09

Interferometry of absolute distances of laser probe relief meters with harmonic wavelength deviation

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Received November 03, 2022
Revised January 13, 2023
Accepted January 17, 2023

The results of modeling a self-mixing laser (autodyne) as a laser probe for controlling microdisplacements are presented. A method for measuring the absolute distance from the ratio of the amplitudes of the harmonics of the autodyne signal spectrum is proposed. The calculation was carried out using the PyCharm IDE software environment and the *numpy* and *matplotlib* software modules. The measurement accuracy was estimated taking into account the accuracy of measuring the power of the autodyne signal and the amplitudes of the spectral harmonics at various deviations of the wavelength of the laser autodyne. It is shown that due to the ambiguity of the Bessel functions included in the algorithm, in order to reliably determine the distance, it is necessary to limit the choice of spectral components to the region of uniqueness, which is located at the end of the significant region of the spectrum. The error in determining the absolute distance from the deviation of the laser wavelength was studied. It was also found that as the distance to the reflector decreases, it is necessary to increase the deviation of the laser radiation wavelength so that the set of measured harmonics is in the high-frequency region. It is shown that in the deviation range from 0.1 nm to 1 nm at a distance of 50 to 100 mm, the measurement accuracy can reach several microns. The promise of using a laser autodyne is due to the task of developing laser probes for monitoring microdisplacements in a narrow range of distances to the reflector.

Keywords: laser interferometry, autodyne, semiconductor laser, laser radiation modulation, external optical feedback, distance measurement, microvibration, spectral signal analysis.

DOI: 10.61011/EOS.2023.06.56657.102-23

Introduction

The interferometry of absolute distance is based on different kinds of current modulation of the laser emission wavelength [1–4]. Most of studies are devoted to the triangular modulation of emission wavelength and the main measured parameter is frequency of interference peaks, which depends on wavelength deviation and distance to the reflector [5,6]. As far as the deviation in modern semiconductor lasers is not greater than a few nanometers, a high accuracy of distance measurements is achieved at long distances.

The most promising for measuring system miniaturization can be lasers with external optical feedback [7] known as laser autodynes. These lasers can be used to determine characteristics of nanovibrations [8–10], microdisplacements [11–13], velocity [14–16] and acceleration [17,18] of the reflector.

One of the application fields of laser autodynes is their use in probe microscopy. In [19,20] the possibility has been discussed to use laser autodynes with current modulation to measure nanodisplacements using interference signal phase. More promising can be the use of multifrequency interferometry methods based on the measurement of spectral component amplitudes of lasers with external optical feedback, which allow measurements of absolute distance to the reflector. Signal from such system is known as autodyne signal. With low levels of feedback the autodyne signal is similar to the interference signal with isolation from the power supply source. The advantages of measuring absolute distances up to 10 cm with harmonic frequency modulation of laser autodyne in comparison with the triangular modulation method have been previously discussed in [21]. However, capabilities of the harmonic modulation method at large wavelength deviations had not been demonstrated before.

The promising potential of laser autodyne application is due to the task of development of laser probes to control microdisplacements in a narrow range of distances to the reflector. With the distance to the surface under measurement fixed in a range of a few millimeters the method of harmonic deviation of laser emission wavelength is relevant with a measurement accuracy of up to units of microns and, in contrast to phase methods of interferometry, the multifrequency modulation method of laser autodyne is free from ambiguity and uncertainty when the direction of shift of the surface relief changes to the opposite. The goal of this study was to numerically model the measurements of absolute distances with a micron accuracy in the mode of harmonic wavelength deviation of semiconductor laser with external optical feedback.

Interference signal model with harmonic deviation of the laser autodyne wavelength

When the laser emission is frequency-modulated, the semiconductor laser power P can be written as amplitude and phase components that depend on the pump current density $j(t)$ [22]:

$$P(j(t)) = P_1(j(t)) + P_2 \cos(\omega(j(t))\tau), \quad (1)$$

where P_1 is the direct power component, P_2 is the amplitude power component depending on the wave phase incursion $\omega(j(t))\tau$ in a system with external reflector, τ is the time it takes the laser radiation to travel the distance to the external reflector and back, $\omega(j(t))\tau$ is the semiconductor laser radiation frequency depending on the pump current density $j(t)$ and the feedback level.

The autodyne signal parameters are affected by the level of external optical feedback [23–25]. As shown earlier, it is possible to choose the feedback level so that the semiconductor laser emission frequency does not change significantly and thereby introduce no distortions into the shape of the interference signal [26,27].

In this case, with harmonic modulation of the pump current density $j(t)$ the semiconductor laser emission frequency takes the following form

$$\omega(j(t)) = \omega_0 + \omega_A \sin(2\pi\nu_1 t), \quad (2)$$

where ω_0 is the natural frequency of the semiconductor laser diode; ω_A is the frequency deviation of the semiconductor laser diode; ν_1 is the frequency of laser diode supply current modulation. Expression (3) for the power of frequency-modulated semiconductor laser emission can be written as follows:

$$P(j(t)) = I_1 \sin(2\pi\nu_1 t) + P_2 \cos(\omega_0\tau + \omega_A\tau \sin(\Omega t)), \quad (3)$$

where $\theta = \omega_0\tau$ is the steady-state phase of the autodyne signal, $\sigma = \omega_A\tau$ is the amplitude of the current modulation phase, $\Omega = 2\pi\nu_1$ is the circular frequency of the laser diode supply current modulation.

Due to the fact that only the phase component of the multifrequency autodyne signal is used to determine the distance, equation (4) can be written as follows:

$$P(j(t)) = P_2 \cos(\theta + \sigma \sin(\Omega t)). \quad (4)$$

To analyze the autodyne signal under conditions of harmonic deviation of the laser diode emission wavelength, a representation of the signal in the form of an expansion in a series of Bessel functions of the first kind J_n and an expansion in a Fourier series with amplitudes of the spectral components S_n will be used. In this case, without taking into account the constant component, $P(t)$ can be written

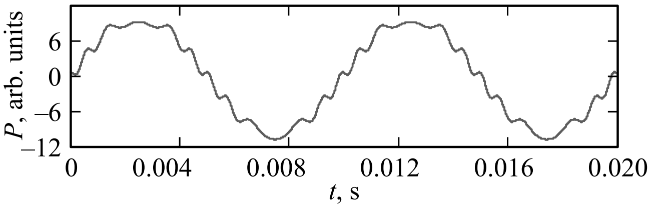

Figure 1. Interference signal model ($\Delta\lambda = 0.01$ nm, $L = 52.8$ mm).

as follows

$$P(t) = P_2 \cos(\theta) J_0(\sigma) + 2P_2 \cos(\theta) \sum_{n=1}^{\infty} J_{2n}(\sigma) \cos(2n\Omega t)$$
$$- 2P_2 \sin(\theta) \sum_{n=1}^{\infty} J_{2n+1}(\sigma) \cos((2n-1)(\Omega t).) \quad (5)$$

Taking into account the relation between J_n and S_n from [22], amplitudes of spectral harmonics of Fourier spectrum S_{2n} and S_{2n+1} for $n = 1, 2, 3\ldots$ can be written in the following form:

$$S_{2n} = 2\cos(\theta) P_2 J_{2n}(\sigma), \quad (6)$$

$$S_{2n+1} = -2\sin(\theta) P_2 J_{2n+1}(\sigma). \quad (7)$$

To determine the distance to target L, which is included in the σ parameter, the ratio of spectral harmonics of Fourier spectrum of the autodyne signal can be used:

$$S_n/S_{n+2} = (J_n(\sigma))/(J_{n+2}(\sigma)). \quad (8)$$

Solving equation (8) for the unknown parameter $\sigma = \omega_A\tau$ requires knowledge of current modulation parameters of the laser autodyne, in particular, the deviation of laser diode emission frequency ω_A. Taking into account the fact that $\tau = 2L/c$, the following relation can be derived to determine the distance to the target:

$$L = \sigma c / 2\omega_A. \quad (9)$$

The autodyne signal was modelled using a program written in Python 3 in the PyCharm IDE software environment. Fig. 1 and 2 show the interference signal model and its Fourier spectrum.

Signal models were built and analyzed using *numpy* software module and graphs were displayed using *matplotlib* module.

The autodyne signal was modelled with the following parameters: $\lambda = 650$ nm, deviation of semiconductor laser diode emission wavelength was $\Delta\lambda = 0.01$ nm, distance to the target was $L = 52.8$ mm, laser emission current modulation frequency was $\nu_1 = 100$ Hz, $\theta = \pi/4$.

Interference signal spectrum (Fig. 2) contains a large number of harmonics, which amplitude ratio allows determining the distance to the target according to (8). Analysis

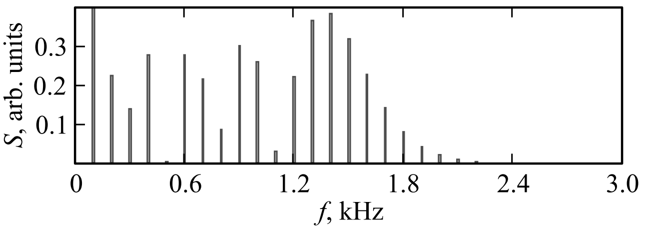

Figure 2. Interference signal spectrum ($\Delta\lambda = 0.01$ nm, $L = 52.8$ mm).

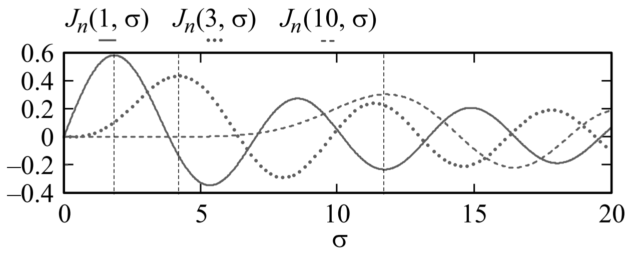

Figure 3. Graphs of dependencies of Bessel functions of different orders J_n on the σ variable.

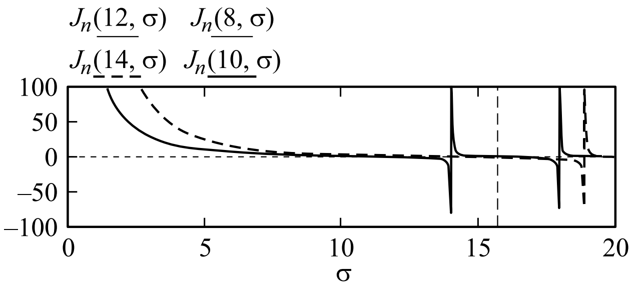

Figure 4. Graphs of dependencies of Bessel functions of different orders used to determine $\sigma = 15.71$ variable ($\Delta\lambda = 0.01$ nm, $L = 52.8$ mm).

of these ratios shows that not all of them have sufficient accuracy. Table 1 shows results of calculation of the distance to the target from the ratio of harmonics with different numbers using relationships (8), (9).

As it follows from the table, high accuracy of the measurements is achieved with the use of a ration of high-order harmonics, in particular, harmonics starting from $n = 11$. Such a relationship is due to properties of Bessel functions. Fig. 3 shows graphs of Bessel functions of different orders J_n as functions of σ variable and Fig. 4 shows graphs of the ratio of these functions used to determine the σ variable from (8).

As can be seen from Fig. 3, value of the σ variable in the region of unambiguity for Bessel functions of the first order is 1.8, for Bessel functions of the third order it is equal to 4.3, for Bessel functions of the tenth order it is equal to 11.8 (shown with vertical dashed lines in Fig. 3).

As can be seen from Fig. 4 and Table 1, the root of equation (8) for the distance of $L = 52.8$ mm is $\sigma = 15.71$, and for the ratio of Bessel functions J_8 and J_{10} it is

Table 1. Results of the distance to reflector calculation based on modeled amplitudes of spectral components and error of the distance determination

n	S_n	S_{n+2}	σ	L, mm	δerror, %	δerror, m
1	9.8	0.14014	0.575343	1.93	> 100	–
2	0.225	0.278	4.34	14.6	> 100	–
3	0.139	0.002	1.07	3.6	> 100	–
4	0.278	0.277	6.4	21.52	> 100	–
5	0.002	0.213	8.75	29.44	> 100	–
6	0.277	0.086	6.6	22.2	> 100	–
7	0.213	0.301	10.03	33.75	> 100	–
8	0.086	0.259	11.71	39.37	> 100	–
9	0.301	0.028	5.9	19.85	> 100	–
10	0.259	0.219	12.52	42.1	25	0.001
11	0.028	0.363	15.708	52.81	$< 10^{-11}$	$< 10^{-14}$
12	0.219	0.382	15.708	52.81	$< 10^{-11}$	$< 10^{-14}$
13	0.363	0.318	15.708	52.81	$< 10^{-11}$	$< 10^{-14}$
14	0.382	0.225	15.708	52.81	$< 10^{-11}$	$< 10^{-14}$
15	0.318	0.140	15.708	52.81	$< 10^{-11}$	$< 10^{-14}$

Table 2. Results of the distance to reflector calculation based on amplitudes of spectral components taking into account the error of their determination

n	S_n	S_{n+2}	σ	L, mm	δerror, %	δerror, μm
11/13	0.028	0.363	15.71	52.81	0.0074	3.9
12/14	0.219	0.382	15.697	52.78	0.07	35
13/15	0.363	0.318	15.687	52.74	0.14	71
14/16	0.382	0.225	16.678	52.71	0.19	100
15/17	0.318	0.140	15.671	52.68	0.23	123

within the ambiguity region, and for the ratio of Bessel functions J_{12} and J_{14} the root of the equation is in the region of unambiguity. Thus, due to the fact that Bessel functions have a region of ambiguity, then to reliably determine the variable, the choice of spectral components of the interference signal must be limited by the region of unambiguity, which is at the end of the significant part of the spectrum.

In practice, the measurement accuracy should be evaluated with the consideration of the accuracy of determining those quantities that are responsible for the maximum error. We used the data on measurement errors for laser autodynes presented in [18–20,22]. We have introduced two noise components: into the formula for calculating the autodyne signal power, relationship (4), and into the measured amplitude of the spectral harmonics used in relationship (8). The results of calculating the distance to the reflector and the error of its determination are shown in Table 2. In this case, 5% error in the power of the autodyne signal and 1% error in the amplitudes of the spectral harmonics taken from the measurements have been chosen.

Taking into account the region of unambiguity of Bessel functions to calculate the absolute distance from the interference signal spectrum shown in Fig. 3, spectral

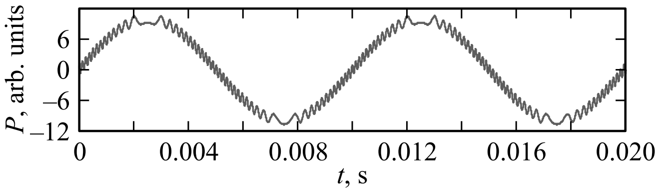

Figure 5. Interference signal model ($\Delta\lambda = 0.05$ nm, $L = 52.8$ mm).

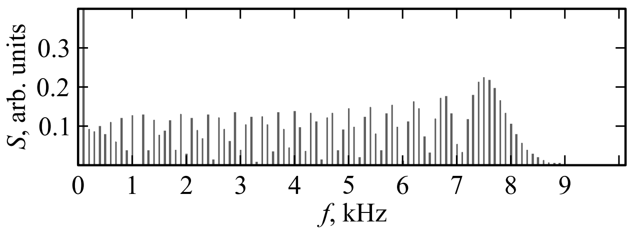

Figure 6. Interference signal spectrum ($\Delta\lambda = 0.05$ nm, $L = 52.8$ mm).

harmonics with n numbers from 11 and more have been chosen. In particular, the ratio of S_{11} and S_{13} harmonics in equation (7) demonstrates the best result and provides error less than $5\,\mu$m.

Interference signal model for variation of deviation of the laser autodyne wavelength

As the laser diode deviation changes, the shape and spectrum of the interference signal changes as well. Fig. 5 and 6 show the model of interference signal and its spectrum at the following parameters: $\Delta\lambda = 0.05$ nm, $L = 52.8$ mm.

As a result of comparative analysis of Fig. 2 and 6, a law can be establish that a change in deviation causes increase in the number of interference peaks, and the spectrum is enriched with high-order harmonics.

To determine the error in measuring the absolute distance, we set 5% error in measuring the power of the autodyne signal and 1% error in measuring the amplitude of the spectral harmonic.

Fig. 7 shows the error of absolute distance determination as a function of the laser autodyne wavelength deviation. The obtained dependence is indicative of the fact that with growth of the deviation the accuracy of distance determination increases and achieves an order of micron at $\Delta\lambda = 0.06$ nm.

If in the process of measuring the distance the deviation value is changed so that the set of measured harmonics is in the high-frequency region, then the measurement accuracy increases with decreasing distance. Taking into account the region of unambiguity of Bessel functions to calculate the absolute distance from the interference signal spectrum shown in Fig. 6, spectral harmonics with n numbers from 70 and more have been chosen. Fig. 8 shows measurement error as a function of the distance to reflector with an unchanged set of spectral components with numbers of $n = 70$ and 72.

The decrease in measurement error observed in Fig. 8 is due to the increase in deviation with decrease in the distance to reflector. Due to the fact that the number of Fourier spectrum harmonics becomes smaller for a constant deviation with a decrease in the distance to the reflector, in order for the set of measured harmonics to be in the high-frequency region, it was necessary to increase the deviation of the laser radiation wavelength. Deviation measurement range was from 0.1 to 1 nm at a measurement accuracy in the range from 3 to $0.2\,\mu$m.

Conclusion

As a result of the performed computer modeling, it has been shown that the proposed method of harmonic frequency modulation of the laser emission allows measurement of absolute distances with micron accuracy. It is found that to calculate the distance from the autodyne signal

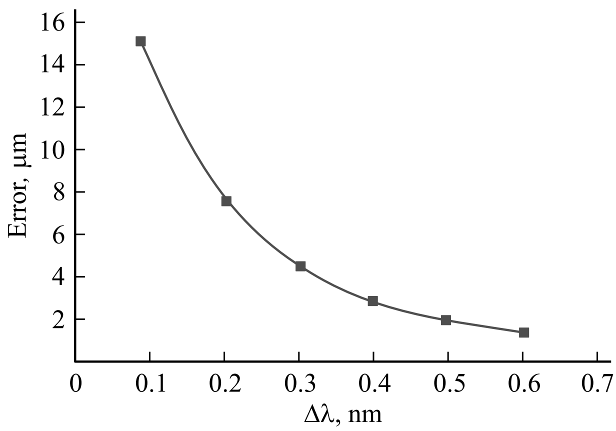

Figure 7. Error of the absolute distance determination as a function of the deviation at a distance of $L = 52.8$ mm.

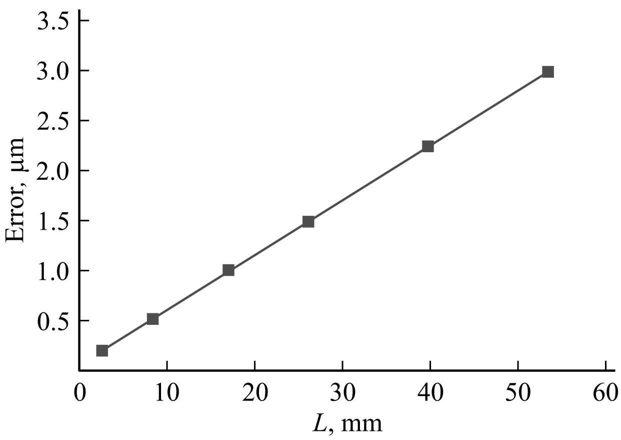

Figure 8. Measurement error as a function of the distance to reflector with an unchanged set of spectral components with numbers of $n = 70$ and 72.

spectrum, high-order harmonics need to be chosen, which is caused by the behavior of the Bessel function in the region of solution unambiguity. With decrease in the distance to reflector the problem of decrease in the number of spectral harmonics of the autodyne signal Fourier spectrum arises, which affects accuracy of the method. The change in the deviation of the emission wavelength allows controlling this process and keeping the accuracy at decreasing distances. The performed calculations substantiate the possibility to use laser autodyne for the development of laser probe measuring instruments to measure surface relief with micron accuracy.

Funding

The study was financially supported by the Innovation Promotion Fund (project №. 171GSSS15-L/78935).

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by Y.Alekseev