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Stabilization of low-threshold two-plasmon parametric decay instability with oblique microwave incidence on the plasma

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Received May 10, 2023
Revised June 4, 2023
Accepted June 7, 2023

It is shown that, when a pump extraordinary wave is obliquely incident onto the plasma in the electron cyclotron resonance heating (ECRH) experiments, an increase takes place in the threshold of parametric decay instability (PDI) leading to generation of two localized upper-hybrid waves. It is proposed to use a small tilt of the heating beam to suppress this PDI which is most dangerous and often observed in the ECRH experiments.

Keywords: parametric decay instability, microwave, upper hybrid wave.

DOI: 10.61011/TPL.2023.08.56685.19623

Recently, experiments on electron cyclotron (EC) plasma heating revealed a number of threshold nonlinear effects, including plasma radiation with the frequency shift relative to the gyrotron frequency [1], which may be interpreted as consequences of the microwave decay. To explain them, a theoretical model of low-threshold parametric decay instability (PDI) was proposed [2], which evolves the established concept of microwave beam propagation in the presence of a monotonic density profile [3]. The model predicted the microwave decay in passing through the plasma region with a local density maximum into a pair of quasi-longitudinal upper-hybrid (UH) waves localized along the non-uniformity direction; later this prediction was many times confirmed by the EC-heating experiments in which the density profile was, for various reasons, non-monotonic [4,5]. The proposed theoretical model allowed reproducing frequency spectra and „radiation temperature" of the anomalous radiation [6,7]. The developed theory predicts the possibility of a considerable anomalous absorption of microwave power [8], which received an indirect experimental confirmation [9,10] and needs searching for ways of controlling and increasing excitation thresholds of low-threshold PDIs. Earlier, the effect of beam focusing and variation in the beam width and power upon the PDI threshold and saturation level was comprehensively studied in [11]. Unfortunately, this effect appeared to be insufficient to suppress or efficiently control the above-mentioned nonlinear process. In this work we studied the influence of the microwave pump beam inclination with respect to magnetic field on the threshold magnitude of PDI possessing the lowest excitation threshold and giving rise to two UH waves trapped in plasma. The results are illustrated by using as an example tokamak TEXTOR where the effect of anomalous radiation revealed during the plasma EC heating was studied in most detail [1].

Consider the extraordinary wave decay into two UH waves localized in the vicinity of the non-monotonic density profile maximum. Taking into account the dimensions of the pump wave decay region, we can ignore both the magnetic field shear and magnetic surface curvature and use the local Cartesian frame of reference (x, y, z) with the origin in the pump beam center and in the local maximum of the UH wave frequency $\omega_{\mathrm{UHR}} = \sqrt{\omega_{ce}^2 + \omega_{pe}^2}$ where ω_{ce}, ω_{pe} are the electron-cyclotron and electron-plasma frequencies. Coordinate x is the flux variable. Axes z and y are directed along the magnetic field line and perpendicular to it on the magnetic surface, respectively. The beam of incident monochromatic microwaves w in radius with power P_0 at frequency ω_0 near the decay point is defined in the Wentzel–Kramers–Brillouin (WKB) approximation as

$$\mathbf{E_0} = \mathbf{e_0}\sqrt{\frac{2P_0}{v_{0g}(x)w^2}}\exp\Big(-\frac{y^2}{2w^2} - \frac{z^2}{2w^2} + i\int_{-\infty}^{x} k_x(\omega_0, k_y, k_z, x')dx' + ik_z z + ik_y y - i\omega_0 t\Big) + \text{c.c.}, \tag{1}$$

where $\mathbf{e}_0$ is the polarization vector, $k_x(\omega_0, k_y, k_z) = k_{0x}$ is the pump wave vector projection in the decay region, v_{0g} is the group velocity, $k_{y,z}w \ll 1$. Potentials of daughter UH waves localized in the vicinity of the UH wave profile maximum may be represented as

$$\varphi_1 = a_m(y, z)\phi_m(x)\exp\big(i(q_z^{m,n} + k_z)z - i\omega_m t\big)/2 + \text{c.c},$$

$$\varphi_2 = a_n(y, z)\phi_n(x)\exp(iq_z^{m,n}z + i\omega_n t)/2 + \text{c.c.}, \tag{2}$$

where eigen functions ϕ_j describe localized modes $j = (m, n)$ and are the solution of equation $\hat{D}_j\phi_j = 0$,

$$\hat{D}_j = l_T^2(\omega_j, x)\hat{q}_x^4 + \varepsilon(\omega_j, x)\hat{q}_x^2 + \omega_j^2 g(\omega_j, x)^2/c^2 + \eta(\omega_j, x)(q_z^{m,n})^2,$$

ε, g, η are the components of the „cold" plasma tensor, $\hat{q}_x^2 = -\partial^2/\partial x^2$, $l_T^2\hat{q}_x^2$ is the thermal correction to ε. Eigen frequency $\omega_m = \omega_0 - \omega_n$ and wave vector component $q_z^{m,n}$ satisfy the quantization conditions. Dimensionless amplitudes a_m, a_n describing the distribution of daughter wave potentials across the plasma non-uniformity direction satisfy the following equations [2]:

$$\left\{\begin{aligned} &\frac{\partial a_m}{\partial t} + i\Lambda_{my}\frac{\partial^2 a_m}{\partial y^2} + i\Lambda_{mz}\frac{\partial^2 a_m}{\partial z^2} + u_m\frac{\partial a_m}{\partial z} \\ &= \gamma_0 \exp\Big(-\frac{y^2}{2w^2} - \frac{z^2}{2w^2} + ik_y y\Big)a_n, \\ &\frac{\partial a_n}{\partial t} - i\Lambda_{ny}\frac{\partial^2 a_n}{\partial y^2} - i\Lambda_{nz}\frac{\partial^2 a_n}{\partial z^2} - u_n\frac{\partial a_n}{\partial z} \\ &= \gamma_0 \exp\Big(-\frac{y^2}{2w^2} - \frac{z^2}{2w^2} - ik_y y\Big)a_m, \end{aligned}\right. \tag{3}$$

where $\gamma_0 \propto \sqrt{P_0/w^2}$ is the instability growth-rate in the limit of an infinitely wide pump beam, which is proportional to the coefficient of nonlinear coupling of two UH waves and pump wave, $\Lambda_{jy} = \langle(2l_T^2(\omega_j, x)\hat{q}_x^2 + \varepsilon(\omega_j, x))\rangle/D_{j\omega}$, $\Lambda_{jz} = \langle\eta(\omega_j, x)\rangle/D_{j\omega}$ are the diffraction coefficients of UH modes $j = m, n$, $D_{j\omega} = 2\langle\omega_j/\omega_{pe}^2\hat{q}_x^2 + q_z^{m,n2}\omega_{pe}^2/\omega_j^3\rangle$, $\langle F(\omega_j, x)\rangle = \int \phi_j(x)^* F(\omega_j, x)\phi_j(x)dx$ represents the operator F averaging over the localization region of respective mode $j = m, n$, $u_m = 2(q_z^m + k_z)\Lambda_{mz}$, $u_n = 2q_z^n\Lambda_{nz}$ are the longitudinal group velocities of the UH wave modes. Fig. 1 demonstrates for typical conditions of the EC-heating experiment at tokamak TEXTOR [1] the dependence of radial wavenumber of the UH wave mode $m = 4$ on the respective coordinate, which was obtained in the WKB approximation ($f_m = 70.09$ GHz, $q_z^{m,n} = 0.21\,\mathrm{cm}^{-1}$) (solid curve); the dashed line represents the sum of wavenumbers of the pump wave at frequency $f_0 = 140$ GHz for the case when it has in the decay region $k_y = 0.1\,\mathrm{cm}^{-1}$, $k_z = 0$, and of the UH wave mode $n = 11$ ($f_n = 69.91$ GHz). In the vicinity of the curves' intersection point, the decay condition is met, and the UH wave eigen modes are generated nonlinearly. The bold solid line represents the UH resonance frequency versus the coordinate. The electron and ion temperatures on the discharge axis and magnetic field in the magnetic island are $T_{e0} = 700$ eV, $T_{i0} = 400$ eV and $B_0 = 1.9$ T, respectively. For the exponentially increasing solutions $a_{m,n} \propto \exp(\gamma t + iS(x, y))$, from equation set (3) we obtain the Hamilton–Jacobi equation [2]:

$$\left(\gamma - i\Lambda_{my}\left(\frac{\partial S}{\partial y}\right)^2 - i\Lambda_{mz}\left(\frac{\partial S}{\partial z}\right)^2 + iu_m\frac{\partial S}{\partial z}\right) \times \left(\gamma + i\Lambda_{ny}\left(\frac{\partial S}{\partial y}\right)^2 + i\Lambda_{nz}\left(\frac{\partial S}{\partial z}\right)^2 - iu_n\frac{\partial S}{\partial z}\right) = \gamma_0^2 \exp\left(-\frac{y^2}{w^2} - \frac{z^2}{w^2}\right), \tag{4}$$

which may be analyzed numerically by the ray-tracing method. Fig. 2 demonstrates the trajectory of one of the daughter waves for $P_0 = 600$ kW and $w = 1$ cm at $k_y = 0.1\,\mathrm{cm}^{-1}$, $k_z = 0$. It corresponds to eigenmodes $p = 1$ and $s = 0$ along axes y and z. One can see that, in addition

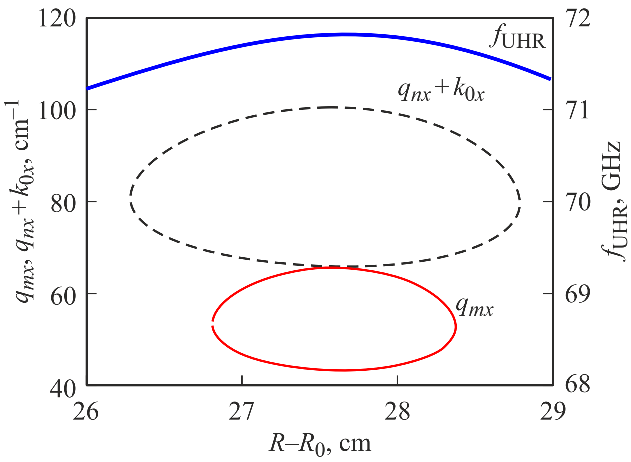

Figure 1. Dependence of the radial wavenumber of the UH wave mode $m = 4$ ($f_m = 70.09$ GHz, $q_z^{m,n} = 0.21\,\mathrm{cm}^{-1}$) on the respective coordinate (solid line). The dashed line represents the sum of wavenumbers of the pump wave ($f_0 = 140$ GHz, $k_y = 0.1\,\mathrm{cm}^{-1}$, $k_z = 0$) and UH-wave mode $n = 11$ ($f_n = 69.91$ GHz). The bold solid line is the UH resonance profile. The electron and ion temperatures on the discharge axis and magnetic field in the magnetic island are $T_{e0} = 700$ eV, $T_{i0} = 400$ eV and $B_0 = 1.9$ T, respectively.

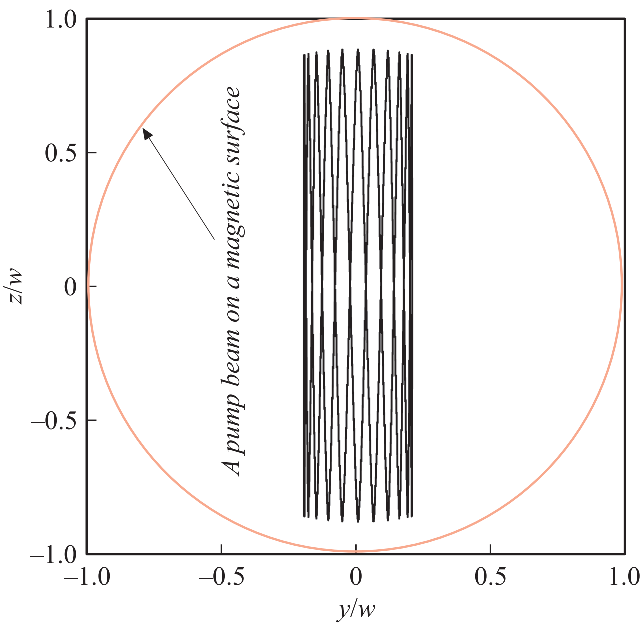

Figure 2. The UH wave trajectory which is the equation (4) solution and relates to fundamental eigen modes $p = 1$ and $s = 0$ along axes y and z. $P_0 = 600$ kW, $w = 1$ cm. Conditions under which $\Lambda_{my} = 5.13 \cdot 10^5\,\mathrm{cm}^2/\mathrm{s}$, $\Lambda_{mz} = 4.3 \cdot 10^7\,\mathrm{cm}^2/\mathrm{s}$, $u_m = 1.1 \cdot 10^6$ cm/s, $\gamma_0 = 1.9 \cdot 10^8\,\mathrm{s}^{-1}$ are the same as in Fig. 1. The arrow points to the UH wave beam within which the trajectory is localized on the magnetic surface.

to localization along axis x due to non-monotonicity of the density profile, UH waves are also localized on the magnetic surface within the pump beam (indicated with arrow) due to a nonlinear additive to the plasma electronic susceptibility. An important simplifying factor for the parameters used in calculation ($\Lambda_{my} = 5.13 \cdot 10^5\,\mathrm{cm}^2/\mathrm{s}$, $u_m = 1.1 \cdot 10^6\,\mathrm{cm/s}$, $\Lambda_{mz} = 4.3 \cdot 10^7\,\mathrm{cm}^2/\mathrm{s}$, $\gamma_0 = 1.9 \cdot 10^8\,\mathrm{s}^{-1}$, see Fig. 1) is a great difference between the frequencies of beam oscillations along the magnetic field and across it, which is caused by domination of the diffraction losses from the decay region along axis z, e. g. when $\Lambda_{my} \ll u_m w \ll \Lambda_{mz}$. This inequality allows applying the adiabatic invariant preservation law and reducing the problem to a quasi-one-dimensional one. Using the quantization procedure for $Q_z = \partial S/\partial z$, obtain the PDI excitation threshold for mode $s = 0$ along axis z in the following form [2]:

$$|\gamma_0(0, P_0^{th})|^2 = \left(3\sqrt{\pi}\Gamma(3/4)/\Gamma(1/4)\right)^{4/3}\Lambda_{mz}\Lambda_{nz}/w^4.$$

Since $\gamma_0 \propto \sqrt{P_0/w^2}$, the threshold pump power increases with decreasing pump-beam radius. However, since the beam radius decrease is restricted by the diffraction limit, this method for its stabilization appears ineffective. Notice that convective losses and diffraction of UH waves across the magnetic field do not affect the instability threshold because of low magnitude. For the same reason, the oblique pump wave propagation along axis y also does not significantly affect the instability threshold.

Consider the effect on the PDI threshold of the pump wave oblique propagation along the magnetic field. When the pump beam tilt angle is sufficiently large, $k_z w \gg 1$, the lowest threshold characterizes the decay process in which diffraction losses dominate for one of the UH wave modes and convective losses with rate $u_m \approx 2k_z\Lambda_{mz}$ are predominant for another mode. In this case $|\gamma_0(0, P_0^{th})|^2 \sim 2k_z\Lambda_{mz}\Lambda_{nz}/w^3$, and dependence of instability threshold P_0^{th} on k_z and angle $\vartheta = \arctan(n_z/\sqrt{1 - n_z^2}) \approx n_z$, $n_z = k_z c/\omega_0$, may be expected. Notice that this angle is not identical to the angle of heating radiation incidence onto plasma but is close to it. Then, assuming the periodical character of boundary conditions, $w = 1\,\mathrm{cm}$, and initial thermal-noise level of daughter waves, let us solve numerically the set of partial-differential equations (3). Every time when $k_z \neq 0$, coefficients of equations (3) are to be recalculated. Fig. 3 demonstrates the solution result that confirms linearity of the instability-threshold dependence on the pump incidence angle (dashed line pointed to with the arrow). When the pump incidence angles are about 20°, the instability threshold appears to be approximately 1 MW; this suggests the possibility of suppressing low-threshold two-plasmon PDI in modern experiments on EC plasma heating in which the heating beam involves radiation from a single gyrotron; however, this issue remains open in the case of the ITER experiment at the total power of about 60 MW.

Thus, we have analyzed the effect of an increase in the pump wave incidence angle relative to the magnetic field upon the threshold of the two-plasmon parametric decay in experiments on EC plasma heating with an extraordinary wave at the second resonance harmonic. The possibility of suppressing the instability via this effect at pump powers achievable in modern experiments in toroidal magnetic traps has been demonstrated. Since the angles necessary for the suppression are small, it is possible to hope that localization of the region of pump wave cyclotron absorption will not change. Therefore, it may be recommended that, in order to decrease the abnormal absorption, a relatively small heating beam tilt relative to magnetic field is used in experiments on additional EC-resonance plasma heating.

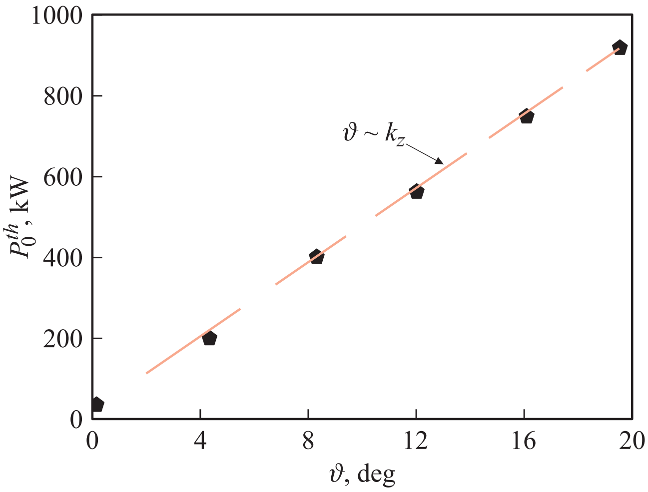

Figure 3. Instability threshold versus the antenna tilt angle. Symbols represent the results of the equation (3) numerical solution. The dashed line represents the linear dependence of the threshold on the tilt angle.

Financial support

Analytical investigation was supported by the Russian Scientific Foundation grant 22-12-00010; numerical simulation was supported by the Ioffe Institute State Contract 0040-2019-0023; the program code for PDI modeling was developed under the Ioffe Institute State Contract 0034-2021-0002.

Conflict of interests

The authors declare that they have no conflict of interests.

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Translated by Solonitsyna Anna