i.e. the total attenuation cross-section of a plane wave incident on the scatterer σ_{ext}, which is equal to the sum of the scattering and absorption cross-sections, is expressed in terms of the imaginary part of the scattering amplitude in the „forward“ direction $\operatorname{Im} A(0)$. This ratio is universal, i.e. it does not depend on the structure of the scatterer. With appropriate minor modifications, it is performed for a wide class of wave problems in optics [1–3], electrodynamics [4], quantum mechanics [5], acoustics [8], elasticity theory [9], seismology [10], etc., covering both bulk and surface [11]waves. Thus, in the case of an electromagnetic field for the plane incident wave with a unit polarization vector $\mathbf{e}_0$, the scalar amplitude $A(0)$ goes into the projection of the corresponding vector amplitude $\mathbf{A}(0)$ onto the complex conjugate value $\mathbf{e}_0^*$, and the optical theorem becomes $\sigma_{\text{ext}} = (4\pi/k_0)\operatorname{Im} \mathbf{e}_0{}^*\mathbf{A}(0)$, where k_0 — wave number.

The optical theorem can also be extended to the case of field excitation by a point source, which in some special settings was considered for the first time in [12]. The description of the point source makes it possible to pass from the usual formulation of the optical theorem for the plane incident wave to the case of an arbitrary incident wave, and to consider, in particular, the energy transfer to the scatterer from distributed sources of radiation close to it. Among the various generalizations of the optical theorem described in many dozens of papers, this case gained particular importance due to the development of nanooptics and photonics, where the use of the optical theorem is often associated with the Purcell factor, which describes changes in the emission of a fluorescent molecule due to the nanoparticles presence near it [13]. Recently, the author of this paper proposed an operator approach to obtain the optical theorem [14,15], which makes it possible to significantly simplify its derivation in the general case, including the case of point source.

In the literature there is often an opinion that for a point source the extinction cross section is expressed in terms of the scattered field near the source, i.e., it is actually connected with scattering in the „backward“ direction. Acceptance of this thesis leads to an obvious „paradox“. Indeed, the plane incident wave can be considered as a limit special case of wave created by an infinitely distant point source. But if for the radiation of the point source the extinction on the scatterer was determined by backscattering, then the transition to the infinitely distant source could not give the usual optical theorem related to „forward“ scattering, since it is difficult to imagine a situation when during simple limit transition the meanings „backward“ and „forward“ exchange places.

Nevertheless, this erroneous opinion is quite widespread. Thus, in the Russian translation of a well-known monograph on nanooptics [13], when discussing the scattering of radiation produced by point dipole source, the following statement is contained: „According to the optical theorem, the relaxation power (the sum of the scattered and absorbed powers) can be expressed in terms of the backscattered field“ ([13]). This formulation is incorrect and obviously contradicts the usual form of the optical theorem, in which the amplitude of „forward“ scattering appears. Comparison with the original shows that the correct formulation of the optical theorem is given in the English text. However, this fragment „corrected“ by the translator, can hardly be attributed to chance. Apparently, the translator kept in mind the case of a point source actually related to the backscattering, which was further considered in [13] in relation to the Purcell factor, and identified this case with the optical theorem characterizing the scatterer.

If the above example can still be attributed to terminological confusion, then there are papers in which attempts were made to directly „explain“ the optical theorem by backscattering. In the case of plane incident wave, as an example of this kind, one can specify the paper [16], in which for scattering by an ideally conducting sphere the optical theorem was related to backscattering. In order to emphasize the radical difference from the usual interpretation of „forward scattering theorem“, the term „backward scattering theorem“ was even introduced. This conclusion was criticized in the paper [17], where the formal nature of the reasoning [16]and the validity of the traditional approach were emphasized.

In the case of point source this erroneous thesis about the decisive role of backscattering was also developed in [18]. In it, an attempt was made to use the optical theorem for the point source to explain the physics of the Purcell effect in terms of the Wheeler–Feynman theory [19]. At the same time, despite the correct description of energy flow balances, the optical theorem for the point source was formulated in [18] as $\sigma_{\mathrm{ext}} = (4\pi/k_0)\,\mathrm{Im}\,\mathbf{e}^*_{sc}\mathbf{p}$, where $\mathbf{e}_{sc}$ is the polarization vector of the backscattered wave, and $\mathbf{p}$ is the dipole moment of the source, i.e., the extinction cross-section σ_{ext}, was expressed in terms of the field scattered back to the source. This relation is incorrect and contradicts the results of papers [12,14,15] for the field of the point source. The absence in literature of the explanation of such a contradiction can confuse the insufficiently prepared reader. The purpose of these methodological notes is to clarify this issue.

Section 1 describes the local laws of energy conservation related to the total field separation into the field of the incident and scattered waves. Here, in contrast to traditional approaches, attention is focused on the separation of radiation (currents) sources into given *a priori* and secondary ones, i.e., induced in the scatterer. The presentation is carried out on the example of a simple scalar wave equation. This simplifies the notations and ensures digress from details that are not essential for the problem under consideration. The final balance relations remain unchanged also for the electromagnetic problem, including the case of anisotropic as well as bianisotropic scatterers (see remarks and references given below). Section 2 considers the transition from local to integral conservation laws. This Section extends the results of the papers [18,20] on the energy balance for the electromagnetic problem, giving a visual description of power flows and the formation of radiation losses on a simpler example of a scalar field. Section 3 compares different approaches to the optical theorem for a point source. In the Conclusion the main conclusions are formulated, revealing the reasons for the occurrence of the indicated „paradox“, and allowing us to avoid such errors in the future.

1. Local energy conservation laws in the scattering problems

The aforementioned „paradox“ is associated with an incorrect interpretation of the energy balance conditions in the scattering problem. We present a brief derivation of these conditions using the scalar wave field model.

Consider the scattering of a monochromatic field (multiplier $e^{-i\omega t}$, where ω — angular frequency, and t — time is omitted everywhere) with a complex amplitude $u_t = u_t(\mathbf{r})$ on scatterer located in free space and limited by a finite volume V_{sc} with an effective scattering potential $\nu = \nu(\mathbf{r})$. In electrodynamic problems, when describing scattering by heterogeneities of the permittivity $\varepsilon(\mathbf{r})$, the value $-k_0^2\delta\varepsilon(\mathbf{r})$ usually acts as ν, where $\delta\varepsilon(\mathbf{r}) = \varepsilon(\mathbf{r}) - 1$ — difference of $\varepsilon(\mathbf{r})$ from the permittivity of the medium (see, for example, [4]).

Let us assume that the field is generated by given distributed (currents) sources $q_0 = q_0(\mathbf{r})$ occupying a finite volume V_0 and located outside the scatterer volume V_{sc}, so that $q_0(\mathbf{r})\nu(\mathbf{r}) \equiv 0$ (see Figure). The case of the point source corresponds to „contracting“ $q_0(\mathbf{r})$ to a delta function localized at some point $\mathbf{r}_0$, $q_0(\mathbf{r}) \sim \delta(\mathbf{r} - \mathbf{r}_0)$. The total field u_t in the case under consideration is expressed as the sum $u_t = u_0 + u_s$ of the field of the incident $u_0 \equiv G_0 q_0$ and scattered $u_s \equiv G_0 \nu u_t$ waves, where $G_0 = (\Delta + k_0^2)^{-1}$ — Green's operator for free propagation corresponding to the scalar wave equation (the explicit form of the operator G_0 and the corresponding integral expressions for u_0 and u_s is given, for example, in [14]).

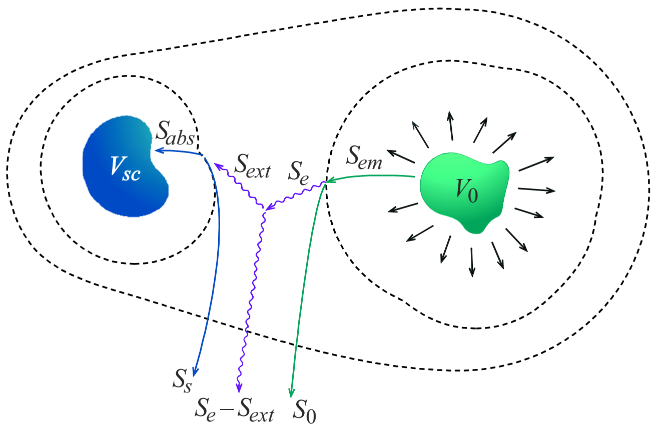
It can be seen from the definition of $u_s = G_0 \nu u_t$ that the scattered wave can be considered as generated by the induced source $q_s \equiv \nu u_t$, i.e. $u_s = G_0 q_s$. In this case, the total field is represented as $u_t = G_0 q_t$, where the total source corresponds to the sum of the given q_0 and induced q_s sources, $q_t = q_0 + q_s$. Each of the fields u_t, u_0 and u_s satisfies the same wave equation of the form $(\Delta + k_0^2)u = q$, but with different sources q_t, q_0 and q_s, and, generally speaking, with different boundary conditions that ensure the uniqueness of the problem solution.

In accordance with the total field division into incident and scattered waves, $u_t = u_0 + u_s$, the quadratic in field time-averaged energy flux density vector

$$\mathbf{s}_t \equiv \mathrm{Im}(u_t^* \nabla u_t) = \mathrm{Im}[(u_0 + u_s)^* \nabla (u_0 + u_s)]$$

is represented as sum of three flows:

$$\mathbf{s}_t = \mathbf{s}_0 + \mathbf{s}_s + \mathbf{s}_e. \tag{1}$$



Balance of energy flows in the scattering problem. Solid lines represent „energy", wavy — „interference" energy flows.

Here $\mathbf{s}_0 = \mathrm{Im}(u_0^* \nabla u_0)$ and $\mathbf{s}_s = \mathrm{Im}(u_s^* \nabla u_s)$ correspond to the incident u_0 and scattered u_s waves separately, and the vector $\mathbf{s}_e = \mathrm{Im}(u_0^* \nabla u_s + u_0^* \nabla u_s)$ can be called an interference flow. This vector has no independent energy meaning and is associated with the overlap of the incident u_0 and scattered u_s waves. Also note that the directly measured value in the problem under consideration is only the flow $\mathbf{s}_t$ associated with the total observed field u_t, while „partial flows" $\mathbf{s}_0, \mathbf{s}_s$ and $\mathbf{s}_e$ play auxiliary role.

Each of the flows $\mathbf{s}_t, \mathbf{s}_0$ and $\mathbf{s}_s$ corresponds respectively to the fields u_t, u_0 and u_s of the sources q_t, q_0 and q_s and satisfies the local laws of energy conservation following from the wave equation

$$\nabla \mathbf{s}_j = w_j, \tag{2}$$

where $j = t, 0, s$. The values here on the right

$$w_j = \mathrm{Im}(u_j^* q_j) \tag{3}$$

have the meaning of local power values describing the energy exchange between the corresponding fields u_j and sources q_j. These values can be both positive and negative, which corresponds to the presence of either sources or drains of field energy (the latter occurs in the case of an absorbing scatterer). Then, all sources q_j are considered as given, although finding the explicit form q_s and q_t requires solving the scattering problem.

It follows from (1)–(3) that the interference flow density $\mathbf{s}_e = \mathbf{s}_t - \mathbf{s}_s - \mathbf{s}_0$ also satisfies equation (2), but with a different form of the right side

$$w_e = \mathrm{Im}(u_0^* q_s + u_s^* q_0). \tag{4}$$

In this case, the total power w_j at each point in space is represented as a sum

$$w_t = w_0 + w_s + w_e, \tag{5}$$

where each of the values w_j corresponds to the corresponding flow $\mathbf{s}_j$ from (1). Thus, all densities of time-averaged flows $\mathbf{s}_j$ $(j = 0, s, t, e)$ satisfy the laws of conservation (2) of the same form, but with different values of volume „sources of flows" w_j. Local conservation conditions (2) contain the entire physics of the energy balance.

For the energy flows $\mathbf{s}_t$, $\mathbf{s}_0$ and $\mathbf{s}_s$, the fulfillment of local laws of conservation (2) seems to be physically obvious, since they reflect the generation (or drain) of the divergence of the corresponding flows at each point where the work of sources (q_t, q_0 and q_s) is performed on the corresponding distinguished part of the field (u_t, u_0 and u_s). In the case of interference flow $\mathbf{s}_e$, the fulfillment of (2) looks less trivial, since the flow $\mathbf{s}_e$ has no direct energy meaning. The corresponding „interference power" is distributed between the source q_0 and the scatterer q_s. In this case, according to (3), for the flow $\mathbf{s}_0$ the source w_0 is concentrated in the area V_0 of the field source q_0, the source w_s of the flow $\mathbf{s}_s$ — inside the scatterer in the region V_{sc}, while for $\mathbf{s}_t$ and $\mathbf{s}_e$ the corresponding sources are w_t and w_e, according to (4) and (5), differ from zero both inside the source (in the region V_0) and inside the scatterer (in region V_{sc}).

Local conservation laws of the form (1) and (2) are also preserved for the electromagnetic problem. In this case, the energy flow is expressed by the Poynting vector $\mathbf{s} = \frac{1}{2}\,\mathrm{Re}\,\mathbf{E} \times \mathbf{H}^*$, and the scalar field u transforms into the electric field strength vector $\mathbf{E}$, $u = \mathbf{E}$ [15,20,21], which for a bianisotropic scatterer is supplemented by the magnetic field strength $\mathbf{H}$, $u = (\mathbf{E}, \mathbf{H})$ [22]. In this case, Maxwell's equations are used instead of the original scalar wave equation. An explicit form of sources w_j for the electromagnetic problem (without introducing induced currents) is given in [20].

2. Integral conservation laws and optical theorem

Integrating local conservation laws (2) over an arbitrary volume V and using the Gauss–Ostrogradsky theorem, it is easy to obtain various integral forms of balance equations. Such equations relate the total energy flows leaving outside through the surface ΣV of volume V with the work of sources W inside V:

$$S_j \equiv \oint_{\Sigma V} \mathbf{s}_j d\Sigma = \int_V \nabla \mathbf{s}_j d\mathbf{r} = \int_V w_j dr \equiv W_j \tag{6}$$

(here $j = t, 0, s, e$). In this case, the negative values of the integral flows S_j will correspond to the predominance of the time-averaged local values of the flows flowing inside V through the surface ΣV. The corresponding negative power values W_j describe the work of the field performed on the sources inside the volume V, i.e., losses of the field energy („drains").

From this obvious consequences follow. If the considered volume V does not contain sources w_j, then the total energy flow S_j through the surface V corresponding to the power w_j will be equal to zero, i.e., flow entering inside V is equal to outgoing. If the volume V completely covers the sources w_j, then further increase in V does not change the total flow S_j through the surface V until this increase affects other sources or drains of energy.

Integrating both sides (2) over an arbitrary volume V according to the left side (6) gives

$$S_t = S_0 + S_s + S_e. \quad (7)$$

Here, each of the total flows S_j is expressed as powers W_j corresponding right-hand side in (6), and

$$W_t = W_0 + W_s + W_e. \quad (8)$$

According to (6), relations (7) and (8) are formally equivalent. However, they focus attention on different physical aspects of the problem. Equality (7) relates to flows propagating in space, while (8) relates powers W_j determined by local densities w_j. The latter correspond to the interaction of the field with given or induced sources at certain points in space.

Choosing different volumes of integration for (7) and (8) it is easy to obtain the results of papers [18,20] related to the energy balance conditions. Let's consider this approach in more details.

2.1. Radiation losses

If we subtract from an arbitrary large volume V containing the sources together with the scatterer the volumes of the sources and the scatterer, i.e., consider $V_1 = V/(V_0 \cup V_{sc})$ (inside V_1 all powers $w_j = 0$, i.e. there is no dissipation or inflow of field energy), then the total flow S_t through the surface V_1 goes to zero, $S_t = 0$. The surface V_1 consists of the outer surface ΣV of the volume V and two inner surfaces V_0 and V_{sc}, the perpendiculars to which are opposite to the perpendiculars to the surfaces V_0 and V_{sc}. Keeping this in mind, the condition $S_t = 0$ takes the form

$$S_t = \oint\limits_{\Sigma V} \mathbf{s}_t \, d\sum - \oint\limits_{\Sigma V_0} \mathbf{s}_t \, d\sum - \oint\limits_{\Sigma V_{sc}} \mathbf{s}_t \, d\sum = 0,$$

so

$$\oint\limits_{\Sigma V} \mathbf{s}_t \, d\sum = \oint\limits_{\Sigma V_0} \mathbf{s}_t \, d\sum + \oint\limits_{\Sigma V_{sc}} \mathbf{s}_t \, d\sum. \quad (9a)$$

This condition leads to a physically obvious energy conservation law relating the total flows of radiated S_{em} and absorbed S_{abs} powers with the flow of radiative losses S_{rad}. Let us write this condition for the case of the absorbing scatterer as:

$$S_{\text{rad}} = S_{em} - S_{abs}. \quad (9b)$$

Here, the radiation loss flow S_{rad} going to infinity, the radiation flow of sources S_{em}, and the flow absorbed by the scatterer S_{abs} are expressed as

$$S_{\text{rad}} = \oint\limits_{\Sigma V} \mathbf{s}_t d\sum, \; S_{em} = \oint\limits_{\Sigma V_0} \mathbf{s}_t d\sum, \; S_{abs} = -\oint\limits_{\Sigma V_{sc}} \mathbf{s}_t d\sum. \quad (10)$$

The sign „minus" in the definition S_{abs} corresponds to the flow going to the scatterer, so that in the presence of absorption $S_{abs} > 0$. The corresponding S_{abs} power absorbed by the scatterer is

$$W_{abs} \equiv -\int\limits_{V_{sc}} \mathrm{Im}(u_t^* q_s) d\mathbf{r}. \quad (11)$$

Relation (9) can be considered as the additivity condition of the corresponding $\mathbf{s}_t$ total fluxes leaving the volumes V_0 and V_{sc}, which are simply summed on the surface enclosing these volumes. It is also met, if we replace in (9a) the flow $\mathbf{s}_t$ by any of the partial flows $\mathbf{s}_0, \mathbf{s}_s$ or $\mathbf{s}_e$, because inside V_1 there are also no sources of each of them. When $\mathbf{s}_t$ in (9a) is replaced by $\mathbf{s}_0$, the integral over the surface ΣV_{sc}, inside which there are no sources $\mathbf{w}_0$ corresponding to $\mathbf{s}_0$, goes to zero, so (9a) becomes

$$\oint\limits_{\Sigma V} \mathbf{s}_0 \, d\sum = \oint\limits_{\Sigma V_0} \mathbf{s}_0 \, d\sum.$$

This relation expresses a physically obvious condition for the conservation of the total flow created by given sources q_0 when radiation propagates from the surface ΣV_0 to any enclosing V_0 surface ΣV in free space. A similar relation is also valid for the flow of scattered radiation $\mathbf{s}_s$.

$\mathbf{s}_e$ substitution in (9a) gives the additivity condition for interference flows that is less obvious than the law of conservation of energy (9b)

$$\oint\limits_{\Sigma V} \mathbf{s}_e \, d\sum = \oint\limits_{\Sigma V_0} \mathbf{s}_e \, d\sum + \oint\limits_{\Sigma V_{sc}} \mathbf{s}_e \, d\sum, \quad (12)$$

which is met for any volume V containing both sources (i.e. volume V_0) and scatterer (volume V_{sc}). In the case of an absorbing scatterer playing the role of a drain, it is natural to take the second term in (12) with minus sign, introducing the extinction flow entering the scatterer (see Section 2.2).

It follows from (12) that far from the scatterer and source the total interference flow is equal to the simple sum of similar flows from given and induced sources. This flow, like the total flow S_{rad}, remains unchanged as the volume V increases, i.e., when moving away at arbitrary distances from the system under consideration. In this case, relations (9a) and (12) are also met for fields in the near zone, when the chosen surface V approaches the surface of the source V_0 or the scatterer V_{sc}.

2.2. Scatterer and optical theorem

For the arbitrary volume V covering only the scatterer but not the sources ($V_{sc} \subset V$ but $V_0 \not\subset V$, so $q_0 = 0$ inside V) in (7) and (8) $W_0 = S_0 = 0$. With this choice of V in the case of the absorbing scatterer the total outgoing flow S_t is equal with minus sign to the absorbed energy flow

S_{abs} (10). Let's introduce notations

$$S_{\text{ext}} \equiv -S_e = -\oint_{\Sigma V} \mathbf{s}_e d\sum, \; W_{\text{ext}} \equiv -W_e = -\int_{V_{sc}} \text{Im}(u_0^* q_s) dr \tag{13}$$

for the flow entering V_{sc} and the corresponding power. As a result, from (7) and (8) we obtain

$$W_{\text{ext}} = W_s + W_{abs}, \quad S_{\text{ext}} = S_s + S_{abs}. \tag{14}$$

Here, the power and flow of the scattered field are expressed as

$$W_s = \int_{V_{sc}} \text{Im}(u_s^* q_s) dr, \quad S_s = \oint_{\Sigma V} \mathbf{s}_s d\sum. \tag{15}$$

Relations (14) are equivalent to the optical theorem expressing the extinction power as the sum of the absorption and scattering powers, which can also be expressed in terms of flows. In the literature, the term „optical theorem" is often used directly in relation to balance relations (14) without going over to the scattering amplitude. Most recently, the optical theorem in the form of integral relations (14) found wide application in obtaining fundamental physical boundaries of absorption and scattering (see, for example, [22–24], as well as the literature referenced there).

Note that the optical theorem (14) for powers also follows directly from the representations W_{abs}, W_{ext} and W_s in the form (11), (13) and (15), and representation of the field as the sum of the incident and scattered waves: $u_t = u_0 + u_s$. Such a generalized form of the optical theorem is also valid for the case of near fields, when the considered volume V approaches the scattering volume V_{sc}.

2.3. Source radiation and amplification factor

Another choice of V, also considered in [18,20], relates to the description of the Purcell effect, which describes the influence of the scattering object presence on the radiation power of given currents q_0. In this case, volume covering only the region V_0 of the source q_0, but not the scatterer is chosen as V. Then $q_s = 0$ inside V and in (7) and (8) the terms corresponding to the scatterer are zero, $W_s = S_s = 0$. In this case (7) and (8) take the form

$$W_{em} \equiv \int_{V_0} \text{Im}(u_t^* q_0) d\mathbf{r} = W_0 + W_e,$$

$$S_{em} \equiv \oint_{\Sigma V} \mathbf{s}_t \, d\sum = S_0 + S_e. \tag{16}$$

Here the notation W_{em} is introduced for the radiation power of the source, leaving it through the surface V_0. At that, in (16) the values

$$W_0 = \int_{V_0} w_0 dr = \int_{V_0} \text{Im}(u_0^* q_0) dr, \; S_0 = \oint_{\Sigma V} \mathbf{s}_0 \, d\sum \tag{17}$$

correspond to the source q_0 radiation if the scatterer is absent. The last terms in (16) describe the interference and are equal to

$$W_e = \int_{V_0} w_e dr = \int_{V_0} \text{Im}(u_s^* q_0) dr, \; S_e = \oint_{\Sigma V} \mathbf{s}_e \, d\sum. \tag{18}$$

The interference power W_e describes the radiation power change of the given source W_{em} due to the scatterer presence, i.e., it represents the Purcell effect. The expression W_e (18) is usually associated with the amplification factor, although the scatterer presence can also lead to decrease in the power of radiation losses due to energy absorption by the scatterer. In the expression (16) for the emitted flow S_{em}, the flows S_0 and S_e are preserved, i.e., do not depend on the choice of volume V enclosing the source, but not affecting the scatterer. Note that, in accordance with the definitions introduced the interference flow going to infinity, which is on the right side of (12), is written as $S_e - S_{\text{ext}}$.

The Figure illustrates the general picture of the balance of energy flows in the scattering problem. This Figure is similar to those given in the papers [18,20], where powers W_j for the electromagnetic problem were considered. In contrast, the Figure gives a more detailed picture of the formation of flows S_j and allows a better understanding of the dynamics of the scattering process.

The flow balance shown in the Figure can be described as follows. The source q_0, on average over time, creates a total power flow S_{em}, leaving it through the arbitrary surface enclosing the source (but not the scatterer). This flux is divided into the radiation flux S_0, which „does not see" the scatterer and goes to infinity, and the interference flow S_e associated with the scattered field, since, according to (16), $S_{em} = S_0 + S_e$. A part of the interference flow S_e, equal to the extinction flow S_{ext}, interacts with the scatterer, being partially absorbed in it in the form of the power of the absorbed flow S_{abs} and partially going to infinity in the form of field scattered by particle S_s, whereby in accordance with the optical theorem (14), $S_{\text{ext}} = S_s + S_{abs}$. Finally, the remaining part $S_e - S_{\text{ext}}$ of the interference flow goes to infinity in accordance with (12), so that the total radiation losses (9a) are $S_{\text{rad}} = S_{em} - S_{abs} = S_0 + S_s + S_e - S_{\text{ext}}$.

The picture shown in Figure is rather conditional, since here we are talking not about local, but about total flows averaged over time and integrated over surfaces. Besides, since interference flows have no independent energy meaning, the value S_{ext} may exceed S_e, so that the corresponding to (12) flow $S_e - S_{\text{ext}}$ can be negative.

There is a new element in the described picture of the energy balance, which earlier was not considered in the literature. Namely, in it, as the energy flow, that excites the scatterer, the incomplete average flow incident on the particle (which, according to (10), is equal to the absorbed flow S_{abs}) is used, as one might expect *a priori*, and the extinction flow is used as part of the interference flow. In this case, the extinction source W_{ext}, in accordance with (13), is due to the work of the incident wave u_0 on the currents q_s induced by it in the scatterer. This is the physical meaning of the optical theorem describing the total energy losses of the incident wave due to scattering and absorption in the scatterer.

When describing the Purcell effect, expressions equivalent to the relation (16) for the power W_{em} with W_e (18) were used in the monograph [13], as well as in many other papers. In the expression (18) for the amplification factor W_e, the value $u_s^* q_0$ appears, which contains the scattered field u_s at the points of the source q_0, i.e., determined by „backward" scattering to source. However, if the optical theorem (14) refers to the properties of the scatterer, then the balance relation (16) refers to the sources q_0 and does not directly depend on the optical theorem.

3. Optical theorem and amplification factor

The erroneous interpretation of the optical theorem indicated in the Introduction for the case of point source [18] is caused by the incorrect identification of the optical theorem with the balance condition (16), which refers not to the scatterer, but to the radiation source. Let us discuss this case in more detail, since the specificity of the optical theorem for the point source can lead to difficulties in its use.

The generalization of the optical theorem to the case of the point source, as noted above, was first obtained for a particular case and in a rather awkward form in [12]. It also follows from the general operator form of the generalized optical theorem proposed in [14,15]. Let us compare the results of the operator approach [14,15] with the results [12], and, following [14], consider the case of arbitrary shape of the incident wave u_0, which, in particular, can correspond to the point source of radiation.

To simplify this comparison, we will use the abbreviated operator notation described in more detail in [14,15] (see also Appendix E from [21]). These notations are quite similar to the abbreviations used in quantum mechanics: for the vector corresponding to the field $u \equiv |u\rangle$ the Hermitian conjugate vector $u^+ = \langle u|$, so that the scalar product of the vectors $|a\rangle$ and $|b\rangle$ is written as $\langle a|b\rangle = a^+ b$, for the operators A and B the Hermitian conjugation gives $(AB)^+ = B^+ A^+$, and $(\mathrm{Au})^+ = u^+ A^+$.

In these notations, the extinction power (13) included in the optical theorem is expressed as

$$W_{\text{ext}} = -\operatorname{Im} \int (u_0^* q_s) dr \equiv -\operatorname{Im} u_0^+ q_s \equiv -\operatorname{Im} q_0^+ G_0^+ q_s. \tag{19}$$

Here the relation $u_0 = G_0 q_0$ is taken into account, so that $u_0^+ = q_0^+ G_0^+$, and formally unlimited integration is actually performed over the volume of the scatterer V_{sc}, where the source q_s is nonzero.

The right side (19) includes the value $G_0^+ q_s$, which differs from the scattered field $u_s = G_0 q_s$ by the presence of the Hermitian conjugation sign for G_0. At the same time, in the paper [12] the similar expression given for the extinction power contained scattered field u_s. For comparison with the results [15], let us discriminate in (13) the part containing the scattered field. For this we write W_{ext} (13) as

$$W_{\text{ext}} = -\operatorname{Im} q_0^+ \big(u_s + (G_0^+ - G_0) q_s\big). \tag{20}$$

This expression, in addition to the scattered field u_s, corresponding to backscattering to the source q_0, also contains integral term related to the induced source q_s. The equivalent term was introduced into [12] when describing the field of the point source using some auxiliary function (the so-called „spherical far-field pattern generator" [12]). The presence of the additional integral term in (20) complicates the physical interpretation and the use of the optical theorem for the point source. In fact, the mistake made in [18] in the formulation of the optical theorem for the point source is equivalent to discarding the term containing q_s in (20). Such discarding is equivalent to identifying the extinction factor W_{ext} (20) with the amplification factor W_e (18).

Expression (20) with distinguished backscattering field looks like some complication of formula (19). Such a distinction seems redundant, since it simultaneously complicates the correct physical interpretation of the optical theorem. Indeed, relation (19) can be interpreted as the work of the incident wave u_0 on induced sources q_s. This work does not coincide with the work of the backscattered field u_s on the given source q_0, which corresponds to the expression for the amplification factor W_e (18).

Conclusion

The paper traces the causes and gives a visual refutation of the widespread misconception that the total attenuation (extinction) of the radiation incident on the scatterer in the case of the point source is determined by backscattering, i.e., by the field scattered towards the source. It is shown that the cause of extinction is not backscattering, but the interaction of the incident wave with secondary currents excited by it in the scatterer. In accordance with the field division into the incident and the scattered wave, the easy-to-use scheme of balances of average power flows is made, which describes the energy exchange between the field sources and the scatterer, taking into account radiation losses. This scheme details the results on the energy balance known from the literature [18,20], focusing on the interference nature of the excitation of the scatterer, and thereby avoiding errors when using the optical theorem and other similar relationships related to the description of the energy exchange between the field and matter in problems of scattering theory.

Conflict of interest

The author declares that he has no conflict of interest.

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