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On the Modeling of Equivalent Transfer Function and Impulse Response of the Fourier-Holography Scheme under the High-frequency Holograms

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An approach for modeling the transfer function and approximating the impulse response in the +1 diffraction order of the $4f$ holography scheme of the Fourier holography under usage of high-frequency holograms characterized by the presence of an inverse section of the dependence of the local diffraction efficiency on the spatial frequency in the frequency range below the frequency of equality of local amplitudes of the reference spectrum and the reference beam by the model of „Difference of Gaussians" is proposed and justified. The expediency of using, when implementing processing models that involve working only with the global maximum of the circuit response, a transfer function that is equivalent in terms of the minimum mean square error of the impulse response, as providing a more accurate approximation of the impulse response compared to the approximation of the direct transfer function, is shown. The validity of the approach is confirmed by comparing the simulation results with experimental data

Keywords: Holography, Fourier-transform, Transfer Function, Impulse Response, Holographic Recording Media, Exposure Characteristic.

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Introduction

When describing and modeling devices as part of the linear systems approximation, two characteristics related to the Fourier transform are traditionally used: the transfer function and the impulse response. In relation to the classical $4f$ Fourier holography scheme with a flat off-axis reference beam (Fig. 1), as the first of them, which will be denoted as $H^P(\nu)$, in practice, the dependence of the local diffraction efficiency (DE) in amplitude $\eta(\nu)$ in the required diffraction order p on the spatial one is used frequency ν, associated with the spatial coordinate in the hologram plane x by the expression $x = \lambda f \nu$, where λ — wavelength, f — focal length of the first Fourier transform lens L_1. As part of this article, we are only interested in the diffraction order coinciding with the direction of propagation of the reference beam +1, in which a field is formed in the output plane Out, described by the mutual correlation function of the object (presented in the input plane In) and the reference fields, therefore, we omit the indication of the diffraction order p and the transfer function of the Fourier holography scheme Fig. 1 for the +1-th order of diffraction, we represent in form

$$H(\nu) = \eta(\nu) = \hat{H}S(\nu), \tag{1}$$

where $S(\nu)$ — spatial-frequency spectrum of reference field amplitudes, $\hat{H}$ — operator taking into account the conditions of hologram recording (frequency of equality of local amplitudes of the signal and reference beams ν_0 and exposure characteristics of the holographic recording medium (EC HRM). Here and below, where possible without compromising the adequacy of the description, to reduce the size of expressions we use the assumption of separability of variables and, accordingly, notation with one-variable functions.

Impulse response of a linear system $h(\xi)$, where ξ — coordinate in the output plane, defined as the response of the system to the delta function — point source in optics — related to the transfer function by the Fourier transform [1]. In holographic practice, the response of the circuit in Fig. 1 is used, in +1-th diffraction order in the output plane Out when a reference field is presented in the input plane In — autocorrelation function (ACF) of the standard, taking into account additional filtering on the hologram due to nonlinearity of the EC HRM, processing mode (development) and hologram recording conditions, first of all — the choice of the frequency of equality of the local amplitudes of the reference spectrum and the flat reference beam [2–12], as well as the choice of the operating section of the dynamic range EC HRM

$$\mathscr{R}(\xi) = \hat{F}\big(S(\nu)\hat{H}\big(S(\nu)\big)\big), \tag{2}$$

where ξ — coordinate in the back focal plane of the second Fourier transform lens L_2 of the output plane of the circuit Out, $\hat{F}$ — Fourier transform operator. This response is a diffraction-limited image of a point reference source δ used when recording the hologram, i.e. represents the point blur function. This definition of the impulse response, as well as the transfer function (1), is specific to holography

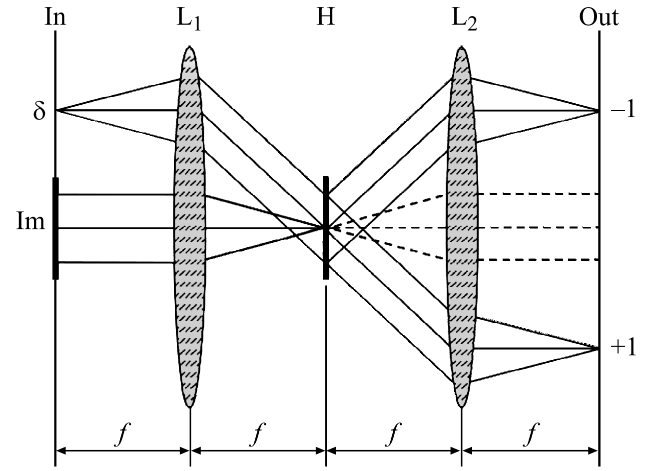

Figure 1. $4f$-Fourier holography scheme with a flat off-axis reference beam: In and Out — input and output planes, respectively; L_1, L_2 — Fourier transform lenses with focal lengths f, H — hologram, Im and δ — image and off-axis point reference source respectively, $+1$-th and -1-th — corresponding diffraction orders.

and differs from its classical definition as a response to an impulse (a point source in optics) at the input of the system, since when a point source is presented in the input plane In, the holographic circuit will reconstruct in the output plane Out in $+1$ the diffraction order is not the diffraction-limited image of the point represented by the global maximum of the ACF (2), but the reference field used to record the Fourier hologram.

Spatial-frequency spectra with a power-law decay [13,14], characteristic of real information, taking into account the nonlinearity of the EC HRM [3–5] and the hologram form factor [10] are often inconvenient with from the point of view of simplicity and clarity of the analytical description. Therefore, a number of papers [3,7–9,15,16] are devoted to the issue of approximation of these characteristics of the Fourier holography circuit, in which the approximation model is consistent with the processing model, in particular, with the measured response characteristic of the circuit — its power, signal-to-noise ratio and/or correlation radius, which is a fundamental information characteristic of the processed field [17,18]. In a number of tasks, for example, when implementing information processing models based on fuzzy-valued logic [19–21], in the output plane Out it is the radius of the correlation response (2) that is measured at a given level β (symbol α in this article, as in [16], is used to denote the level of DE measurement and the transfer function $H(\nu)$ in the hologram plane).

An important aspect of the problem of approximating experimentally measured characteristics (1) and (2) — is the convenience of the approximation model, both in terms of simplicity and clarity, and in terms of minimizing the requirements for computing resources in relation to digital holography methods [22–24]. The article [16] shows the opportunity of approximating the transfer characteristic of the circuit in Fig. 1 with exponential functions for the case of recording low-frequency holograms, i.e., holograms in which the frequency of the local amplitudes of the signal and reference beams is equal to $\nu_0 = 0$, since the amplitude of the reference beam is equal to or greater than the value of the amplitude spectrum of the standard at zero spatial frequency. As a result, in such holograms the maximum diffraction efficiency (DE), which is also the maximum transfer function (1), is localized at zero spatial frequency.

If the frequency of equality of the local amplitudes of the reference beam and the amplitude spectrum ν_0 is different from zero, then the maximum of the transfer function $H(\nu)$ is localized in the area of this frequency ν_0, and in the frequency range $\nu < \nu_0$ there is usually an inverse dependence of the DE on the frequency with respect to the standard spectrum $\eta(\nu)$: $\frac{1}{S(\nu)}$. For such holograms, called high-frequency ones, the paper [15] proposed an approximation by the sum of two symmetrically shifted Gaussian functions, the magnitude of the function shift corresponds to the frequency of the maximum DE of the hologram.

Approach (15) is convenient from the point of view of analytical description and numerical modeling, but has a number of practically significant limitations due to the insufficient flexibility of the model, since it has only two parameters for matching with experimental data $\eta(\nu)$: frequency of equality of local amplitudes of the signal and reference beams ν_0 and Gaussian function parameter $\nu_{0.606}$. In particular, this approach does not always allow to approximate adequately the transfer function in two important areas of the frequency range: the inverse dependence of the DE in the low-frequency range and the high-frequency one.

A significant „dip“ of DE in the area of low frequencies, up to their rejection, arises due to the limited dynamic range of the EC HRM [6–8] when choosing a sufficiently high frequency of equality of the local amplitudes of the signal and reference beams ν_0 [2,9]. Taking into account the inverse section of the $\eta(\nu)$ dependence in the model is important both in the case of implementing dynamic models of information processing, i.e. for holographic circuits of resonance architecture, since a strong attenuation of low spatial frequencies leads to a change in the type of system dynamics from convergent to intermittent mode or to a restructuring of stable type of solution [20,21], and in terms of changes in the ACF model (impulse response of the system) — growth and subsequent dominance of side maxima [21].

The high-frequency range of the $\eta(\nu)$ dependence should be taken into account in the model, since it makes the main contribution to the formation of the global maximum of the autocorrelation function. The course of the $\eta(\nu)$ dependence in this area is determined not only by the reference spectrum, but also by the modes of recording the hologram and displaying the HRM. In particular, when recording high-frequency holograms under the condition $\nu_0 \gg 0$, the high-frequency components of the spectrum fall into the range of the exposure characteristics of the HRM with the maximum transmission coefficient, as a result, the specific gravity of the high-frequency components recorded on the hologram relative to the reference spectrum increases.

The type of dependence $\eta(\nu)$ is also significantly influenced by the choice of the working section of the dynamic range of the EC HRM. In particular, if the local exposure at a frequency equal to the local amplitudes of the reference spectrum and the reference beam ν_0 corresponds to the saturation sub-range of the EC HRM, then in a certain vicinity of ν_0 the dependence $\eta(\nu)$ flattens — instead of a clearly pronounced maximum $\eta(\nu_0)$ there is a plateau, width which is determined by local overexposure.

Therefore, adequate approximation of the transfer function of the Fourier holography scheme with one convenient function for all hologram recording conditions (options for choosing the ν_0 value and manifestation) is extremely problematic. But there is a number of tasks in which only the global maximum of the response and its shape are important. For example, when implementing fuzzy-valued logics, the measured characteristic of the response is the correlation radius at a given level [19]. For such a limited class of tasks, in this article, in development of the papers [15,16], an approach to modeling the transfer function of a circuit with strong, even rejection, attenuation of low spatial frequencies on a hologram, is proposed and justified by the DOG function (Difference of Gaussians), widely used in a number of practical applications: wavelet analysis [25], digital image processing [26], holographic particle capture schemes [27], and etc.

1. Approach and approximation model

Let us represent the transfer characteristic (1) of the circuit in Fig. 1 by the difference of two exponential functions

$$H(\nu) = H_1(\nu) - H_2(\nu) = a_1 \exp\left(-\ln(\alpha)\left(\frac{\nu}{\nu_1}\right)^{D_1}\right) - a_2 \exp\left(-\ln(\alpha)\left(\frac{\nu}{\nu_2}\right)^{D_2}\right), \quad (3)$$

where D_1 and D_2 — exponents, ν_1 and ν_2 — model parameters — frequencies measured by level

$$\alpha_i = \frac{H_i(\nu_\alpha)}{H_i(0)}.$$

The approximating function (AF) (3) is the development of a transfer characteristic suitable for approximation in the case of low-frequency holograms of the model [16] by adding a second term (subtracted), required to represent the inverse section of the dependence of DE on frequency in the range $[0, \nu_0]$. The model (3) includes six parameters, and to find their values, both the numerical approach presented in [15] and the analytical one can be used. To analytically find the parameters, we impose the following conditions on the approximating function (AF) (3) in terms of matching $H(\nu)$ with experimental data $\eta(\nu)$:

A) non-negativity in the approximation range

$$\nu \in [0, \nu_{\max}]: \quad H(\nu) \geq 0, \quad (4)$$

where $\nu_{\max}$ — determined by the EC HRM or the aperture in the Fourier plane H is the upper bound of the frequency range recorded on the hologram;

B) localization:

B.1.) of maximum transfer characteristic

$$\frac{dH(\nu_0)}{d\nu} = 0, \quad (5)$$

where ν_0 — frequency of localization of the DE dependence maximum on frequency $\eta(\nu)$, in the first approximation, sufficient for the approximation task, coinciding with the equality frequency of the local amplitudes of the reference and signal beams when recording a hologram [3,7];

B.2) and the minimum DE at zero frequency

$$\frac{d^2H(0)}{d\nu^2} > 0; \quad (6)$$

C) normalization of the maximum transfer characteristic at the maximum frequency DE ν_0:

$$\max(H(\nu_0)) = \eta(\nu_0) = 1; \quad (7)$$

D) for the coincidence of the values of AF (3) and the experimentally measured DE at the point $\nu = 0$

$$H(0) = \eta(0) = a_1 - a_2; \quad (8)$$

E) intersection of the experimental dependence of DE and AF at a given level α

$$H(\nu_\alpha) = \eta(\nu_\alpha). \quad (9)$$

We are interested in the option of strong attenuation of low frequencies, i.e., taking into account in the model the inverse decline of DE in the $\nu < \nu_0$ area, so selection of $\alpha > \eta(0)$ makes sense. This implies the requirement that condition (9) be satisfied at two frequencies: $\nu_{\alpha Inv} < \nu_0$ in the inverse and $\nu_{\alpha Match} > \nu_0$ ranges consistent (to some approximation) with the spectrum of the standard $S(\nu)$ depending on DE versus frequency $\eta(\nu)$.

As a result, we have 7 conditions, i.e. they are redundant, since AF (3) includes only 6 parameters. At the same time, in analytical modeling, function (3) in its general form gives very lengthy and inconvenient expressions. Therefore, the „convenient" option $\alpha = 0.606$ and $D = 2$ is of practical interest, i.e. the DOG function

$$H(\nu) = \mathscr{J}_1(\nu) - \mathscr{J}_2(\nu) = a_1 \exp\left(-0.5\left(\frac{\nu}{\nu_1}\right)^2\right) - a_2 \exp\left(-0.5\left(\frac{\nu}{\nu_2}\right)^2\right). \quad (10)$$

The DOG function is attractive both from the point of view of reducing the dimension of the task (four parameters), which is important when using numerical methods, and in terms of analytically determining approximate parameter values, which, if required, can then be refined or corrected numerically.

Since $\nu_{\alpha Inv} < \nu_{\alpha Match}$, and from the property of non-negativity DE (4) and condition (8) follows $a_1 > a_2$, then analysis of the roots of the equation solved with respect to the parameter ν_1 when substituting the DOG function (10) into condition (7) shows that in approximation, acceptable taking into account the usual requirements for the accuracy of analogue methods, the matched AF section in a certain neighborhood α can be described only by the first term (10), i.e.

$$\alpha \approx a_1 \exp\left(-0.5\left(\frac{\nu_{\alpha Match}}{\nu_1}\right)^2\right).$$

This implies an approximate estimate of the parameter of the first term in AF (10)

$$\nu_1 \approx \nu_{\alpha Match}\sqrt{\frac{1}{2\big(\ln(a_1)-\ln(\alpha)\big)}}\Bigg|_{\alpha=0.606} = \nu_{0.606 Match}\sqrt{\frac{1}{1+2\ln(a_1)}}, \tag{11}$$

from which we obtain an approximate estimate of the value of the parameter a_1

$$a_1 \approx \sqrt[1-\frac{\nu_0^2}{\nu_{\alpha Match}^2}]{\exp\left(0.5\,\frac{\nu_0^2}{\nu_{\alpha Match}^2}\right)}. \tag{12}$$

The value of the a_2 parameter is found by substituting (12) into condition (8).

The value of the parameter ν_2 is obtained from condition (9) for $\nu_{\alpha Inv}$

$$\nu_2 = \frac{\nu_{0\,Inv}}{\sqrt{-2\ln\left(\frac{a_1}{a_2}\exp\left(-0.5\,\frac{\nu_{0\,Inv}^2}{\nu_1^2}\right)-\frac{\alpha}{a_2}\right)}}. \tag{13}$$

Thus, we have approximate values of all four parameters AF (10) for approximating the dependence $\eta(\nu)$ in the range from zero to some frequency greater than ν_0.

But it is quite obvious that this approach does not allow us to fully reflect the features of recording and processing holograms taking into account EC HRM — the course of the $\eta(\nu)$ dependence both in the high-frequency range and in the frequency region where the local amplitudes of the reference spectrum and the reference beam are equal ν_0 when recording a hologram in overexposure mode. These ranges of the $\eta(\nu)$ dependence are not presented separately as part of the DOG (10) model; there are no tools for adjusting AF (10) for approximation in these ranges in the model. The traditional approach to approximation using the integral criterion of the minimum mean-square error (MSE) does not solve the problem of representing the high-frequency range, since due to the small proportion of high-frequency components they make an insignificant contribution to the value of the MSE. But it is impossible to ignore the high-frequency part of the dependence of DE on frequency, since it is precisely this that dominates in the formation of the global maximum of the ACF, primarily — its peak and the practical values of the response radius measurement level β. As a result, using model (10) as presented can lead to an inadequate model of the response of the holographic circuit, and ultimately it is the response of the circuit that is of interest.

To solve this problem, it is meaningful to use the approximation criterion not in the Fourier space H (hologram plane), but in the response space — at the output plane of the circuit Out. Accordingly, the task of finding the transfer function no longer appears as an approximation, but as a simulation — finding a function equivalent to the transfer function according to a given criterion, for example, the criterion of the minimum error of the global maximum impulse response and/or GM ACF, and/or the response radius according to required level β.

Impulse response of a circuit as a Fourier transform of the approximating function (10)

$$h(\xi) = \hat{F}\big(H(\nu)\big) = \hat{F}\left(a_1\exp\left(-\frac{1}{2}\left(\frac{\nu}{\nu_1}\right)^2\right) - a_2\exp\left(-\frac{1}{2}\left(\frac{\nu}{\nu_2}\right)^2\right)\right) = a_1\exp\left(-\frac{1}{2}\left(\frac{\xi}{\xi_1}\right)^2\right) - a_2\exp\left(-\frac{1}{2}\left(\frac{\xi}{\xi_2}\right)^2\right), \tag{14}$$

where the parameters

$$\xi_1 = \frac{1}{2\pi\nu_1} \quad \text{and} \quad \xi_2 = \frac{1}{2\pi\nu_2}$$

are measured at the level $\beta = 0.606$ [28].

The presence in the response space of only one criterion — minimum RMS or response radius error at a given leve β — does not allow solving the four-parameter optimization problem analytically. Taking into account normalization (7), the problem can be reduced to a three-parameter one by adopting $a_1 = 1$, but this does not solve the problem of analytically finding the parameters. A transition to numerical techniques is required.

2. Numerical simulation and experimental verification

To check the validity of the presented approach to simulation of the equivalent transfer function, determined

Mean-square errors in approximation of the DE dependence on frequency and the global maximum of the impulse response

Hologram	RMS of DE approximation		RMS of GM IE approximation	
	Analytically	Numerically	Analytically	Numerically
I	0.164	0.259	0.208	$3.75 \cdot 10^{-3}$
II	0.249	0.087	$1.8 \cdot 10^{-2}$	$1.027 \cdot 10^{-3}$

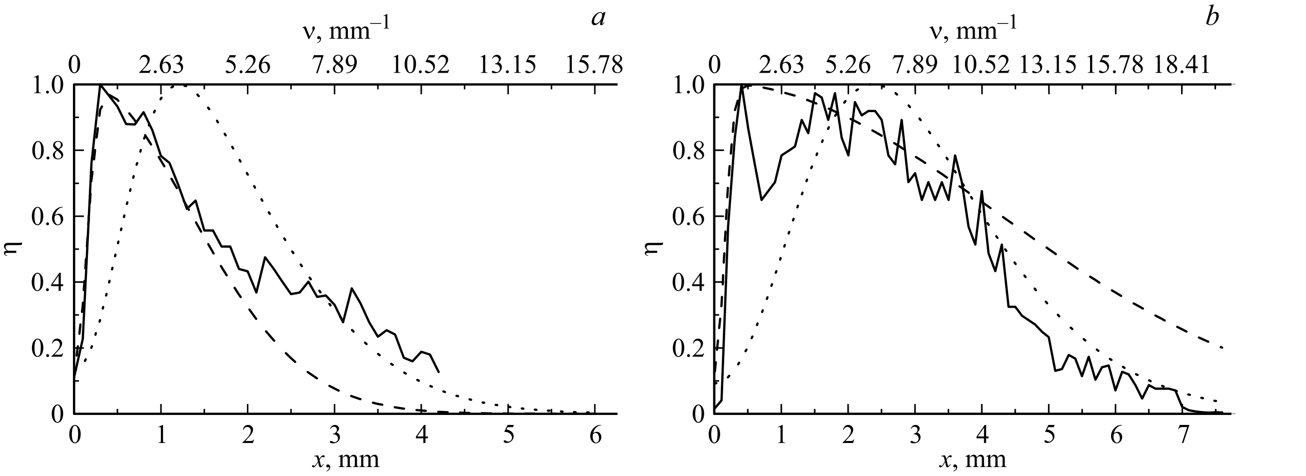

Figure 2. Dependences of the normalized local diffraction efficiency η of holograms I (a) and II (b) on the spatial coordinate x and spatial frequency ν: solid line — experimental data, dashed line — analytically found approximating DOG function (10), dotted line — DOG function (10), equivalent in terms of the minimum error criterion to the global maximum of the impulse response (14).

by the criterion of minimum RMS for the GM response in the output (correlation) plane, Fourier holograms were used, recorded under different conditions from a reference image representing the implementation of two-dimensional fractal Brownian agitation with a power law spectrum [29]

$$S(\nu) = \nu^{2H+1},$$

where $H \in [0, 1]$ — Hurst exponent. The holograms were recorded on a PFG-03m HRM (JSC Slavich Company), developer GP-8, at values of frequency equal to the local amplitudes of the signal and reference beams ν_0, ensuring the presence of a clearly expressed section of the inverse dependence of the DE in the low-frequency range $\nu < \nu_0$. As an example, in Figs. 2, a and b for two characteristic cases of recording holograms, experimental dependences of the DE on the spatial coordinate x (lower abscissa axis) are given, associated with that presented on the upper axis abscissa spatial frequency ν by $x = \lambda f \nu$, where $\lambda = 633$ nm — wavelength, $f = 600$ mm — focal length of the first Fourier transform lens L_1, as well as functions of the DOG model (10): approximating and modeling, equivalent in terms of the minimum error criterion to the global maximum of the impulse response (14).

Hologram I was recorded when the local exposure at frequency ν_0 was matched with the upper limit of the operating section of the dynamic range of the EC HRM — there is a pronounced maximum of the dependence $\eta(\nu)$ in the area ν_0, while in the area of high spatial frequencies (approximately $\nu > 3.5\,\mathrm{mm}^{-1}$) there is a clear inadequacy approximation of the $\eta(\nu)$ dependence by the DOG (10) model.

Hologram II was recorded with overexposure — due to the choice of local exposure at frequency ν_0, corresponding to the saturation range of the EC HRM; in the area ν_0 there is not a maximum of the dependence $\eta(\nu)$, but a kind of plateau. As a result of overexposure in area ν_0:

— „DE dip“ in the area of zero frequencies is deeper than that of hologram I;

— the high-frequency range is significantly wider than that of hologram I, since it fell into the working section of the dynamic range of EC HRM, while the high-frequency part is quite adequately approximated by the DOG function (10).

Figure 3, a, b shows the impulse responses of the circuit in Fig. 1 for these holograms; the designations correspond to those in Fig. 2. Response modules are given, since only their powers are actually measured.

The table shows the RMS of the DE measured dependence on the frequency $\eta(\nu)$ and the global maximum of the experimental impulse response (IR): by the analytical model (10) and numerically according to the criterion of the RMS (14) for these holograms.

Figure 4 shows the absolute values of the relative errors in approximation of the response radius depending on the β level. For hologram I, the dependence of the relative

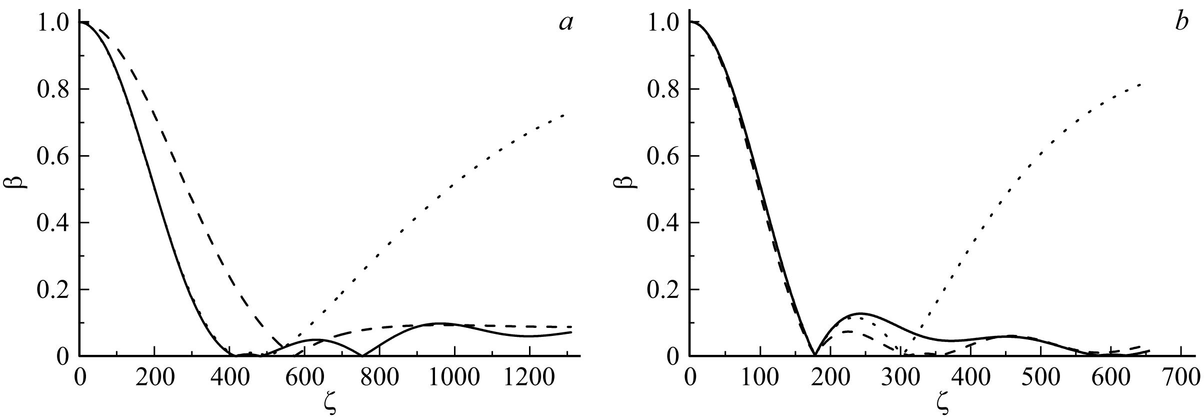

Figure 3. Impulse responses (normalized values) of the circuit in Fig. 1 for the holograms I (a) and II (b) shown in Fig. 2, ξ — coordinate in the correlation plane: solid line — for experimental data, dashed line — for the analytically found approximating dependence $\eta(\nu)$ of the DOG function (10), dotted line — DOG function (14), satisfying the criterion of minimum error of the global maximum impulse response.

error is given only for the numerical selection of the equivalent function, since in the analytical approximation of the transfer function the relative error, as is clearly seen from Fig. 3, a, is clearly large — its values are in the range $0.4-0.67$.

Similar results were obtained for other high-frequency holograms — in all cases, the numerical selection of the parameters of the function (14) gave a better approximation of the impulse response of the circuit, compared with the approximation of the transfer function itself by the DOG (10) model.

Conclusion

Thus, in case of high-frequency Fourier holograms, characterized by the presence of a clearly expressed section of the inverse dependence of the diffraction efficiency on the spatial frequency in the frequency range below the frequencies of equality of local amplitudes of the reference spectrum and the reference beam, simulation of the equivalent in terms of the minimum error criterion for approximating the impulse response of the transfer function circuit with the DOG function gives an adequate model for the global maximum of the impulse response, taking into account the generally accepted requirements for analog processing error of maximum 0.1. The advantage of this approach — is the simplicity and clarity of the analytical description of the holographic circuit, which is practically important from the point of view of reducing the requirements for computing power and processor memory in computer implementations, and the use of discrete space-time modulators and sensors [22–24].

Fig. 3, a, b clearly demonstrate that the presented approach to approximating the impulse response by simulation of the equivalent transfer function is valid only within the limits of the global maximum of the impulse response.

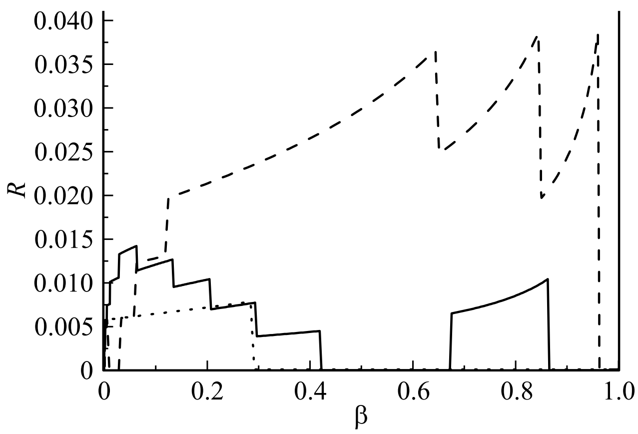

Figure 4. Dependences of the absolute values of the relative error R of the radius approximation of the impulse response within the global maximum on its level β: solid line — hologram I, numerical selection of parameters of the DOG model (14), dashed line — hologram I , analytical approximation of the transfer function by the DOG (10) model, dotted line — hologram II, numerical selection of the parameters of the DOG (14) model.

Accordingly, the approach is applicable only in information processing problems that involve working exclusively with the global maximum of the circuit response, for example, in tasks of correlation pattern recognition [22–24], including computer methods, implementation of fuzzy logic [21] etc.

Information on the authors contribution

The contribution of A.V. Pavlov is in the formulation of the problem, the formation of an approach, the development of software, the planning and implementation of numerical and natural experiments, the processing of their results, writing the article, the contribution of A.O. Gaugel consists in the participation in numerical simulation.

Conflict of interest

The authors declare that they have no conflict of interest.

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