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The influence of noises with different spectra on viscous incompressible fluid flows

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The results of an experimental study of the effects of noises on flows in a rotating spherical layer are presented. Noise is introduced into the flow in the form of random fluctuations of the inner sphere rotational rate relative to a time constant mean value. The flow velocity was measured with a laser Doppler anemometer. It was found that the noises of the equal amplitude, but with different spectra cause the generation of mean flows of different intensity. It is shown that the increase in the average velocities under the action of the noise is larger for the stable flows compared to unstable ones

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All real-world flows and technical systems are subjected to the influence of noise. The potential use of external noise is taken into account in the engineering of process techniques and devices for energy extraction from wind and wave flows [1]. The addition of „white" noise in the form of random fluctuations of the rotation rate of the inner sphere to flows in a rotating spherical layer, which are examined in the present study, may induce a change in the wave number with loss of stability [2], lower the stability limit, and lead to generation of mean flows [3]. Mean flows may also be generated under the influence of time-periodic modulation of the rotation rate of one of the spherical boundaries [4].

The amplitudes of spectral components of „white" noise remain equal throughout the entire used frequency range f; if the spectrum slope is presented as $1/f^{\alpha}$, „white" noise has $\alpha = 0$. In addition to „white" noise, other types of noise [5–7] or separate bounded regions of the „white" noise spectrum [8–10] have been used lately to control the state of technical systems and flows. Compared to „pink" ($\alpha = 1$) and „white" ($\alpha = 0$) noise, „red" noise ($\alpha = 2$) exerts a more significant influence on the nature of structure oscillations [7]. „White" noise in a low-frequency band may induce a more efficient suppression of high-frequency instability in a combustion chamber than single-frequency modulation [8,9], while noise in a high-frequency band may enhance this instability. If the high-frequency part of „white" noise is removed, anomalously high amplification of the wave component of the flow energy is corrected in flows with rotation of a stratified fluid [10].

The aim of the present study is to examine experimentally the influence of the spectrum type of added noise on the variation of properties of a spherical Couette flow (flow of a viscous incompressible fluid induced within a gap between coaxially positioned spheres by the rotation of one or both boundaries). The outer sphere was stationary, and noise was added in the form of random fluctuations of the rotation rate of the inner sphere. Spheres were made of an optically transparent material; the radius of the inner sphere was $r_1 = 0.075$ m, and the outer sphere had $r_2 = 0.150$ m. Silicone oil with kinematic viscosity $\nu = 5 \cdot 10^{-5}\,\mathrm{m^2/s}$ at a temperature of 22°C was used as the working fluid in a spherical gap. In order to limit the deviations of temperature of the fluid layer from the specified values to ±0.05°C, spheres were introduced into a thermostat with a controllable temperature of silicone oil; a temperature sensor was positioned at the equator of the outer sphere. Azimuthal velocity component u_{φ} [m/s] of flow in the layer was measured with a laser Doppler anemometer at a point located 0.078 m away from the equator plane and 0.105 m away from the rotation axis. Flow velocity signal u_{φ} was recorded at a sampling rate of 25.45 Hz for subsequent processing. The specified rate of angular rotation of the inner sphere $\Omega(t)$ was maintained with an accuracy of ±0.05% by a digital control system with a designated signal processor. Instantaneous actual values $\Omega(t)$ were calculated by performing time differentiation of the phase signal from a rotation rate sensor positioned on the drive shaft. Control inputs to the drive were produced based on the difference between actual and specified angular rotation rates. If a nonzero noise amplitude is set in the control system, actual value $\Omega(t)$ starts varying in time: perturbations of a normalized value with a zero mean are added at each operating cycle of the system (each 0.04 s). These perturbations may be both positive and negative and are produced by a random number generator. This is the way in which „white" noise was added in experiments [2,3] to the rotation rate signal (with the same spectrum amplitudes at all the frequencies used; see Fig. 2 in [2] and Fig. 1 in [3]). Just as in [8,9], only a part of the working frequency band of the spectrum used in the present study was occupied

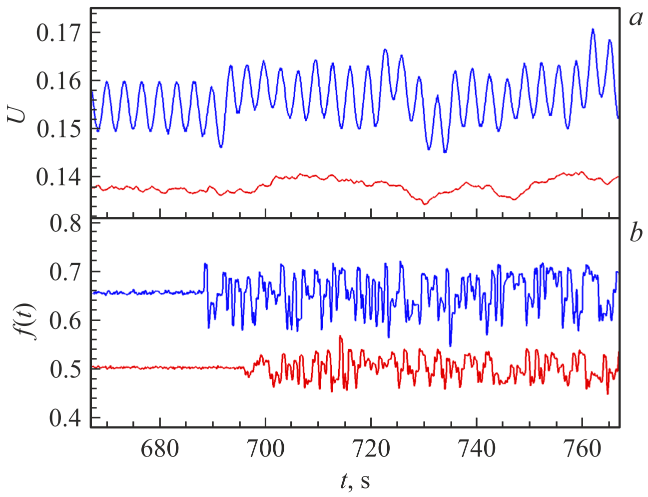

Figure 1. Dependence of normalized flow velocity U (a) and rotation rate $f(t)$ (b) on time t at cutoff frequency $f_{2c} = 1$ Hz. The upper curve corresponds to unstable flow, $\mathrm{Re}_1/\mathrm{Re}_c = 1.0046$, $N = 0.085$, $m = 3$; the lower curve represents steady-state flow, $\mathrm{Re}_1/\mathrm{Re}_c = 0.772$, $N = 0.047$.

by „white" noise. The rotation control system allows one to produce such noise by introducing the above-mentioned perturbations in time intervals longer than 0.04 s. Thus, „white" noise is retained in the spectrum only at low frequencies, and the amplitudes of spectral components at higher frequencies (starting from cutoff frequency f_c) decay in a near-exponential manner. Experiments were conducted in accordance with the following procedure. The flow at a chosen Reynolds number $\mathrm{Re} = \Omega_0 r_1^2/\nu$ was first allowed to reach steady state without the addition of noise (Ω_0 is the mean rate of angular rotation of the inner sphere). Measurements of the azimuthal velocity of steady-state flow were performed for 15 min, and additional noise (with its intensity remaining fixed throughout each experiment) was then added to the rotation rate signal. All the results reported below are presented as functions of number Re and noise amplitude N, where $N = \frac{1}{\Omega_0}\sqrt{\frac{1}{K-1}\sum_{i=1}^{K}\left(\Omega(t_i) - \Omega_0\right)^2}$ and K is the length of the time sample. It should be noted that fluctuations of rotation rate signal $\Omega(t)$ with $N < 0.0095$ are observed even when no additional perturbations are introduced. The spectrum of these fluctuations corresponds to „white" noise within the entire range of frequencies used [2]. We examined the influence of noise with two types of spectra with different cutoff frequencies f_c: $f_{1c} = 3$ Hz and $f_{2c} = 1$ Hz; at $f < f_c$, a region of „white" noise remained in the spectrum. Noise amplitude N varied from 0.038 to 0.116. Rotation rate $f_0 = \Omega_0/2\pi$ of the inner sphere varied from 0.5 to 0.658 Hz, and condition $f_0 < f_c$ was satisfied at all times.

All measurements in the present study were performed near the flow stability limit, which corresponds (if no additional noise is present) to critical Reynolds number $\mathrm{Re}_c = 460 \pm 2$ [11]. A periodic flow compose in the form of travelling azimuthal waves with wave numbers $m = 3$ or 4 forms at the stability limit in a spherical layer with the considered parameters without additional noise [11] and with it [2]. These waves have frequencies $f_3 = 0.31$ Hz at $m = 3$ and $f_4 = 0.41$ Hz at $m = 4$. The direction of wave propagation is the same as the direction of rotation of the inner sphere. When additional noise is present, the velocity field prior to loss of stability varies in time, but retains the same flow symmetries that are found in the case of zero additional noise: symmetry with respect to the equator and to the rotation axis [3]. Figure 1 presents the dependences of rotation rate $f(t) = \Omega(t)/2\pi$ and normalized azimuthal flow velocity$U = u_\varphi/(\Omega_0 r_1)$ on time t obtained when noise with cutoff frequency $f_{2c} = 1$ Hz was added to the rotation rate signal. The velocities of unstable and steady-state flows are indicated. In both cases, the application of noise resulted in the emergence of low-frequency (relative to f_3) variations of velocity U. The amplitude of these low-frequency oscillations is comparable to the amplitude of azimuthal waves (the upper curve in Fig. 1, a) and is substantially higher than the amplitude of variation of U in a steady-state flow (the lower curve in Fig. 1, a). These dependences differ considerably from the results obtained earlier [3] in the presence of additional „white" noise. For example, the application of „white" noise resulted only in a change in the average flow velocity at the same measurement point, but the nature of time dependence remained the same. It may be concluded that the fluctuations of flow velocity in [3] induced by „white" noise were below the anemometer sensitivity threshold at the measurement point.

Figure 2 shows the spectra of noise added to the rotation rate signal and the spectra of azimuthal flow velocity prior to loss of stability. The noise spectra in the frequency interval from 0.01 to 1 Hz represent „white" noise. The

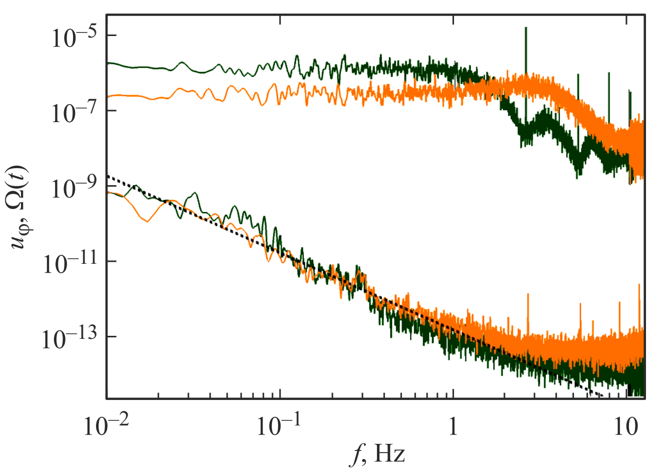

Figure 2. Spectra of $\Omega(t)$ (top) and u_φ (bottom) at $\mathrm{Re}/\mathrm{Re}_c = 0.772$. Orange curves: f_{1c}, $N = 0.074$; green curves: f_{2c}, $N = 0.076$. The dashed line is the approximated slope of flow velocity spectra. A color version of the figure is provided in the online version of the paper.

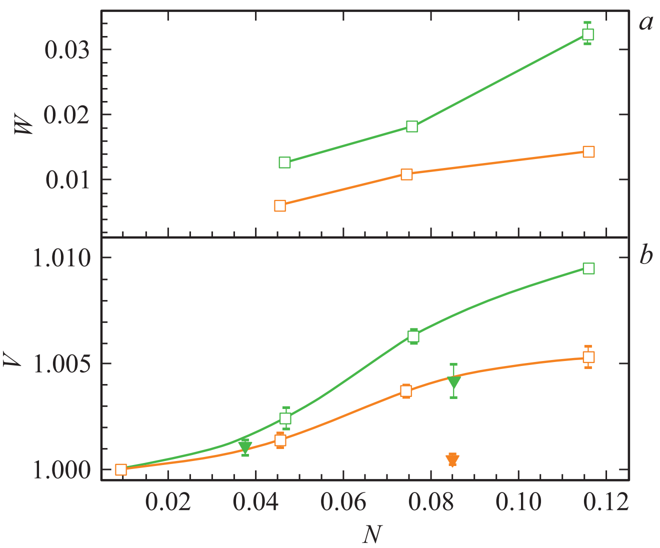

Figure 3. *a* — Normalized fluctuations of flow velocity W; *b* — relative growth of average flow velocity V ($V = 1$ corresponds to the case of zero additional noise). Squares and triangles correspond to steady-state and unstable flows. Variations under the influence of noise at $f_{1c} = 3$ Hz and $f_{2c} = 1$ Hz are colored orange and green, respectively. A color version of the figure is provided in the online version of the paper.

amplitudes of spectral components of two types of spectra start decreasing at $f > f_c$; at $f > 10$ Hz, they become more than two orders of magnitude lower. Interestingly, the qualitative appearance of spectra of the measured flow velocity is almost independent of the noise spectrum type. At frequencies $0.01 < f < 1$ Hz, the slope of u_φ spectra may be presented in the form of $1/f^\alpha$, where $\alpha = 2.06$. Thus, the considered „white" noise with different f_c in the rotation rate signal induces „red" noise in the u_φ spectrum away from the noise source. Velocity fluctuations were found to decay faster with distance from the noise source at higher frequencies. This agrees with the calculated data from [12], where a similar result was obtained for periodic oscillations of the rotation rate of the inner sphere with two frequencies.

Let us examine the influence of cutoff frequency f_c on the measured azimuthal velocity. Figure 3 presents the normalized fluctuations of azimuthal flow velocity W and the relative growth of average flow velocity V:

$$W = \frac{1}{\bar{u}_{\varphi N}} \sqrt{\frac{1}{K-1} \sum_{i=1}^{K} \left(u_{\varphi N}(t_i) - \bar{u}_{\varphi N}\right)^2}, \quad V = \bar{u}_{\varphi N}/\bar{u}_{\varphi 0}.$$

Here, $\bar{u}_{\varphi N}$ and $\bar{u}_{\varphi 0}$ are the averaged flow velocities determined with and without additional noise, respectively, and $u_{\varphi N}(t_i)$ are the instantaneous flow velocity values. It can be seen that noise with a lower cutoff frequency f_{2c} (green curves in Figs. 2 and 3) induces a more pronounced enhancement of both fluctuations of the flow velocity (Fig. 3, *a*) and its average values (Fig. 3, *b* with an increase in N than noise with a higher cutoff frequency f_{1c} (orange curves in Figs. 2 and 3). Steady-state flows undergo a more pronounced enhancement of W and V than unstable flows at the same noise intensity.

Thus, the sensitivity of flows to random in time fluctuations of the rotation rate is governed by the spectrum type of these fluctuations (specifically, the width of the frequency interval with „white" noise). The removal of even a small high-frequency part of the „white" noise spectrum (i.e., the shift of f_c from 3 to 1 Hz) at a constant spectrum amplitude induces an enhancement of both fluctuations of the flow velocity and its average values.

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Conflict of interest

The authors declare that they have no conflict of interest.

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