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Entanglement between an isolated qubit and a qubit in a cavity with Kerr media

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We found the exact solution for a model consisting of two dipole-coupled qubits, one of which is an isolated, and the other interacts with the thermal mode of a cavity with the Kerr medium. The results showed that Kerr nonlinearity may greatly enhance the degree of entanglement induced by a thermal field both for separable and entangled initial states of qubits. We also showed the possibility of the disappearance of the sudden death of entanglement for a model with a Kerr nonlinearity.

Keywords: qubits, thermal field, Kerr nonlinearity, dipole-dipole interaction, entanglement, sudden death of entanglement.

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Introduction

The Jaynes−Cummings model, which describes the interaction of a two-level atom with a lossless resonator field isolated quantum mode, is the simplest fully quantized exactly solvable model of quantum optics [1]. Despite its simplicity, the Jaynes−Cummings model and its generalizations, which take into account several transitions in atoms, the presence of several modes of a quantized field, several atoms, dipole-dipole and other types of interactions between atoms, the presence of various types of nonlinear media in a resonator, make it possible to describe all the main quantum effects of radiation-matter interaction [2]. Recently, various generalizations of the Jaynes−Cummings model have found wide application in the physics of quantum computing and quantum communications. This is due to the fact that to generate and control the entangled states of natural and artificial two-level atoms (qubits), such as superconducting rings with Josephson junctions, ions in magnetic traps, impurity spins, etc., electromagnetic fields of resonators are usually used [3–5]. At present, the dynamics of qubit entanglement induced by resonator fields has been studied for a large number of different Jaynes−Cummings-type multi-qubit models (see references in [6–11]). The use of entangled states for quantum computing and communications suggests the need to choose appropriate measures to quantify the degree of qubit entanglement. Although the general properties of entangled states have been studied in sufficient detail, rigorous quantitative criteria for qubit entanglement have so far been introduced only for two-qubit systems. These criteria include consistency (Wootters test) [12] and negativity (Peres−Horodecki test) [13,14]. As for multi-qubit systems, it has not been possible to introduce similar quantitative criteria for them so far. Therefore, at present, special attention is paid to the study of the dynamics of entanglement of two-qubit systems.

One of the obstacles to the implementation of efficient and reliable protocols for the physics of quantum computing and quantum communications is the effect of sudden death of qubit entanglement. Therefore, the study of mechanisms that contribute to the disappearance or weakening of the effect of sudden death of qubit entanglement becomes one of the priority tasks for quantum informatics. The entanglement sudden death phenomenon consists in the disappearance of qubit entanglement at times shorter than the decoherence time. The effect was theoretically predicted for the first time by Yu and Eberly [15] while studying the unitary dynamics of two qubits in a resonator. Later, this effect was observed experimentally [16]. In Ref. [17], the effect of entanglement sudden death was studied using the example of a system of two spatially separated two-level atoms: an isolated atom and an atom interacting with the quantum electromagnetic field mode of a lossless resonator (Janes−Cummings atom). The authors restricted themselves to considering the case when the atoms at the initial moment of time are in an entangled state, and the resonator field is in the Fock state. At the same time, the authors predicted the possibility of sudden death of entanglement of atoms in the process of their evolution. Differences in the behavior of various quantitative measures of qubit entanglement [17], as well as the influence of the quantum phase on the entanglement sudden death effect [18,19], were also investigated within the framework of the model proposed in [20]. The influence of the transition frequency detuning in the Jaynes−Cummings atom and the thermal resonator mode, as well as the direct dipole-dipole interaction of atoms on the nature of the manifestation of the entanglement sudden death effect in this model, is considered in [21,22].

At present, entangled states of qubits of different physical nature have been experimentally obtained in resonators at various temperatures from nK to room temperature [4,5]. This means the presence of thermal photons in the resonators of such quantum devices. It has been shown in a number of papers that the interaction of qubits with the thermal radiation fields in resonators can lead to the sudden death of qubit entanglement (see references in [23,24]). Therefore, it is of considerable interest to study the mechanisms that prevent the sudden death of qubit entanglement caused by the thermal radiation fields of resonators. It has now been shown that thermal noise-induced entanglement sudden death is eliminated by including qubit and field frequency detuning, direct dipole-dipole and Ising interactions between qubits, Stark shift, etc. (see references in [25]). The disappearance of the effect can also be facilitated by the use of nonlinear resonator media, in particular, the Kerr nonlinearity [26]. For atomic systems, the Kerr nonlinearity parameter Ξ is small compared to the photon loss rate from the resonator κ, so the impact of the Kerr medium on the dynamics of atoms at relaxation times is insignificant. However, for superconducting qubits in coplanar resonators, it was possible to create conditions under which the ratio between the Kerr nonlinearity and the loss rate satisfies the inequality $\Xi\kappa > 30$ [27]. Under such conditions, the Kerr nonlinearity can have a significant effect on the entanglement dynamics of superconducting qubits.

Here we report the exact dynamics of a system of dipole-coupled isolated two-level atom (qubit) and Jaynes–Cummings atom interacting with the thermal radiation field of a resonator containing a Kerr medium. Based on the exact solution, we have studied the time dependence of the qubit entanglement parameter, i.e., the negativity for separable and entangled initial states of qubits. The conditions for the disappearance of the sudden death of qubits entanglement are analyzed.

1. Model and its exact solution

Consider a system consisting of two identical dipole coupled natural or artificial two-level atoms (qubits) Q_1 and Q_2 with a transition frequency between the excited and ground energy levels equal to $\hbar\omega_0$. In this case, the first qubit is trapped in a single-mode ideal resonator and resonantly interacts through single-photon transitions with the resonator field of frequency $\omega = \omega_0$, while the second qubit is free. Note that for artificial atoms, e.g., superconducting rings with Josephson junctions, the direct dipole-dipole interaction constant can exceed the atom-field interaction constant [28]. Let us also assume that there is an additional Kerr medium in the resonator. Then the Hamiltonian of the considered model in the interaction picture can be written in the form

$$H = \hbar g(\sigma_1^+ a + \sigma_1^- a^\dagger) + \hbar J(\sigma_2^+\sigma_1^- + \sigma_1^+\sigma_2^-) + \hbar\Xi\, a^{\dagger 2}a^2, \tag{1}$$

where $\sigma_1^- = |-\rangle_{11}\langle+|$ and $\sigma_1^+ = |+\rangle_{11}\langle-|$ are the transition operators between excited $|+\rangle_1$ and ground $|-\rangle_1$ state in the first qubit, $a^\dagger$ and a are operators of creation and annihilation of resonator mode photons, g is the constant of coupling between the qubit and the resonator field, J is the constant of the dipole-dipole interaction of qubits and Ξ is the Kerr nonlinearity constant.

We assume that initially the qubits are prepared in one of the separable states of the form

$$|\Psi(0)\rangle_{Q_1 Q_2} = |+,-\rangle, \tag{2}$$

$$|\Psi(0)\rangle_{Q_1 Q_2} = |+,+\rangle, \tag{3}$$

or in an entangled Bell-type state

$$|\Psi(0)\rangle_{Q_1 Q_2} = \cos\theta|+,-\rangle + \sin\theta|-,+\rangle, \tag{4}$$

where θ is a parameter that determines the degree of initial entanglement of qubits Q_1 and Q_2. The maximum degree of qubit entanglement corresponds to the value $\theta = \pi/4$. Such initial states for qubits in resonators can be obtained using microwave pulses of a certain duration.

As the initial state of the field, we choose a single-mode thermal radiation state with a density matrix of the form

$$\rho_F\,(0) = \sum_n p_n|n\rangle\langle n|.$$

Here the weight functions p_n have the form

$$p_n = \bar{n}^n/\,(1+\bar{n})^{n+1},$$

where $\bar{n}$ is the average number of thermal photons given by the Bose–Einstein formula

$$\bar{n} = (\exp\,[\hbar\omega/k_{\mathrm{B}}T] - 1)^{-1},$$

where k_{B} is the Boltzmann constant and T is the temperature of the microwave resonator. Depending on the physical nature of a natural or artificial atom interacting with the resonator field, the resonator temperature can vary from room temperature for nitrogen-substituted vacancies in diamond to nanokelvins in the case of neutral atoms and ions in magnetic traps.

We set the task of finding the exact dynamics of the model under consideration. For this purpose, we first find the exact solution of the time-dependent Schrodinger equation for the model under consideration in the case of the initial state of the electromagnetic field with a certain number of photons, and then generalize the results obtained to the case of a thermal field.

For a field state with a certain number of photons, the wave function is

$$|\Psi(0)\rangle_F = |n\rangle \quad (n = 0, 1, 2, \ldots).$$

To obtain the exact solution of the time-dependent Schrödinger equation for the model under consideration, we use the so-called „dressed" state representation, i.e. eigenfunctions of the Hamiltonian (1). Let us assume that the number of excitations of the „two qubits + cavity field" system is equal to $N = n + 2$ $(n \geq 0)$. Then the evolution of the time-dependent state vector will take place in a 4-dimensional Hilbert space with the basis

$$|-,-,n+2\rangle,\ |+,-,n+1\rangle,\ |-,+,n+1\rangle,\ |+,+,n\rangle.$$

Then, using the expansion of the time-dependent state vector in terms of basis vectors

$$|\Psi(t)\rangle_n = X_{1,n}(t)|-,-,n+2\rangle + X_{2,n}(t)|+,-,n+1\rangle + X_{3,n}(t)|-,+,n+1\rangle + X_{4,n}(t)|+,+,n\rangle \quad (5)$$

together with the time-dependent Schrodinger equation, we get

$$\dot{\tilde{X}} = -(\imath H/\hbar)\tilde{X},$$

where

$$\tilde{X}(t) = \begin{pmatrix} X_{1,n}(t) \\ X_{2,n}(t) \\ X_{3,n}(t) \\ X_{4,n}(t) \end{pmatrix}. \quad (6)$$

The solution of Eq. (6) for excitation numbers $N \geq 2$ and initial states of qubits $|-,-\rangle, |+,-\rangle, |-,+\rangle, |+,+\rangle$ is

$$X_{i,n}^{(m)}(t) = \sum_{j=1}^{4} U_{ij,nm}(t) X_{j,n}^{(m)}(0), \quad (7)$$

where

$$U_{ij,nm}(t) = \sum_{l=1}^{4} t_{inm}^{l} (t_{jnm}^{l})^{*} e^{-\imath\lambda_{ln} g t}.$$

Here the index $m = 1, 2, 3, 4$ enumerates the initial states of qubits of the form $|-,-\rangle$, $|+,-\rangle$, $|-,+\rangle$, $|+,+\rangle$ respectively, and $\lambda_{ln} = E_{ln}/(\hbar g)$ $(l = 1, 2, 3, 4)$ are the normalized eigenvalues of the Hamiltonian (1):

$$\lambda_{1n} = (1/2)\chi + n\chi + n^2\chi - V_n - W_n,$$

$$\lambda_{2n} = (1/2)\chi + n\chi + n^2\chi - V_n + W_n,$$

$$\lambda_{3n} = (1/2)\chi + n\chi + n^2\chi + V_n - W_n,$$

$$\lambda_{4n} = (1/2)\chi + n\chi + n^2\chi + V_n + W_n,$$

where

$$V_n = \frac{1}{2}\sqrt{-2c_n/3 + d_n^2/4 + 2^{1/3}G_n/(3F_n) + F_n/(3\,2^{1/3})},$$

$$W_n = \frac{1}{2}\sqrt{-2c_n/3 + d_n^2/4 - 2^{1/3}G_n/(3F_n) - F_n/2^{1/3}},$$

$$F_n = \Big(27b_n^2 - 72a_nc_n + 2c_n^3 - 9b_nc_nd_n + 27a_nd_n^2 + \sqrt{\big(-4(12a_n + c_n^2 - 3b_nd_n)^3 + (27b_n^2 - 72a_nc_n + 2c_n^3 - 9b_nc_nd_n + 27a_nd_n^2)^2\big)}\Big)^{1/3},$$

$$G_n = 12a_n + c_n^2 - 3b_nd_n,$$

$$H_n = -(2b_n - c_nd_n + d_n^3/4) \Big/ \sqrt{-2c_n/3 + d_n^2/4 + 2^{1/3}G_n/(3F_n) + F_n/(3\,2^{1/3})},$$

$$a_n = \sqrt{1+n}\sqrt{2+n}(n+1) - n(2 + 7n + 9n^2 + 5n^3 + n^4)\chi^2 + -n^2(n^2-1)\sqrt{1+n}\sqrt{2+n}\chi^2 + n\alpha^2(2 + n - 2n^2)\chi^2 - n^4\alpha^2\chi^2 - 2n^3\chi^4 - 5n^4\chi^4 - 2n^5\chi^4 + 4n^6\chi^4 + 4n^7\chi^4,$$

$$b_n = n^8\chi^4\big(2\chi + 6n\chi + 6n^2\chi + 2n^3\chi + 2n^2\sqrt{1+n}\sqrt{2+n}\chi + 2\alpha^2\chi + 2n\alpha^2\chi + 2n^2\alpha^2\chi + 2n^2\chi^3 - 10n^4\chi^3 - 12n^5\chi^3 - 4n^6\chi^3\big),$$

$$c_n = -1 - n - \sqrt{1+n}\sqrt{2+n} - \alpha^2 + 2n\chi^2 + 8n^2\chi^2 + 12n^3\chi^2 + 6n^4\chi^2,$$

$$d_n = -2\chi - 4n\chi - 4n^2\chi, \quad \chi = \Xi/g, \quad \alpha = J/g$$

and

$$X_{1,n}^{(1)}(0) = 1,\ X_{2,n}^{(1)}(0) = 0,\ X_{3,n}^{(1)}(0) = 0,\ X_{4,n}^{(1)}(0) = 0,$$

$$X_{1,n}^{(2)}(0) = 0,\ X_{2,n}^{(2)}(0) = 1,\ X_{3,n}^{(1)}(0) = 0,\ X_{4,n}^{(1)}(0) = 0,$$

$$X_{1,n}^{(3)}(0) = 0,\ X_{2,n}^{(3)}(0) = 0,\ X_{3,n}^{(3)}(0) = 1,\ X_{4,n}^{(3)}(0) = 0,$$

$$X_{1,n}^{(4)}(0) = 0,\ X_{2,n}^{(4)}(0) = 0,\ X_{3,n}^{(4)}(0) = 0,\ X_{4,n}^{(4)}(0) = 1.$$

The explicit form of the coefficients t_{inm}^{l} for the initial states of atoms (2)−(4) is not presented here because it is excessively cumbersome.

For the number of system excitations $N = 1$, the solution of the time-dependent Schrödinger equation can be presented as follows:

a) If the initial state of the system is $|-,-,0\rangle$, then

$$|\Psi(t)\rangle_1 = Y_1^{(1)}(t)|-,-,1\rangle + Y_2^{(1)}(t)|+,-,0\rangle + Y_3^{(1)}(t)|-,+,0\rangle, \quad (8)$$

where

$$Y_1^{(1)} = \sum_{j=1}^{3} \varepsilon_j e^{\imath\varepsilon_j t}/q_j, \quad Y_2^{(1)} = -\sum_{j=1}^{3} \varepsilon_j^2 e^{\imath\varepsilon_j t}/q_j,$$

$$Y_3^{(1)} = -\alpha \sum_{j=1}^{3} \varepsilon_j e^{\imath\varepsilon_j t}/q_j,$$

$$q_j = 1 + \alpha^2 - 3\varepsilon_j^2,$$

$$\varepsilon_1 = \mathrm{Re}\big[(-\sqrt[3]{2}\zeta/\xi + \xi/\sqrt[3]{2})\big]/3,$$

$$\varepsilon_2 = \mathrm{Re}\big((1+\imath\sqrt{3})\zeta/2^{2/3}\xi - (1-\imath\sqrt{3})\xi/2\sqrt[3]{2}\big)/3,$$

$$\varepsilon_3 = \mathrm{Re}\big((1-\imath\sqrt{3})\zeta/2^{2/3}\xi - (1+\imath\sqrt{3})\xi/2\sqrt[3]{2}\big)/3,$$

$$\xi = 2^{1/3}\imath\sqrt{3(\alpha^2+1)}, \quad \zeta = -3(1+\alpha^2);$$

b) If the initial state of the system is $|-,-,0\rangle$, then

$$|\Psi(t)\rangle_2 = Y_1^{(2)}(t)|-,-,1\rangle + Y_2^{(2)}(t)|+,-,0\rangle + Y_3^{(2)}(t)|-,+,0\rangle, \quad (9)$$

where

$$Y_1^{(2)} = -\alpha\sum_{j=1}^{3} e^{\imath\varepsilon_j t}/q_j, \quad Y_2^{(2)} = -\alpha\sum_{j=1}^{3}\varepsilon_j e^{\imath\varepsilon_1 t}/q_j,$$

$$Y_3^{(2)} = -\alpha\sum_{j=1}^{3} e^{\imath\varepsilon_j t}/q_j;$$

c) If the initial state of the system is $|-,-,1\rangle$, then

$$|\Psi(t)\rangle_3 = Y_1^{(3)}|-,-,1\rangle + Y_2^{(3)}|+,-,0\rangle + Y_3^{(3)}|-,+,0\rangle, \quad (10)$$

where

$$Y_1^{(3)} = \sum_{j=1}^{3}(\alpha^2 - \varepsilon_j^2)e^{\imath\varepsilon_1^2)t}/q_j, \quad Y_2^{(3)} = \sum_{j=1}^{3}\varepsilon_j e^{\imath\varepsilon_1 t}/q_j,$$

$$Y_3^{(3)} = -\alpha\sum_{j=1}^{3} e^{\imath\varepsilon_j t}/q_j.$$

Finally, for the number of excitations $N = 0$ and the initial state of the system $|-,-,0\rangle$, the time-dependent wave function remains unchanged in time

$$|\Psi(t)\rangle_4 = |-,-,0\rangle. \quad (11)$$

Using expressions (5)−(11), we can obtain the time-dependent density matrix for the total system for the thermal initial state of the resonator field having the form

$$\rho(t) = \sum_n p_n |\Psi(t)\rangle_{n\,n}\langle\Psi(t)|.$$

To find the quantitative criterion of cubit entanglement, the negativity, it is necessary to calculate the reduced cubit-cubit density matrix. For this purpose, we should average the total density matrix over the field variables, $\rho_{Q_1Q_2}(t) = Tr_F\rho(t)$.

2. Calculation of negativity

For a two-qubit system described by the density operator $\rho_{Q_1Q_2}(t)$, the measure of entanglement of qubits, the negativity, can be determined through negative eigenvalues ν_i^- of the reduced two-qubit density matrix $\rho_{Q_1Q_2}^{T_1}$ [13,14]:

$$N(t) = -2\sum_i \nu_i^-. \quad (12)$$

For separable initial states of qubits (2), (3) and entangled initial state (4), the two-qubit density matrix has the form

$$\rho_{Q_1Q_2}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & 0 \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{23}(t)^* & \rho_{33}(t) & 0 \\ 0 & 0 & 0 & \rho_{44}(t) \end{pmatrix}. \quad (13)$$

Correspondingly, the density matrix partially transposed over the variables of one qubit for (13) is

$$\rho_{Q_1Q_2}^{T_1}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{23}(t)^* \\ 0 & \rho_{22}(t) & 0 & 0 \\ 0 & 0 & \rho_{33}(t) & 0 \\ \rho_{23}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}. \quad (14)$$

The elements of matrix (14) for the initial state of the cubits $|+,+\rangle$ are

$$\rho_{11} = \sum_{n=0}^{\infty} p_n |X_{4,n}^{(4)}(,t)|^2, \quad \rho_{22} = \sum_{n=0}^{\infty} p_n |X_{2,n}^{(4)}(,t)|^2,$$

$$\rho_{33} = \sum_{n=0}^{\infty} p_n |X_{3,n}^{(4)}(,t)|^2, \quad \rho_{44} = \sum_{n=0}^{\infty} p_n |X_{1,n}^{(4)}(,t)|^2,$$

$$\rho_{23} = \sum_{n=0}^{\infty} p_n X_{2,n}^{(4)}(t) X_{3,n}^{(1)}(t)^*.$$

For the initial state $|+,-\rangle$ they take the form

$$\rho_{11} = \sum_{n=0}^{\infty} p_n |X_{4,n-1}^{(2)}(,t)|^2,$$

$$\rho_{22} = \sum_{n=0}^{\infty} p_n |X_{2,n-1}^{(2)}(,t)|^2 + p_0 Y_2^{(1)}(t),$$

$$\rho_{33} = \sum_{n=0}^{\infty} p_n |X_{3,n-1}^{(2)}(,t)|^2 + p_0 Y_3^{(1)}(t),$$

$$\rho_{44} = \sum_{n=0}^{\infty} p_n |X_{1,,n}^{(2)}(,t)|^2 + p_0 Y_1^{(1)}(t),$$

$$\rho_{23} = \sum_{n=0}^{\infty} p_n X_{2,n-1}^{(2)}(t) X_{3,n-1}^{(2)}(t)^*.$$

For the entangled initial state (4), we do not present the matrix elements here because they are too cumbersome.

Matrix (14) has only one eigenvalue, which can take negative values. As a result, for negativity (12) we have

$$N(t) = \sqrt{(\rho_{11}(t) - \rho_{44}(t))^2 + 4|\rho_{23}(t)|^2} - \rho_{11}(t) - \rho_{44}(t). \quad (15)$$

The results of numerical calculations of negativity (15) for various initial states of qubits and model parameters are shown in Figs. 1–4.

3. Results and discussion

Figure 1 shows the dependence of the entanglement parameter on the dimensionless time gt for an disentangled initial state of qubits $|+,-\rangle$ and different values of the

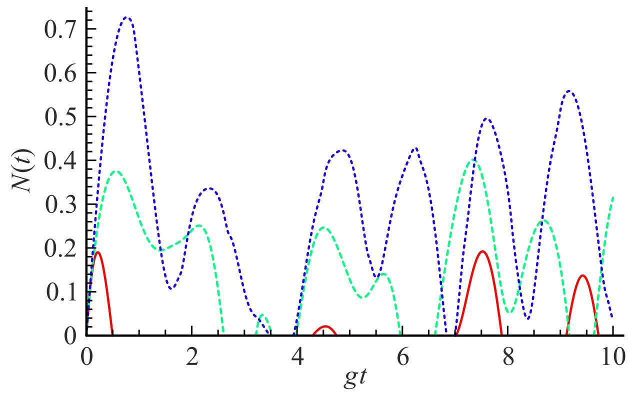

Figure 1. Negativity as a function of the dimensionless time gt for the separable initial state of qubits $|+,-\rangle$. The average number of photons in the mode is $\bar{n} = 3$. The dimensionless parameter of the dipole-dipole interaction is $\alpha = 0.75$. The dimensionless parameter of Kerr nonlinearity is $\chi = 0$ (solid line), $\chi = 1$ (dashed line) and $\chi = 3$ (dotted line).

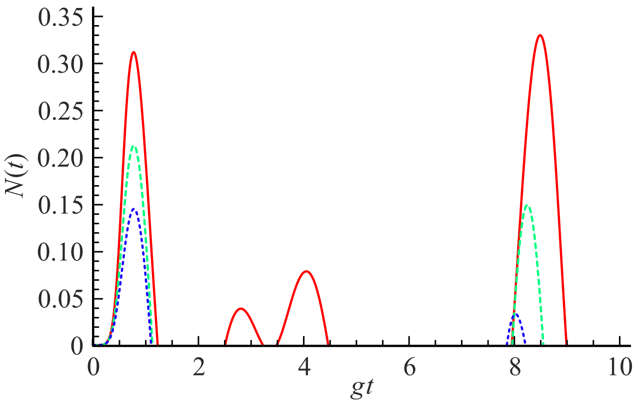

Figure 2. Negativity as a function of the dimensionless time gt for the separable initial state of qubits $|+,+\rangle$. The average number of photons in the mode is $\bar{n} = 3$. The dimensionless parameter of the dipole-dipole interaction is $\alpha = 0.75$. The dimensionless parameter of Kerr nonlinearity is $\chi = 0$ (solid line), $\chi = 0.2$ (dashed line) and $\chi = 0.3$ (dotted line).

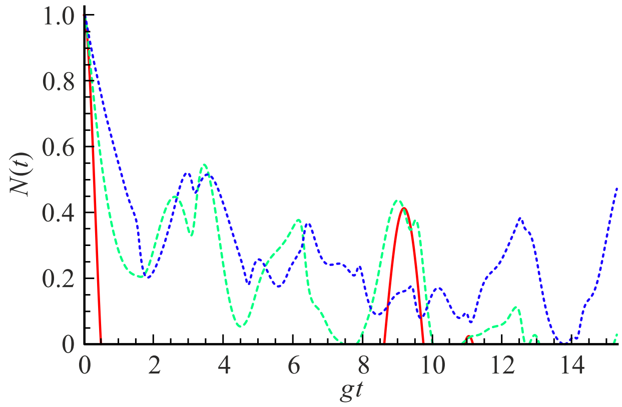

Figure 3. Negativity as a function of the dimensionless time gt for the entangled initial state of qubits (4) at $\theta = \pi/4$. The average number of photons in the mode is $\bar{n} = 3$. The dimensionless parameter of the dipole-dipole interaction is $\alpha = 0$. The dimensionless parameter of the Kerr nonlinearity is $\chi = 0$ (solid line), $\chi = 1$ (dashed line) and $\chi = 2$ (dotted line).

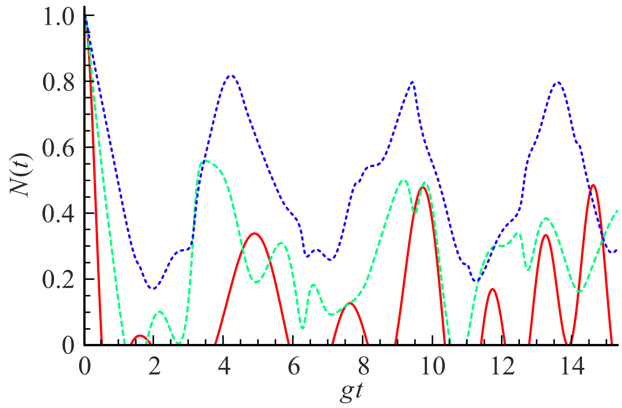

Figure 4. Negativity as a function of the dimensionless time gt for the entangled initial state of qubits (4) at $\theta = \pi/4$. The average number of photons in the mode is $\bar{n} = 3$. The dimensionless parameter of the dipole-dipole interaction is $\alpha = 0.75$. The dimensionless parameter of the Kerr nonlinearity is $\chi = 0$ (solid line), $\chi = 1$ (dashed line) and $\chi = 2$ (dotted line).

Kerr nonlinearity. The curves are plotted for a model with an average number of thermal photons in the mode $\bar{n} = 3$ and a dimensionless parameter of the dipole-dipole interaction of qubits $\alpha = 0,75$. For numerical simulation of the entanglement parameter, a rather intense thermal field of the resonator is considered, since at low intensities the influence of the Kerr nonlinearity on the degree of entanglement of qubits is insignificant.

It is easy to see that as the dimensionless Kerr nonlinearity parameter χ increases, the maximum degree of qubit entanglement increases. A similar result also takes place for qubits interacting with the common thermal field of the resonator [25]. However, for the model under consideration, the occurrence of entanglement of qubits in the case of disentangled initial states is possible only in the presence of their dipole coupling. Figure 1 also shows that in the absence of the Kerr nonlinearity, the effect of sudden death of entanglement takes place, that is, the disappearance of entanglement during times shorter than the decoherence time. As the parameter of the Kerr nonlinearity increases, the sudden death of entanglement disappears. Thus, in the case of $\alpha = 0.75$ and $\bar{n} = 3$, the sudden death disappears for values of the Kerr nonlinearity parameter $\chi > 3.5$.

The time dependence of the negativity for the initial atomic state $|+,+\rangle$ and different values of the Kerr nonlinearity is shown in Fig. 2. As in the previous Figure, the curves are plotted for the value of the qubit dipole coupling parameter $\alpha = 0.75$ and the average number of thermal photons in the mode is $\bar{n} = 3$. For the considered initial state of qubits, an increase in the Kerr nonlinearity parameter leads to the opposite effect: the maximum degree of entanglement of qubits decreases in this case.

Figures 3 and 4 show the time dependence of negativity for the Bell entangled state of qubits$1/\sqrt{2}(|+,-\rangle + |-,+\rangle)$ and different values of the Kerr nonlinearity parameter. The average number of thermal photons in the mode is $\bar{n} = 3$, and the qubit dipole coupling parameter is chosen to be $\alpha = 0$ (Fig. 3) and $\alpha = 0.75$ (Fig. 4). It can be seen from the figures that, for entangled initial states of qubits, the inclusion of the Kerr nonlinearity leads to stabilization of the negativity oscillations both for the model with dipole−dipole interaction and for the model without such interaction. In this case, at large values of the nonlinearity parameter, the effect of the entanglement sudden death disappears.

Conclusion

We studied the dynamics of a free qubit and a qubit trapped in a single-mode ideal resonator with a Kerr medium in the presence of direct dipole−dipole interaction of qubits and resonant interaction of the qubit with the field. An exact solution of the quantum Liouville equation of the considered model is obtained in the case of initial separable and entangled states of qubits and the thermal radiation field of the resonator. Based on the exact solution, an analytical expression for negativity is found. We have studied the influence of the Kerr nonlinearity and the dipole-dipole interaction on the degree of qubit entanglement. For qubits prepared in the separable state $|+,-\rangle$, the Kerr nonlinearity leads to an increase in the maximum degree of entanglement of qubits and can suppress the effect of sudden death of entanglement. For a separable initial state of qubits $|+,+\rangle$, the inclusion of the Kerr nonlinearity, on the contrary, leads to the disappearance of entanglement of qubits. For entangled initial states of qubits, the nonlinearity leads to stabilization of the negativity oscillation amplitudes both for the model with dipole−dipole interaction and for the model without such interaction. For models with large values of the Kerr nonlinearity parameter, the effect of entanglement sudden death disappears.

Conflict of interest

The author declares that he has no conflict of interest.

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