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Effect of large single-ion anisotropy on the dynamic and static properties of a non-heisenberg ferrimagnet

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The influence of a large single-ion anisotropy of the „easy plane" type on the phase states and spectra of elementary excitations of a ferrimagnet with sublattices $S = 1$ and $\sigma = 1/2$ and non-Heisenberg exchange interaction for a sublattice with $S = 1$ is studied. It is shown that for different ratios of the material parameters of the system, only one phase state is possible, characterized by both vector and tensor order parameters (quadrupole-ferrimagnetic). The condition for sublattice spin compensation is determined, as well as the behavior of the spectra of elementary excitations near the spin compensation line. In the vicinity of the spin compensation line, the magnon spectra are „antiferromagnetically similar".

Keywords: large single-ion anisotropy, biquadratic exchange interaction, ferrimagnet.

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1. Introduction

Modern electronics is no longer pure electronics, but becomes spintronics, i.e. is based on the fact that energy or information in solid body may be transferred by „spin" current, rather than electronic current [1–4]. Compensated magnets, i.e. antiferromagnets, are the main materials used in modern spintronic devices [5–7]. Such choice is based on the exchange enhancement effect inherent in antiferromagnets that considerably increases the resonance frequency up to terahertz range and enhances other dynamic characteristics of the system such as speed limits of domain walls [8] and magnetic vortices [9,10]. It should be emphasized that spin current has a significant influence on the properties of compensated magnets [11–18], and this is what makes them so attractive for spintronics applications.

However, with all benefits of antiferromagnets, they have one significant disadvantage — their magnetic properties are very sensitive to any crystal structure defects. This fact severely impedes their use. But this problem may be bypassed. The matter is that there is another class of magnetic-ordered systems that may be considered as compensated magnets (in appropriate conditions) having the exchange enhancement effect — these are ferrimagnetics in the vicinity of the compensation point of magnetic sublattices [19–21]. In addition, ferrimagnets are not so sensitive to the quality of crystal lattice and their behavior in the vicinity of the compensation point is „antiferro-like". Therefore, ferrimagnets in the vicinity of the compensation point of sublattices may be used for spintronics devices. Thus, dynamical properties of domain walls of ferrimagnets and high-frequency ferrimagnetic vortices were studied in [22–25], and a subterahertz nanogenerator setup based on ferrimagnets with spin current pumping was offered in [26]. In addition, ferrimagnets have another essential effect: an ultrafast (about few picoseconds) sublattice magnetization flip when exposed to laser pulse with a duration of at least 100 femtoseconds was detected for a rare-earth and transition metal alloy - GdFeCo [27,28]. And according to [29,30], variation of magnetic moment modules of sublattices plays a significant role here. Thus, for superfast remagnetization, longitudinal evolution of sublattice magnetic moments plays a significant role [31,32].

longitudinal behavior of magnons is directly associated with quantum spin reduction effect [33]. This effect is observed in magnets with „easy plane" single-ion anisotropy as well as in so called non-Heisenberg magnets that take into account high spin invariants in the exchange Hamiltonian [33–49]. Description of longitudinal behavior of anisotropic and non-Heisenberg magnets is out of the scope of the Landau–Lifshitz equation and requires to take into account the dynamics of tensor variables, which are quantum average values of bilinear operators in terms of spin components [34–40].

Generally, the energy associated with high spin invariant and single-ion anisotropy energy is considerably lower than the Heisenberg (bilinear) exchange interaction. However, there is a set of magnetic-ordered systems whose properties cannot be described within standard models. These are primarily rare-earth magnets [38,42]. Thus, for example, EuSe at temperatures about 3 K is ferrimagnetically ordered with biquadratic exchange interaction (non-Heisenberg interaction) much higher than bilinear (Heisenberg) interaction, which significantly influences static and dynamic propertiesof the system [50,51]. Thus, the properties of non-Heisenberg ferrimagnets taking into account the influence of high „easy plane“ type single-ion anisotropy (comparable or exceeding the exchange interaction constant) is of not only of academic interest, but also of great practical importance.

2. Model

Two-sublattice ferrimagnet will be studied herein. Magnetic moment spin of the first sublattice is $S=1$ and of the second sublattice is $\sigma = 1/2$. Moreover, in the first sublattice, both bilinear and biquadratic exchange interactions are taken into account, as well as high „easy plane“ type single-ion anisotropy. „High“ anisotropy means that the single-ion anisotropy constant is comparable or even exceeds the exchange interaction constants. Hamiltonian of such system may be written as

$$H = -\frac{1}{2}\sum_{n,n'}\Big[J^{(2)}(n-n')(\mathbf{S}_n\mathbf{S}_{n'}) + K(n-n')(\mathbf{S}_n\mathbf{S}_{n'})^2\Big]$$

$$-\frac{1}{2}\sum_{m,m'}J^{(1)}(m-m')(\sigma_m\sigma_{m'})$$

$$-\frac{1}{2}\sum_{n,m'}A(n-m)(\boldsymbol{\sigma}_m\mathbf{S}_n) + \frac{\beta}{2}\sum_n (S_n^x)^2, \qquad (1)$$

where $J^{(1)} > 0$ is the exchange interaction constant for a sublattice with spin $\sigma = 1/2$; $J^{(2)} > 0$, $K > 0$ are bilinear and biquadratic exchange interaction constants for sublattice in $S=1$; $A < 0$ is the intersublattice interaction constant; $\beta > 0$ is the „easy plane“ type single-ion anisotropy constant (basal plane ZOY). Hereinafter, $\beta > J, K$ is assumed. For further consideration, the case of low temperatures ($T \ll T_N$, T_N — Neel temperature) will be used.

The model described by Hamiltonian (1) was studied in detail in [52]. But for this model, unlike [52], high single-ion anisotropy was addressed ($\beta \gg J, K$). Moreover, the properties of highly anisotropic ferrimagnet, but without biquadratic exchange interaction, were studied in [34]. We are interested both in static and dynamic properties of highly anisotropic non-Heisenberg ferrimagnet ($\beta > J, K$) in the vicinity of the compensation point of sublattice spins.

The model described by Hamiltonian (1) will be reviewed by mean field approximation using the Hubbard operator technique [53].

Using the findings of [52], we can omit the description of some mathematical operations and proceed directly to energy states of magnetic ion

$$E_1 = -B_2^0 + \frac{\beta}{4} - \bar{H}_s\cos 2\alpha - B_2^2\sin 2\alpha + \Delta,$$

$$E_0 = -2B_2^0 + \frac{\beta}{2} + \Delta,$$

$$E_{-1} = -B_2^0 + \frac{\beta}{4} + \bar{H}_s\cos 2\alpha + B_2^2\sin 2\alpha + \Delta,$$

$$\varepsilon_{1/2,-1/2} = \mp\bar{H}_\sigma\langle\sigma^z\rangle, \qquad (2)$$

The following notations are introduced here:

$$\bar{H}_s\left(J_0^{(2)} - K_0/2\right)\langle S^z\rangle - \frac{1}{2}A_0\langle\sigma^z\rangle,$$

$$\bar{H}_\sigma = J_0^{(1)}\langle\sigma^z\rangle - \frac{1}{2}A_0\langle S^z\rangle, \quad B_2^0 = \frac{K_0}{6}q_2^0,$$

$$B_2^2 = \frac{K_0}{2}q_2^2 - \frac{\beta}{4},$$

$$\Delta = \frac{1}{2}J_0^{(1)}\langle\sigma^z\rangle^2 + \frac{1}{2}\left(J_0^{(2)} - \frac{K_0}{2}\right)\langle S^z\rangle^2$$

$$+ \frac{K_0}{4}\left(\frac{(q_2^0)^2}{3} + (q_2^2)^2\right) - \frac{1}{2}A_0\langle S^z\rangle\langle\sigma^z\rangle. \qquad (3)$$

In (3, $q_j^i = \frac{1}{2}\langle S^iS^j + S^jS^i\rangle$ — quadrupole moment tensor components. In this case, as it follows from the problem symmetry, $q_2^0 = 3\langle(S^z)^2\rangle - S(S+1)$ and $q_2^2 = \frac{1}{2}\langle(S^x)^2 - (S^y)^2\rangle = \langle(S^+)^2\rangle + \langle(S^-)^2\rangle$ are nonzero tensor components q_j^i. Sublattice wave functions written as

$$\psi(1) = \cos\alpha|1\rangle + \sin\alpha|-1\rangle;$$

$$\psi(0) = |0\rangle \text{ and } \psi(-1) = -\sin\alpha|1\rangle + \cos\alpha|-1\rangle;$$

$$\Phi\left(\frac{1}{2}\right) = \left|\frac{1}{2}\right\rangle \text{ and } \Phi\left(-\frac{1}{2}\right) = \left|-\frac{1}{2}\right\rangle. \qquad (4)$$

Using wave functions (4), construct the Hubbard operators for each of the sublattices $X^{M'M} = |\psi(M')\rangle\langle\psi(M)|$, $Y^{m'm} = |\Phi(m')\rangle\langle\Phi(m)|$, which are associated with the operators as follows [53]:

$$S_n^z = \cos 2\alpha(X_N^{11} - X_n^{-1-1}) - \sin 2\alpha(X_n^{1-1} + X_n^{-11});$$

$$S_n^+ = \sqrt{2}\Big[\sin\alpha(X_n^{01} - X_n^{-10}) + \cos\alpha(X_n^{0-1} + X_n^{10})\Big],$$

$$S_n^- = (S_n^+)^+,$$

$$\sigma^z = \frac{1}{2}\left(Y^{\frac{1}{2}\frac{1}{2}} - Y^{-\frac{1}{2}-\frac{1}{2}}\right), \ \sigma^+ = Y^{\frac{1}{2}-\frac{1}{2}}, \ \sigma^- = (\sigma^+)^+.$$

Here, α is unitary transformation u–v parameter determined by

$$\bar{H}_s\sin 2\alpha = B_2^2\cos 2\alpha.$$

Correlation between spin operators and Hubbard operators allows to determine order parameters as function of α [54]:

$$\langle S^z\rangle = \cos 2\alpha, \quad q_2^2 = \sin 2\alpha, \quad q_2^0 = 1.$$

Since the second sublattice is isotropic and Heisenberg, then it is described only by vector (dippole) order parameter $\langle\sigma^z\rangle$ and plays a role of a „bias“ field.

3. Free energy density of highly anisotropic non-Heisenberg ferrimagnet

Since, a ferrimagnet in low-temperature limit ($T \to 0\,\mathrm{K}$) is addressed herein, then the free energy density with high accuracy coincides with the ground state energy of the magnetic ion. According to relations (7), the lowest energy levels of the first and second sublattices are E_1 and $\varepsilon_{1/2}$, respectively. Consequently, the free energy density of the ferrimagnetic in question can be written as $F = E_1 + \varepsilon_{1/2}$. Taking into account relations (4) and (7), the following expression is obtained for free energy density

$$F = -\frac{1}{12}K_0 - \frac{1}{4}\beta - \frac{1}{2}J_0^{(1)}\langle\sigma^z\rangle^2 - \frac{1}{2}\left[J_0^{(2)} - K_0\right]\langle S^z\rangle^2 + +\frac{1}{2}A_0\langle\sigma^z\rangle\langle S^z\rangle + \frac{\beta}{4}\sin 2\alpha.$$

At low temperatures $\langle\sigma^z\rangle = 1/2$, and $\langle S^z\rangle = \cos 2\alpha$. Then, for free energy density:

$$F = -\frac{1}{4}\left[\beta + \frac{4}{3}K_0 + \frac{1}{2}J_0^{(1)}\right] - \frac{1}{4}|A_0|\cos 2\alpha + \frac{\beta}{4}\sin 2\alpha - \frac{1}{2}\left[J_0^{(2)} - K_0\right]\cos^2 2\alpha. \quad (5)$$

Intersublattice interaction constant $A < 0$ is taken into account here. By minimizing relation (5), we derive the equation that allows to define phase states of the ferrimagnet at various relations of material parameters

$$\frac{|A_0|}{2}\sin 2\alpha + \frac{\beta}{2}\cos 2\alpha + 2\left(J_0^{(2)} - K_0\right)\cos 2\alpha \sin 2\alpha = 0. \quad (6)$$

Formally, equation (6) allows to define parameter u–v of transformation α, but as it follows from the expressions for vector and tensor order parameters ($\langle S^z\rangle = \cos 2\alpha$; $q_2^2 = \sin 2\alpha$), then this equation actually defines phase states for system with various relations of material parameters.

According to equation (6), the magnetization of sublattice with S = 1 depends to a large extent on the ratio of material parameters, and the magnetization of sublattice with spin 1/2 remains constant and plays the role of a „bias" field. It should be noted that condition $\langle\sigma^z\rangle = 1/2$ occurs naturally from relation of the$z$-th operator component σ with Habbard operators $Y^{m'm}$ and is accurate in case of $T = 0$.

It should be also noted that equation (6) describes astroid in space ($|A_0|, \beta$). As shown in [55], this closed curve (astroid) divides the plane ($|A_0|, \beta$) into two parts, in one of which metastable state is possible, and in the second one — metastable state is impossible. Consider the solutions of equation (6) at various relations of material parameters and low temperatures, i.e. in what conditions stable state is achieved in the system.

4. Phase states and excitation spectra of non-Heisenberg highly anisotropic ferrimagnet

Consider equation (10) with following relations of material parameters, i.e. with$\beta > J > K$, or with $\beta > K > J$, assuming that the temperature is close to 0 K.

First, consider the situation when the Heisenbergexchange interaction constant exceeds the biquadratic exchange interaction ($J_0^{(2)} > K_0$) taking into account that $\beta > J > K$. In this case, solution of equation (6) may be written as

$$\cos 2\alpha = \frac{|A_0|}{\beta - 4\left(J_0^{(2)} - K_0\right)}.$$

As mentioned before, with $T \to 0$ $\cos 2\alpha$ defines the average spin $\langle S^z\rangle$ of the sublattice with S = 1. Thus, in this case (at $\beta > J > K$), magnetization of the first lattice becomes lower than the maximum possible value. This is associated with the quantum spin reduction effect of the first sublattice [38] caused by the influence of high single-ion anisotropy and biquadratic exchange interaction.

If the biquadratic exchange interaction is higher than bilinear exchange interaction ($K_0 > J_0^{(2)}$), with prevailing single-ion anisotropy, the average magnetization (per lattice point) is equal to

$$\langle S^z\rangle = \cos 2\alpha = \frac{|A_0|}{\beta + 4\left(K_0 - J_0^{(2)}\right)},$$

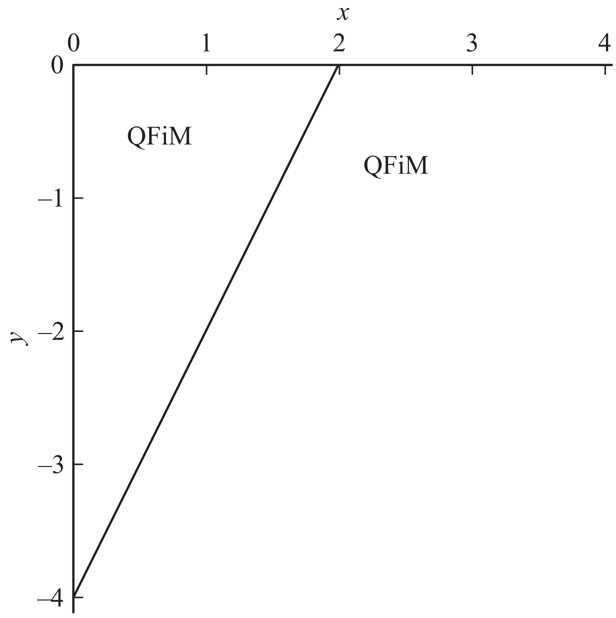
i.e. the average magnetization also remains lower than the maximum possible value. Both in the first and second cases, the average magnetic moment is nonzero ($\langle S^z\rangle \neq 0$), which is attributable to the influence of the „bias" field of sublattice $\sigma = 1/2$.

Therefore, both with $\beta \gg K_0 > J_0^{(2)}$ and with $\beta \gg J_0^{(2)} > K_0$, a state with magnetization of the first sublattice much lower than the maximum possible is achieved in the system, while the second sublattice maintains the saturated magnetization ($|\langle\sigma^z\rangle| = 1/2$). Vector and quadrupole order parameters of the first sublattice, in this case, are written as

$$\langle S^z\rangle < 1, \quad q_2^2 = \sin 2\alpha < 1, \quad q_2^0 = 1.$$

According to the expressions for $\langle S^z\rangle$, the average magnetic moment (per point) decreases with growth of the biquadratic exchange interaction constant.

Thus, the system achieves the phase in which both the vector order parameter of the first sublattice ($\langle S^z\rangle$) and the quadrupole moment tensor components (q_2^2) of the first sublattice take intermediate values in the range from zero to one, and the second sublattice plays a role of a constant „bias field". Thus, at considerable single-ion anisotropy, the quantum spin reduction effect appears in the first sublattice [34,35,46]. Such the state will be referred to as a quadrupole-ferrimagnetic state (QFiM).



Sublattice spin compensation line $z = 4$.

Since in this model, the intersublattice exchange interaction $a < 0$, magnetic moment vectors of the first and second sublattices are anticollinear and, therefore, sublattice spin compensation is possible in this state taking into account the quantum spin reduction of first sublattice [33,35,46]. From $\langle S^z \rangle = -\langle \sigma^z \rangle$, and taking into account that $|\langle \sigma^z \rangle| = 1/2$, the following equation describing the sublattice spin compensation surface in space $(A, J^{(2)}, K, \beta)$ is derived:

$$|A_0| = 2\left(J_0^{(2)} - K_0\right) - \beta/2. \tag{7}$$

Equation (7) is more convenient to rewrite in relative variables $y = |A|/K$, $x = J/K$, $z = \beta/K$. In these variables, the compensation surface equation is written as

$$y = 2(x - 1) - z/2. \tag{8}$$

Thus, equation (7) (or (8)) describes the surface in variables (J, K, A, β) on which the total average sublattice spin is equal to zero ($\langle S^z + \sigma^z \rangle = 0$). It should be emphasized that the sublattice spin compensation, rather that sublattice magnetic moment compensation is addressed herein. Since the magnetic moment is associated with the spin moment of sublattices by relation $\mathbf{M} = -g\mu_{\mathrm{B}}\mathbf{S}$, where g is the Lande factor (g-factor), and in the model in question, sublattices are nonequivalent, then it should be taken into account that g-factors of sublattices are not equal and, therefore, sublattice magnetic moments on the compensation plane are not equal [20]. Thus, although the spin moments compensate each other with certain relations of material parameters, but the integral magnetic moment in this case may be nonzero and may reach a sufficiently high value. Moreover, this resulting magnetic moment is parallel to the antiferromagnetism vector, and the ferrimagnet behavior in the compensation point can be considered as „antiferromagnetic“ [20]. It should be noted that equation (7) or (8) exactly corresponds to the compensation surface equation for weakly anisotropic ($\beta \ll J < K$) non-Heisenberg ferrimagnet [34,52], but taking into account that, in the ferrimagnet with low anisotropy, the quantum spin reduction effect is associated with the presence of biquadratic exchange interaction. In case addressed herein, this effect is attributable to the influence of biquadratic exchange interaction as well as to high „easy plane“ single-ion anisotropy.

These findings allow to build a compensation surface of the studied system (see equation (7) or (8)), and it is more convenient to represent it in the given variables in plane (x, y) with fixed value z, i.e. with fixed single-ion constant values β This diagram is shown in the Figure.

Behavior of the elementary system excitation spectra in the vicinity of the sublattice spin compensation surface will be investigated below. Elementary excitation spectra are known to be defined by the Green's function poles [56] which will be defined as follows within the Hubbard operator technique [53,56–58]:

$$G^{\lambda\lambda'}(n, \tau; n', \tau') = -\langle \hat{T}\tilde{X}_n^{\lambda}(\tau)\tilde{X}_{n'}^{\lambda'}(\tau')\rangle,$$

where $\tilde{X}_n^{\lambda}(\tau) = \exp(H\tau)X_n^{\lambda}\exp(-H\tau)$ is the Hubbard operator in Heisenberg's representation, $\hat{T}$ is the Wick operator, λ are root vectors defined by the algebra of Hubbard operators [48,50,51]. Derivation of the dispersion equation is described in detail in [53,57,59]. Dispersion equation that defines the magnon spectra is true with arbitrary relation of material constants. In this case, $T \to 0$.

The system in question has three elementary excitation branches: two „transverse“ excitation branches associated with precession of sublattice magnetic moments and one „longitudinal“ branch associated with magnetic moment module variation of sublattice with S = 1. The longitudinal excitation branch attract the most interest. It is defined by quadrupole order parameters and quantum spin reduction. Spectrum of this branch generally is written as

$$\varepsilon_1^2(k) = (E_{1-1} + K_k)\left(E_{1-1} + K_k + 2\left(J_k^{(2)} - K_k\right)\sin^2 2\alpha\right), \tag{9}$$

where

$$E_{1-1} = E_1 - E_{-1}$$
$$= -K_0 - 2\left(J_0^{(2)} - K_0\right)\cos^2 2\alpha + \frac{A_0}{2}\cos 2\alpha + \frac{\beta}{2}\sin 2\alpha.$$

Taking into account that in the vicinity of the compensation line the average magnetic moment of sublattice with S = 1

$$\langle S^z \rangle = \cos 2\alpha = \frac{|A_0|}{\beta + 4\left(K_0 - J_0^{(2)}\right)} = \frac{1}{2},$$

„longitudinal" excitation spectrum will be

$$\varepsilon_1^2(k) = \left(-\frac{1}{2}(J_0^{(2)} - K_0) - \frac{|A_0|}{4} - \frac{\sqrt{3}\beta}{4} - (K_0 - K_k)\right) \times \left(-\frac{|A_0|}{4} - \frac{\sqrt{3}\beta}{4} - (K_0 - K_k) + (J_k^{(2)} - K_k)\right).$$

Using equation (9), longitudinal excitation spectrum ε_1 may be written as

$$\varepsilon_1^2(k) \approx \left((K_0 - K_k) + \frac{\beta}{4}(\sqrt{3} - 1)\right)\left(\frac{|A_0|}{2} + \frac{\beta}{4}\left(\sqrt{3} + \frac{1}{2}\right)\right). \tag{10}$$

According to (10), the energy gap in the longitudinal excitation spectrum is proportional to the single-ion anisotropy constant, but enhanced by the intersublattice exchange interaction. Such behavior of the activation energy is specific to antiferromagnets. Therefore, in the vicinity of the compensation line (surface), the non-Heisenberg highly anisotropic ferrimagnet behaves as antiferrimagnet [20,52].

In addition to the longitudinal excitation branch described above, the system also contains two „transverse" elementary excitation branches. These excitations are associated with precession spin motion of sublattices with S = 1 and $\sigma = 1/2$, respectively. Energies of these excitations are defined by solutions of biquadratic equation

$$\varepsilon^4(k) + b(k)\varepsilon^2(k) + c(k) = 0, \tag{11}$$

where

$$b(k) = \left[\left(\frac{A_k}{2}\right)^2 \cos 2\alpha - \left(E_{\frac{1}{2}-\frac{1}{2}} + \frac{J_k^{(1)}}{2}\right)^2 - (E_{10} + J_k^{(2)})^2 + (J_k^{(2)} - K_k)^2 \sin^2 2\alpha\right],$$

$$c(k) = \left(E_{\frac{1}{2}-\frac{1}{2}} + \frac{J_k^{(1)}}{2}\right)^2 \left[(E_{10} + J_k^{(2)})^2 - (J_k^{(2)} - K_k)^2 \sin^2 2\alpha\right] + \left(\frac{A_k}{2}\right)^2 \left[\frac{1}{4}\left(\frac{A_k}{2}\right)^2 \cos^2 2\alpha - \left(E_{\frac{1}{2}-\frac{1}{2}} + \frac{J_k^{(1)}}{2}\right)\left(E_{10} + J_k^{(2)} - (J_k^{(2)} - K_k)\sin^2 2\alpha\right)\right],$$

$$E_{10} = -K_0 - (J_0^{(2)} - K_0)\cos^2 2\alpha + \frac{A_0}{4}\cos 2\alpha - \frac{\beta}{4}(1 - \sin 2\alpha); \; E_{\frac{1}{2}-\frac{1}{2}} = -\frac{J_0^{(1)}}{2} + \frac{A_0}{2}\cos 2\alpha.$$

Solutions of equation (11) may be written as

$$\varepsilon_{2,3}^2(k) = \frac{|b(k)|}{2} \pm \sqrt{\left(\frac{b(k)}{2}\right)^2 - c(k)}. \tag{12}$$

Solutions of equation (11) are rather cumbersome and analysis of these solutions is difficult, therefore they will be not addressed here, but spectra ε_2 and ε_3 may be analyzed from the following considerations.

Taking into account that in the vicinity of sublattice spin compensation surface $\cos 2\alpha = 1/2$, $\sin 2\alpha = -\sqrt{3}/2$, it can be shown that coefficients $c(k)$ and $b(k)$ of equation (11) in the long-wavelength limit (at $k \to 0$) are written as

$$c(0) \approx \left(\frac{1 + \sqrt{3}/2}{16}|A_0|\beta\right)^2, \; b(0) = -\frac{1 + \sqrt{3}/2}{4}|A_0|\beta.$$

By isolating in solutions (12) the obvious dependence on the wave vector (in the long-wavelength limit at $k \to 0$), we obtain

$$\varepsilon_{2,3}^2(k) = \frac{|b(0)|}{2} \pm \frac{|b(0)|}{2}\sqrt{1 - \alpha k^2} \approx \frac{|b(0)|}{2} \pm \frac{|b(0)|}{2}\left(1 - \frac{\alpha k^2}{2}\right).$$

Thus, one of the „transverse" excitation branches is zero-gap and is proportional to k ($\varepsilon_2(k) \sim k$), and in the second branch, energy gap appears $b(0)$ ($\varepsilon_3(0) \sim |b(0)|$), which is proportional to the single-ion constant enhanced by the intersublattice exchange interaction.

5. Discussion of findings

In [34,50–52] it was shown that in an anisotropic non-Heisenberg ferrimagnet (with weak single-ion anisotropy $\beta \ll J, K$) as well as in an isotropic ferrimagnet, depending on the relations of exchange integrals, both a phase characterized by a dipole order parameter (FiM-phase) and a phase characterized by vector and tensor order parameters (QFiM-phase) may be achieved. Moreover, in a weakly anisotropic ferrimagnet, unlike the isotropic case, the QFiM phase existence region increases which id associated with the influence of the single-ion anisotropy, which like biquadratic exchange interaction, tends to establish quadrupole (or nematic) order. In addition, the impact of even low (compared with exchange interactions) single-ion anisotropy results in the variation of the phase transition type between FiM- and QFiM-phases.

As shown herein, the impact of high „easy plane" single-ion anisotropy ($\beta \gg J, K$) makes the implementation of the truly ferrimagnetic state characterized by the vector order parameters of sublattices ($\langle S^z \rangle$, $\langle \sigma^z \rangle$) disadvantageous and the system achieves only quadrupole-ferrimagnetic ordering. This state is stable at any relations of exchange integrals, i.e. it exists both at $J > K$ and in the opposite case. This phase state is characterized by vector and tensor order parameters such as

$$\langle S^z \rangle = \cos 2\alpha < 1, \; \langle \sigma^z \rangle = 1/2, \; q_2^0 = 1, \; q_2^2 = \sin 2\alpha < 1.$$

The presence of sublattice $\sigma = 1/2$ and exchange interaction between sublattices prevent occurrence of nematic state in sublattice S = 1, i.e. $\langle S^z \rangle \neq 0$, $q_2^2 \neq 1$.

It should be also noted that consideration of high „easy plane“ type single-ion anisotropy and biquadratic exchange interaction in sublattice $S = 1$ results in the situation when a sublattice spin compensationline (surface) occurs in the system with an appropriate relation of material parameters. Moreover, equation of this line(see (7), (8)) exactly coincides with the identical equation of the compensation line of a weakly anisotropic non-Heisenberg ferrimagnet [52]. Therefore, the biquadratic exchange interaction is the main sublattice spin compensation mechanism.

In addition, we have studied the ferrimagnet excitation spectra behavior in the vicinity of the spin compensation line. These investigations have shown that the elementary excitation spectra in the vicinity of this line have „antiferro-like“ behavior.

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Conflict of interest

The authors declare that they have no conflict of interest.

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