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Response of a Josephson junction to a current pulse with the energy of a microwave photon

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A theoretical analysis of the efficiency of detecting photon-like pulses at frequencies of the order of 10 GHz by the Josephson junction is performed with parameters available for aluminum technology. Numerical simulation of the junction switching dynamics under the influence of a switching pulse is carried out within the framework of a linear resistive model of the Josephson junction. For comparison with simulations, the experimental data of a sample made using aluminum technology by the shadow evaporation technique are used. The times between dark counts for which the junction is sensitive to single photons are determined. Ways to increase sensitivity are considered.

Keywords: RCSJ model, lifetime, photon absorption.

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1. Introduction

Photon absorption in the Josephson junction (JJ) may be considered as an increase in junction energy by photon energy [1]. This increase is short-term, because the energy in the Josephson junction dissipates over time. But when certain conditions described herein are met, short-term exposure may result in a change of the junction state — switch from the zero voltage state (s state) to the resistive state (r state).

In quantum models, photon absorption is an immediate process that suddenly increases the system energy [2,3]. The approach used herein represents a photon as a current pulse $p(t)$ through JJ, see equation (1). In this case, the energy does not vary suddenly, but grows smoothly with the photon oscillation frequency ω_{ph}

$$p(t) = \sqrt{N}\sqrt{\frac{\hbar\omega_{ph}}{R_N t_{ph}}}\, e^{-\frac{1}{2}(\frac{t}{t_{ph}})^2} \cos(\omega_{ph} t). \qquad (1)$$

Integral of current pulse over time includes the energy of N photons. Pulse width t_{ph} is the pulse variable, i. e. the number of semioscillations at the photon frequency executed during the pulse time. The less oscillations, the higher switching pulse amplitude.

This model seems feasible for the systems interacting with the external thermostat when the quantum description is significantly complicated due to the presence of dissipation. Here, dissipation in JJ is considered due to the current flow trough a fixed resistance R_N (linear RCSJ model) or voltage-dependent resistance $R(V)$ (non-linear RCSJ model). As an alternative to this approach, the Lindblad equation may be, for example, used, which describes the evolution of an open quantum system density that was studied in [4,5] with respect to the Josephson junction.

According to the linear RCSJ model, the Josephson junction may be represented in the form of four circuit elements (nonlinear inductance, capacitance C, resistance and fluctuating current source $I_F(t)$) connected in parallel with respect to the current produced by the external source. Expression for the total current flowing through the junction with the Josephson phase ϕ and voltage V is written as:

$$I_b + p(t) = I_c \sin\phi + \frac{V}{R_N} + C\frac{dV}{dt} + I_F(t). \qquad (2)$$

Fluctuating current is supplied to simulate thermal noise by the Langevin method as a random external force. In the RCSJ model, a junction-shunting resistance is the source of this noise, therefore spectral density is defined by the Nyquist equation for current fluctuations through the resistor at T. For such noise current, the following relations are true:

$$\langle I_F(t)\rangle = 0, \quad \langle I_F(t)\rangle\langle I_F(t+\tau)\rangle = \frac{kT}{\pi R_N}\,\delta(\tau). \qquad (3)$$

The fluctuation force acting on the Josephson junction is characterized by the relation between thermal energy kT and bond energy E_J.

2. Switching scenarios in the RCSJ model

The Josephson junction has a stored energy equal to two Josephson energies $E_J = \hbar I_c/(2e)$ which is caused by overlapping of wave function of superconducting electrodes

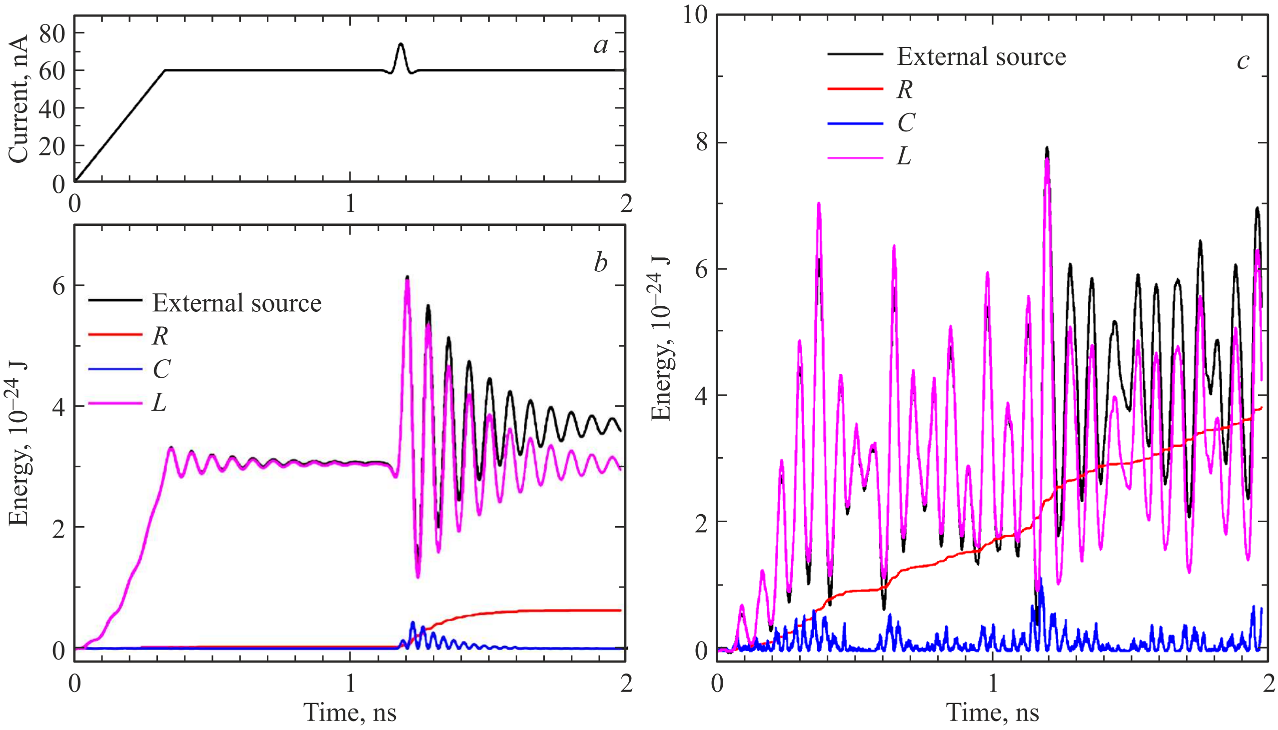

Figure 1. *a* — dependence of current through JJ on time: assignment of linear operating current from 0 to $0.3I_c$, waiting for photon arrival at permanent current, photon-induced current pulse; *b* — energy distribution in JJ when exposed to external current (*a*) without thermal fluctuations: black — external source energy (current source), red — resistance-dissipated energy, blue — capacitor energy, pink — energy in inductance (superconducting current energy); *c* — similar to illustration (*b*), but with thermal fluctuations with the intensity corresponding to 20 mK.

similar to the atom bond energy in a molecule. This quantity defines the typical energy scale separating the zero voltage state and finite voltage state. Let us consider the junction switching process from s state to r state under the action of the external photon-like current pulse from the point of view of energy redistribution.

When current is applied from the external source, voltage appears on the junction. The energy equal to the time integral of the product of external current and junction voltage is spent to increase the energy of the superconducting condensate that accelerates up to the rate depending on the set current and junction parameters. Or, in other terms, conversion of the potential energy into kinetic energy takes place.

Assume that the current develops linearly from 0 to the set value I_b causing plasma oscillations of the JJ phase with frequency $\omega_p = \sqrt{2eI_c/\hbar C}\,(1 - I_b^2/I_c^2)^{1/4}$ depending only on current I_b. Then, after damping of plasma oscillations, a current pulse with the photon energy arrives (Figure 1, *a*). Figures 1, *b* and *c* show the energy change in three elements R, L and C which form JJ when the external current is applied without thermal fluctuations and at a finite temperature of 20 mK, respectively. The external force energy spent to increase the current flowing through the junction from zero to the fixed value $I_b < I_c$ results in the increase in the superconducting current energy (current through inductance L). Also, when the steady state is established, capacitance C is recharged. In addition, a part of the energy dissipates in resistance $R = R_N$.

The simulation is carried out herein for JJ with $I_c = 200$ nA, $R_N = 1500\,\Omega$ and $C = 80$ fF. These variables were taken from considerations that such junction in the temperature range of interest from 20 to 100 mK has no phase diffusion that greatly complicates the switching dynamics. For this JJ, the phase diffusion occurs above 200 mK. Such model and calculation method description are given in [6].

The current in Figure 1, *a* develops from 0 to $0.3I_c$. The pulse of one 9 GHz photon increases the current at a maximum by another 14 nA. At the zero temperature, photon-induced energy oscillations are clearly seen both in L and C channel. At the temperature not equal to zero, the phase oscillates at the plasma frequency, even when the set current is unchanged and there are no other external impacts. For this junction, the average current variation amplitude due to thermal fluctuations at 20 mK is 0.84 nA. In this case, the maximum current pulse amplitude with the energy of one photon is equal to 14 nA at the shift current of 180 nA. Though these values differ by an order of magnitude, thermal oscillations of the superconducting current energy and capacitor energy are so high that the

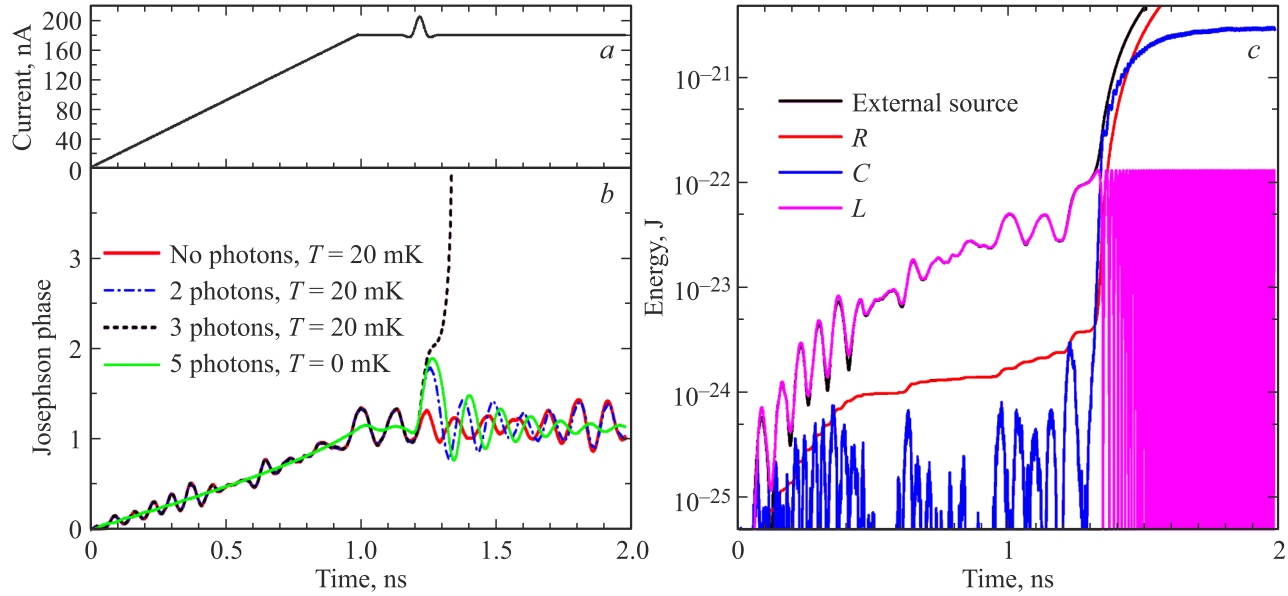

Figure 2. Evolution of current (a) and JJ phase (b) when 0, 2, 3 and 5 photons are absorbed. During the period from 0 to 1 ns, the current increases to 180 nA $(0.9I_c)$. At the time point of 1, 2 ns, a pulse from several in-phase photons arrive. b — red curve — 0 photons, blue curve — 2 photons, black curve — 3 photons at 20 mK, green curve — 5 photons without fluctuations. With 3 photon signal and temperature of 20 mK, JJ switched to resistive state. At the zero temperature,even 5 photons are not enough for switching. c is the energy change in elements R, C, L of the Josephson junction when switching to the resistive state after absorption of 3 photons.

current development process and the photon arrival time are barely noticeable against their background.

The described case demonstrates the role of thermal fluctuations and how photon effect is degraded during photon absorption. At a shift current of 60 nA, a single photon do not cause JJ switching to the resistive state. To observe switchings, let us set the same JJ variables, but increase the shift current. Let the current develop linearly up to 180 nA slowly enough (Fogure 2, a) so that not to excite plasma oscillations. This is achieved at the scales of several nanoseconds. Plasma frequency at 180 nA is equal to 9 GHz. The arriving photon (equation (1)) also has a frequency of 9 GHz and a standard width of $t_{ph} = 0.02$ ns. Pulses with such shape were studied, for example, in [7] and [8]. Let us first consider the case without thermal fluctuations. Then, to switch the junction from the state with a shift current of 180 nA $(0.9I_c)$, minimum 6 in-phase photons are necessary, 5 photons are not enough (green curve in Figure 2, b). When thermal fluctuations with a temperature of 20 mK are applied, switching may occur from 3 photons already, but provided that the fluctuation and photon phases coincide (black dashed line in Figure 2, b).

It is obvious that when there are thermal plasma oscillations, the photon interaction result will depend on the phase of the arriving signal. The photon phase in the considered approximation of the external exciting force is set relative to the plasma oscillation phase by the pulse start time $p(t)$. Photon effect id too short, therefore the plasma oscillation and photon phases shall coincide for switching to the resistive state to occur. If the photon arrives in the reversed phase, the phase oscillations are damped for some time. If the plasma oscillation and photon phases coincide, the oscillation amplitude increases. Figure 2, b shows the plasma oscillation amplitude induced by a temperature of 20 mK without photons (red curve). The amplitude of these oscillations is comparable with the amplitude from the absorption of 3 photons followed by switching to the resistive state.

Figure 2, c shows the energies in R, C and L elements and the external force energy for JJ switching to the resistive state after the absorption of three photons (corresponds to the dashed black curve for the phase in Figure 2, b). After the switching, the inductance energy changes from 0 to $2E_J$ with the Josephson frequency.

On the one hand, dependence of the detector on the photon phase sharply decreases the sensitivity, because all photons with unsuitable phases are passed. On the other hand, in this even weak signals whose influence is comparable with the fluctuation amplitude can be detected, while not all, but with the certain phase.

In the example above, switching with the increase in the energy of junction to the energy of one or more than one photons is demonstrated for the coinciding photon frequencies and plasma oscillations. In JJ, the plasma oscillation frequency varies with the increasing shift current I_b. This is particularly apparent at shift currents close to the critical current when even a small increase in the current amplitude increases the oscillation period significantly. In this respect, the photon frequency is not so important for successful detection, because even when it is equal to the plasma

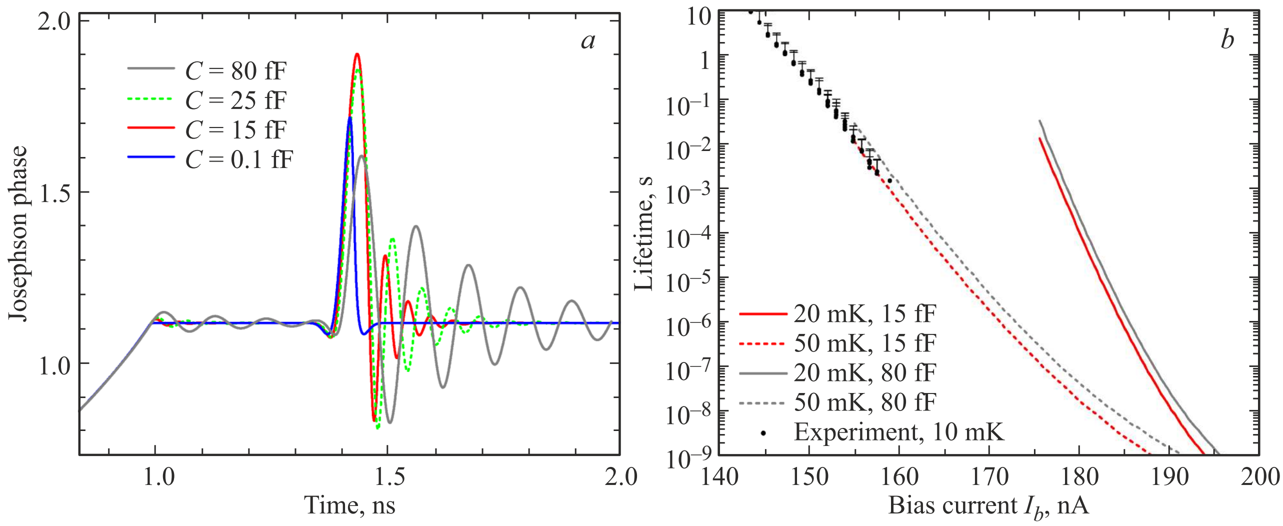

Figure 3. *a* — phase evolution after absorption of three 9 GHz photons with different capacitances. Case without fluctuations. *b* — life time for the best capacitance of 15 fF as simulated and technologically available capacitance of 80 fF at 20 and 50 mK. Black markers are experimental data for JJ with similar parameters and cryostat temperature 10 mK.

frequency at the waiting current, a small current increase results in desynchronization. The only requirement for the photon frequency is that it shall not exceed the plasma oscillation frequency. The amplitude of the current induced by the photon arrival is more important.

Quantum fluctuations are another factor complicating the detecting process. Experimental characteristics of JJ show that, beginning from some temperature, further temperature decrease is not accompanied with decreasing fluctuations in the junction. According to the literature data, the minimum temperature of the quantum crossover is about 50 mK [9,10]. We observe approximately the same fluctuation saturation temperature in our measurements. An illustrative example includes life time measurements from the shift current. The curve slope is set by the relation between the fluctuation energy and barrier height. It can be seen that the experimental curve at 10 mK fits well the calculated curve with 50 mK.

Theoretical life time at 180 nA and 20 mK is equal to 0.2 ms. If the fluctuation temperature is set to 50 mK like in the experiment, switching occurs from two and more photons and the life time in this case is only 40 ns.

3. Switching efficiency improvement

In the practical implementation of the photon detector, we are limited by the existing process variables, i. e. by the aluminium process for tunneling junctions. This sets the critical temperature as well as capacitance per unit length formed by the aluminium oxide between aluminium electrodes. Capacitance can be adjusted by the oxide thickness, but this method enables the changes to be made by max. 30–50% without loss of the barrier quality. Capacitance also depends on the tunneling junction area. But the first and second methods result in simultaneous variation of both junction resistance and critical current.

The RCSJ model allows all JJ variables to be varied independently and thus to find their best combination to detect photons of the required frequency range. Further, if this combination is not available for the existing technologies, this may promote the search for new technologies and materials to be used in the tunneling junctions.

Let us define the role of capacitance. For this, only the capacitance will be varied with unchanged normal resistance and critical current through the transition (or critical electrode temperature).

According to the results shown in Figure 3, *a*, there is an optimum capacitance at which the photon response is maximum and other variables are fixed. For the selected variables, this is −15 fF. Response to the same number of photons (3) increases by a factor of 1.5 compared with 80 fF obtained for the standard aluminium process. It should be also considered that the capacitance variation affects the JJ life time in its superconducting state. Figure 3, *b* shows the life time for two capacitances and two temperatures. A temperature of 50 mK gives life times which are very close to the experimental data measured at 10 mK (black markers with errors) indicating that the fluctuation temperature in the experiment does not decrease below 50 mK. At 20 mK and shift current 180 nA, the life time with 15 fF is twice as low as that with 80 fF: instead of 200 we have 100 μs, which still may be demanded for some applications, where the photon waiting time is much lower 100 μs.

Increase in sensitivity to single photons occurs at temperature decrease. As mentioned above, the JJ temperature in the experiment does not decreases below some boundary which corresponds to quantum fluctuations [11]. But this boundary depends on the sample variables. Such variables may be found at which MQT exhibits below 20 mK or

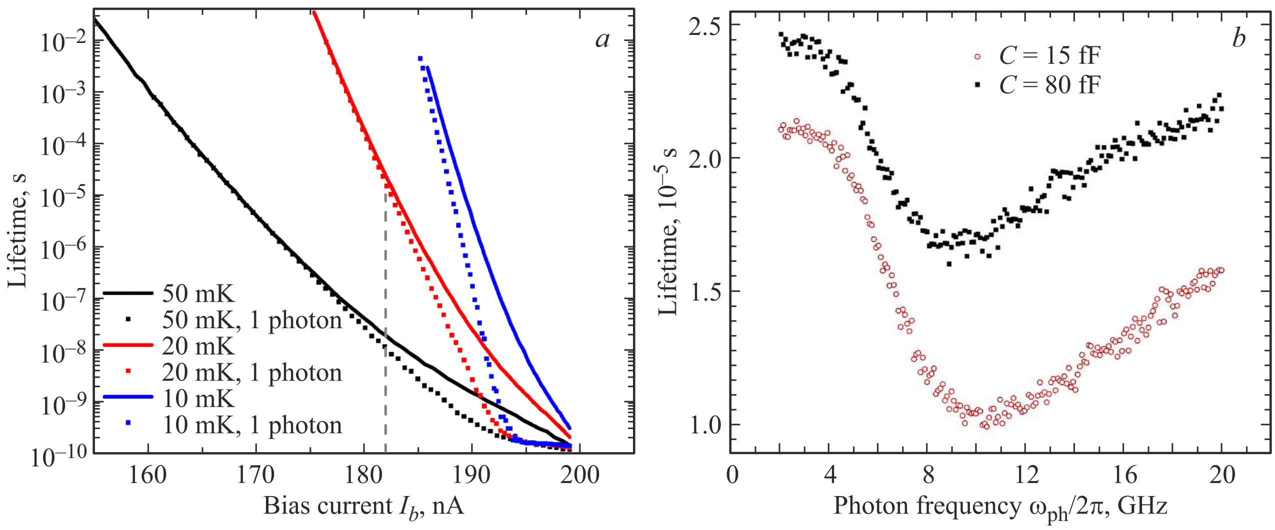

Figure 4. *a* — simulation of JJ life time with capacitance 80 fF at 50, 20 and 10 mK. Lines — life time defined by thermal fluctuations, markers — life time during photon arrival at time point = 0. The dashed line shows the shift current for which the right curve was calculated. *b* — simulation of life time at 20 mK from the photon frequency for two JJ capacitances. Shift current 182 nA.

JJ variables with $I_c = 200$ nA and $R_N = 1500\,\Omega$ when switching occurs as a result of absorption of 9 GHz photons

Capacitance C, fF	Set current I_b, nA	Temperature T, nK	Life time τ	Number of photons for switching
80	180	0 mK	∞	≥ 6
80	180	50 mK	40 ns	≥ 3
80	180	20 mK	0.2 ms	≥ 3
15	180	20 mK	0.1 ms	≥ 2
80	190	10 mK	1 ms	≥ 1

even below 10 mK. But even in this case, the actual JJ temperature may be higher than the cryostat temperature due to background noise and cable noise. To solve this problem, there are sample shielding methods and powder filters showing high efficiency for measurements below 100 mK. Thus, it is also worth considering the temperature decrease as a method of sensitivity optimization. Life time response to a single photon at different temperatures is shown in Figure 4, *a*. At 50 mK, a single photon can be distinguished at currents I_b, at which life time is lower than 0.1 μs. The use of a detector with such life time is limited by applications where photon waiting time is less than 0.1 μs. When the sample temperature decreases down to 10 mK, single photons can be distinguished at life times exceeding 1 ms, which approximately corresponds to the experimental data on detecting single photons [12].

Photon frequency is another variable by which the sensitivity per photon can be optimized. In nonlinear systems exposed to a periodic signal, a resonance activation effect is well known [6], which is defined as the life time decrease for a certain external signal frequency. Such effect also takes place for a short pulse from a single photon. Simulation of the life time vs. absorbed photon frequency is shown in Figure 4, *b*. The life time approximately twice as low, if the photon frequency coincides with the plasma frequency which is equal to 9 GHz for this shift current.

The main results of this section are listed the table, which shows that the single-photon sensitivity may be achieved at standard variables for the aluminium process, if the noise level is lower than the thermal noises at 10 mK.

4. Conclusion

We have considered detecting of microwave band single photons using the Josephson junction and RCSJ model and representing a photon in the form of a ascending and then descending external current pulse. The energy spent for operation of the external current source generating such pulse is equal to the photon energy. The role of thermal fluctuations in three JJ current channels has been investigated and it has been shown that at the fixed shift current through the junction, the highest oscillations take place in the overcurrent energy due to the combination of the fluctuation energy with the external current source energy.

A parameter region has been found where single photons can be detected at the mean JJ life time of about 1 ms (Table).

It should be noted that the situation with photon detection is much more complicated for the junctions in the phase diffusion conditions. In this case, even a successful phase transfer over the barrier does not guarantee that a resistive state will occur, but only causes short-term voltage increase on the junction. In order to detect this, time resolution shall be not worse than the plasma oscillations at such current. Effect of the phase diffusion on the JJ switching times and efficiency during photon absorption will be discussed in an other paper.

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Conflict of interest

The authors declare that they have no conflict of interest.

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