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Features of polarized monochromatic Cherenkov radiation in the UV, optical and terahertz range

© M.V. Bulgakova [1,2], V.S. Malyshevsky [1,¶], G.V. Fomin [1]

[1] Southern Federal University, Rostov-on-Don, Russia
[2] Belgorod State National Research University, Belgorod, Russia
¶e-mail: vsmalyshevsky@sfedu.ru

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The spectral-angular characteristics of polarized Cherenkov radiation of relativistic particles in quartz glass are analyzed in the ultraviolet, optical and terahertz spectral ranges at grazing angles of entry into the target. The radiation generated in thin quartz targets can be used both for creating monochromatic directional radiation sources in various spectral ranges and for diagnosing the angular characteristics of beams of accelerated relativistic particles.

Keywords: Cherenkov radiation, transition radiation, relativistic particles, quartz glass.

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The Vavilov–Cherenkov radiation discovered in the papers [1,2] and the transition radiation of accelerated relativistic charges as they pass through matter have recently obtained a great incentive for further research. The emerging technical opportunities for accelerating multiply charged ions have expanded the scope of these effects and posed new problems in this area. The opportunities of generating Cherenkov radiation in various frequency ranges are actively discussed in the literature. In particular, it is known that in some substances the dielectric constant in the X-ray frequency range near the edges of the absorption lines can be greater than unity[1]. In this case, it turns out to be possible to perform the threshold for the appearance of Vavilov–Cherenkov X-ray radiation. In particular, such an effect can be observed in a number of substances, such as titanium, vanadium [3], beryllium [4], etc. In a softer frequency range, for example, in the ultraviolet or optical, to obtain monochromatic radiation, it is required, on the one hand, to have a target with sufficient dispersion in the frequency region of interest, and on the other hand, the opportunity of collimating radiation to isolate the corresponding area of the spectrum. In particular, for generation in the ultraviolet part of the spectrum, the opportunity of using gaseous targets of inert gases (helium, neon, argon), in which the absorption is weak [5], was studied. In the optical range, there was monochromatic radiation for the first time in fused quartz [6] and diamond [7] targets. In a number of recent papers it has been proposed to use pulsed beams of relativistic electrons to generate Cherenkov radiation in the terahertz range [8]. The use of accelerated relativistic multiply charged ions for these purposes can significantly increase the radiation yield, since the power of the resulting radiation is proportional to the square of the charge of the particles entering the target.

Among the materials considered for the generation of monochromatic Cherenkov radiation in various frequency ranges, quartz glass is the most studied. Quartz glass contains a very small amount of impurities of other chemical elements, and its basis is silicon oxide. Therefore, quartz glass has a very wide transmission spectrum, low absorption, high homogeneity, and resistance to ionizing radiation. All these properties can be useful for using quartz glass as a target for generating Cherenkov and transition radiation in various spectral ranges. The dispersion properties of this material have been studied in sufficient detail in a wide range of wavelengths — from vacuum ultraviolet to the terahertz area. For example, in the paper [9] the results of measurements of the complex refraction index of quartz glass in the spectral range from 30 nm to 1000 μm are collected and summarized. In addition, useful formulas are proposed for the real and imaginary parts of the complex refraction index of quartz glass, which provide a good approximation of the experimental data. The table shows some values of the real and imaginary parts of the complex dielectric constant of fused quartz $\varepsilon = \varepsilon' + i\varepsilon''$. As shown below, these data make it possible to analyze the parameters of Cherenkov and transition radiation in a very wide spectral range.

To solve the problem of transition and Cherenkov radiation in an absorbing target of finite thickness, we use the well-known results of solving Maxwell's equations based on the method of matching the normal and tangential components of fields at the media interfaces. This method allows obtaining the most complete information on the spectral-angular distribution of the emerging electromagnetic radiation and is described in detail in [10]. It should be noted that at normal incidence of relativistic particles on a target whose real part of the refraction index is greater

[1] Modern data on the frequency dependence of the complex permittivity of various substances are contained in the databases of the X-ray Optics Center of the Berkeley National Laboratory named after Lawrence (http://henke.lbl.gov/optical.constants/index).

The values of the real and imaginary parts of the complex dielectric constant of fused quartz according to [9]

Wavelength, μm	Re(ε)	Im(ε)
0.14	3.51	0.077
0.16	2.79	$2.74 \cdot 10^{-4}$
0.45	2.15	$2.05 \cdot 10^{-7}$
0.65	2.12	$2.78 \cdot 10^{-7}$
800	4.22	0.965
900	5.59	0.994

than 1.40, the output of Cherenkov radiation of relativistic particles is impossible due to total internal reflection from the second boundary of the target. To observe Cherenkov radiation in the direction of particle motion, in this case, the particle should fall on the target at a certain angle ψ between the direction of its motion and the normal to the surface. Then the Cherenkov radiation cone takes an asymmetric shape, and for a sufficiently large angle ψ, part of the Cherenkov cone will experience total internal reflection at the target boundary of thickness L, and part will pass in the direction of particle motion.

It is well known that in the case of a normal incidence of a fast charged particle on the interface, the resulting electromagnetic radiation is polarized in the so-called radiation plane containing the radiation wave vector **k** and the normal to the surface. Following the paper cited above [10], we will call this „longitudinal" polarization[2]. When particles fall at an angle to the interface between two media, an additional „transverse" radiation polarization occurs in a plane orthogonal to the radiation plane. It should be expected that the spectral-angular distribution of electromagnetic radiation from different polarizations will depend differently on the angle of incidence of particles on the target. Let us denote the charge of the particle flying into the medium as Ze and assume that the magnetic permeability of the medium is $\mu = 1$. Then the corresponding analytical expressions for the spectral-angular density of radiation with longitudinal $dI^{\parallel}(\mathbf{k}, \omega) = I^{\parallel}(\mathbf{k}, \omega) d\omega d\Omega$ and transverse $dI^{\perp}(\mathbf{k}, \omega) = I^{\perp}(\mathbf{k}, \omega) d\omega d\Omega$ polarization in the frequency range $\omega, \omega + d$ in the solid angle $d\Omega$ in the direction of particle motion in a medium with a complex dielectric constant ε will have the following form [10]:

$$I^{\parallel}(\mathbf{k}, \omega) = \frac{Z^2 e^2}{\pi^2 c} \frac{\beta_z^2 \cos^2\theta_z}{\sin^2\theta_z} \frac{|\varepsilon - 1|^2}{|[A^2 - \beta_z^2 x^2][A^2 - \beta_z^2 \cos^2\theta_z]|^2} \times \frac{|B_\psi^{\parallel}(\omega, \mathbf{k}, \mathbf{v})|^2}{|(x+y)^2 e^{-i\frac{\omega L}{c}x} - (x-y)^2 e^{i\frac{\omega L}{c}x}|^2}, \quad (1)$$

where $x = \sqrt{\varepsilon - \sin^2\theta_z}$, $y = \varepsilon\cos\theta_z$, $A = 1 - \beta_x \cos\theta_x$

$$\begin{aligned} B_\psi^{\parallel}(\omega, \mathbf{k}, \mathbf{v}) &= (x+y)(A + \beta_z x)[(A - \beta_z^2 - \beta_z x)\sin^2\theta_z \\ &+ \beta_x\beta_z x\cos\theta_x]e^{-i\frac{\omega L}{c}x} + (x-y)(A - \beta_z x) \\ &\times [(A - \beta_z^2 + \beta_z x)\sin^2\theta_z - \beta_x\beta_z x\cos\theta_x]e^{i\frac{\omega L}{c}x} \\ &- 2x[(A + \beta_z y)(A - \beta_z^2)\sin^2\theta_z + \beta_z(\beta_x\cos\theta_x - \sin^2\theta_z) \\ &\times (\beta_z x^2 + Ay)]e^{-i\frac{\omega L}{v_z}A}. \end{aligned} \quad (2)$$

The spectral-angular radiation density of a component with a polarization perpendicular to the radiation plane is given by the following formula:

$$\begin{aligned} I^{\perp}(\mathbf{k}, \omega) &= \frac{Z^2 e^2}{\pi^2 c} \frac{\beta_x^2 \beta_z^4 \cos^2\theta_y \cos^2\theta_z}{\sin^2\theta_z} \\ &\times \frac{|\varepsilon - 1|^2}{|[A^2 - \beta_z^2 x^2][A^2 - \beta_z^2\cos^2\theta_z]|^2} \\ &\times \frac{|B_\psi^{\perp}(\omega, \mathbf{k}, \mathbf{v})|^2}{|(x + \frac{y}{\varepsilon})^2 e^{-i\frac{\omega L}{c}x} - (x - \frac{y}{\varepsilon})^2 e^{i\frac{\omega L}{c}x}|^2}, \end{aligned} \quad (3)$$

where

$$\begin{aligned} B_\psi^{\perp}(\omega, \mathbf{k}, \mathbf{v}) &= (A + \beta_z x)(x + \frac{y}{\varepsilon})e^{-i\frac{\omega L}{c}x} \\ &+ (A - \beta_z x)(x - \frac{y}{\varepsilon})e^{i\frac{\omega L}{c}x} - 2x(A + \beta_z\cos\theta_z)e^{-i\frac{\omega L}{v_z}A}. \end{aligned} \quad (4)$$

The direction of the electron velocity makes an angle ψ with the axis z, so that $\beta_x = \beta\sin\psi$, $\beta_z = \beta\cos\psi$, $\beta = \frac{v}{c}$. The direction of radiation is determined by the direction cosines about the axes x, y and z: $\cos\theta_x = \sin\theta\cos\varphi$, $\cos\theta_y = \sin\theta\sin\varphi$, $\cos\theta_z = \cos\theta$, where θ and φ — polar and azimuthal angles, respectively.

Examples of some calculations of the angular distribution of the relativistic particle polarized in different planes Cherenkov radiation in a thin quartz glass plate with a given wavelength are shown in Figs 1 and 2. It is convenient to present the results of calculations in variable direction cosines $\cos\theta_x$ and $\cos\theta_y$, because in this way, in contrast to the variables θ and φ, the visibility of the spatial-angular distribution of radiation is achieved. The maximum in the region of positive direction cosines — is the maximum of transition radiation, which increases with increasing energy of the particle, concentrated in a narrow interval of angles along the direction of particle motion $\Delta\vartheta \simeq \frac{1}{\gamma}$, where $\gamma = E/mc^2$. Let us note that when particles fall obliquely on the target (in contrast to the normal one), the angular distribution of transition radiation becomes asymmetric with respect to the direction of particle motion [4]. At the angle of incidence chosen in Fig. 2, one shoulder in the angular distribution is strongly suppressed.

[2] It should be noted that the terms „longitudinal" and „transverse" polarization are traditionally used only in relation to Cherenkov and transition electromagnetic radiation. Following modern terminology, it can be spoken differently, namely, about σ- and π - components of polarized electromagnetic radiation.

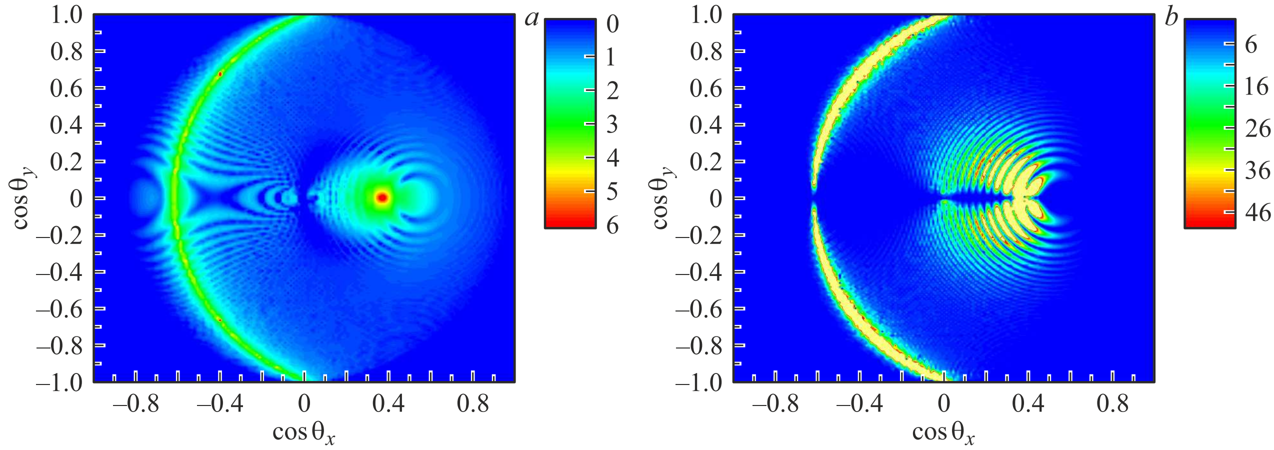

Figure 1. Calculated angular distribution of Cherenkov radiation with longitudinal polarization of relativistic particles ($\gamma = 250$) in the optical range ($\lambda = 0.45\,\mu$m) in a fused quartz target with thickness $L = 100\lambda$. Angle between the direction of particle motion and the normal to the target surface $\psi = 22^\circ$. *a* — longitudinal polarization, *b* — transverse polarization.

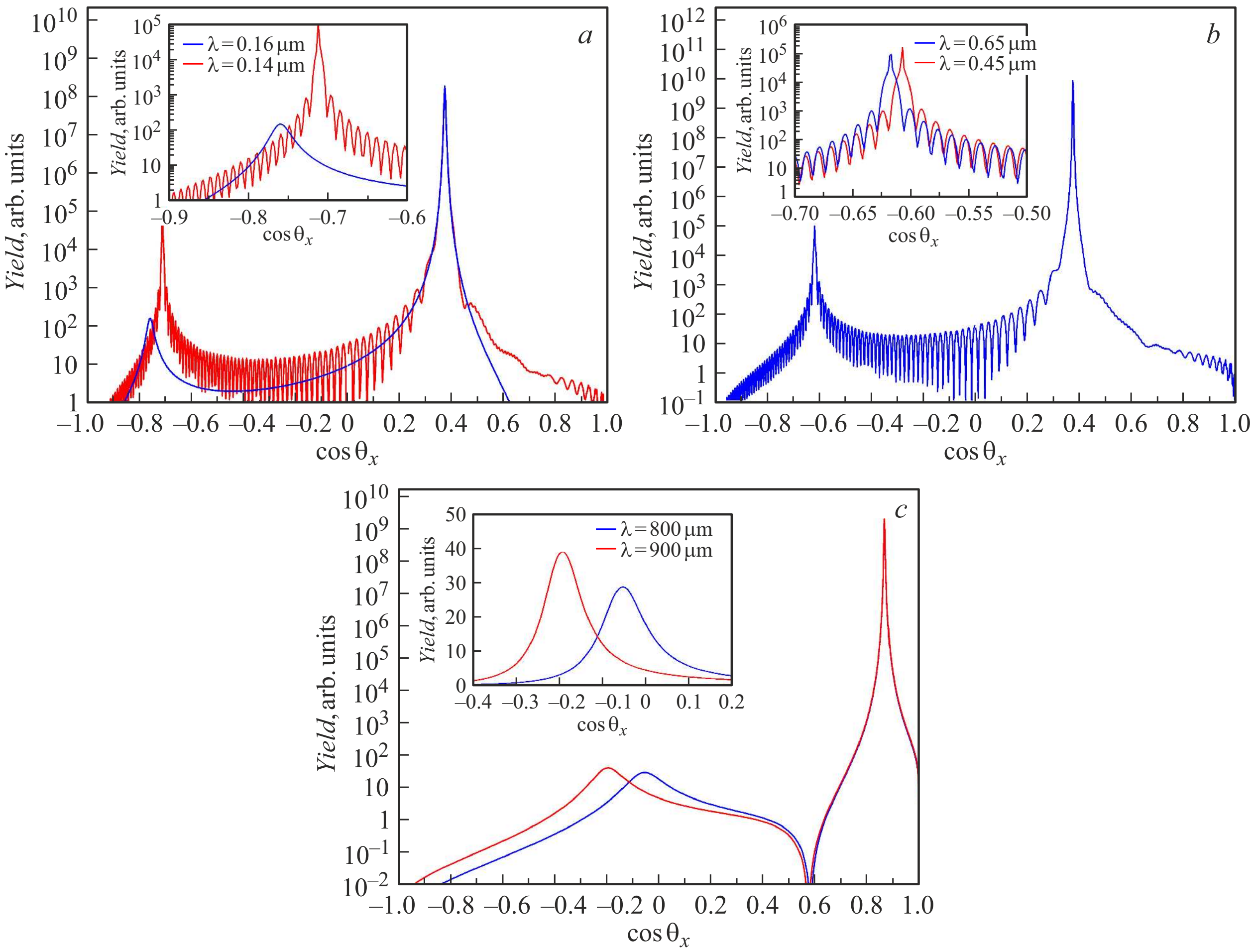

Figure 2. The calculated angular distribution over the polar angle in the plane of Cherenkov radiation (azimuthal angle $\varphi = 0$) with longitudinal polarization of relativistic particles ($\gamma = 250$) in various frequency ranges in a fused quartz target: *a* — in the near ultraviolet range, $\psi = 22^\circ$, $L = 100\lambda$; *b* — in the optical range, $\psi = 22^\circ$, $L = 100\lambda$; *c* — in the terahertz range, $\psi = 60^\circ$, $L = 10\lambda$.

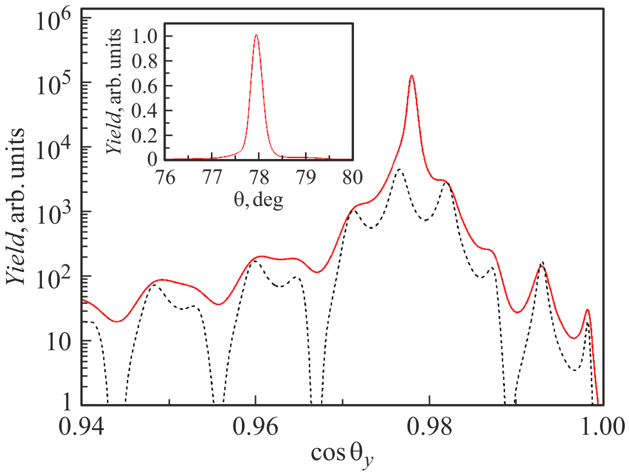

Figure 3. The calculated angular distribution of Cherenkov radiation over the polar angle in the plane orthogonal to the radiation plane (azimuthal angle $\varphi = 90°$), with longitudinal polarization (solid curve) and transverse polarization (dashed curve) of relativistic particles ($\gamma = 250$) in the optical range ($\lambda = 0.65\,\mu$m) in fused silica targets, $\psi = 22°$, $L = 100\lambda$.

In the area of negative direction cosines, a fragment of the cone of Cherenkov radiation emerging from the target is visible. Pronounced oscillations in the polar and azimuthal angles are caused by the interference of the incident wave and the wave reflected from the target boundaries. Calculations using the above formulas show that the maximum intensity of radiation with transverse polarization is much less than with longitudinal polarization. The degree of longitudinal polarization depends both on the angle of incidence of particles on the target and on the azimuthal angle of observation. In particular, at the azimuthal observation angle $\varphi = 0$, i.e. in the radiation plane, the degree of longitudinal polarization is 100% at any angle of incidence.

The angular distribution of radiation with longitudinal polarization in the radiation plane in dimensionless units Z^2e^2/π^2c in the ultraviolet, optical and terahertz areas of the spectrum is shown in Fig. 2. The angular width of the Cherenkov maxima does not exceed 10^{-2} rad, and the radiation maxima themselves with different wavelengths are separated, which allows us to conclude that monochromatic lines can be distinguished in the selected spectral range. The absence of oscillations in the polar angle in some cases is explained by the large value of the imaginary part of the permittivity and, as a consequence, the attenuation of the waves reflected from the target boundaries.

Fig. 3 shows the angular distribution of Cherenkov radiation over the polar angle in the plane orthogonal to the radiation plane (azimuthal angle $\varphi = 90°$) calculated by formulas (1) $-$(4), with longitudinal and transverse polarization. The intensity of the component with transverse polarization, as mentioned above, is much less than the intensity of the component with longitudinal polarization. The inset to Fig. 3 shows the normalized angular distribution of the longitudinally polarized component. The maximum radiation under the chosen conditions is achieved at a polar angle of 78°, and the angular width does not exceed 0.5°.

In conclusion, we note that the performed analysis of the spectral-angular characteristics of the Cherenkov radiation of relativistic particles during oblique incidences on thin quartz targets shows the fundamental opportunity of using the effect to create monochromatic directional sources of polarized radiation in various spectral ranges, and the small angular width of radiation can be useful for diagnosing angular characteristics of beams of accelerated relativistic particles.

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Conflict of interest

The authors declare that they have no conflict of interest.

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