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Comparison of the laser generation parameters in the coherent and in the standard incoherent passive mode locking regime

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Received April 17, 2023
Revised April 17, 2023
Accepted May 10, 2023

A comparison of the generation efficiency of short pulses with coherent and standard passive (incoherent) mode locking with a saturable absorber is made on the basis of numerical simulations. The advantage of the coherent mode locking in comparison to the standard mode locking in lasers with a saturable absorber is shown. Coherent mode locking in two-section lasers is based on the coherent interaction of the radiation with the absorbing and amplifying media. It allows the generation of ultrashort laser pulses with a duration shorter than the polarisation relaxation time T_2 of the absorbing and amplifying media. In conventional lasers with standard (incoherent) passive mode locking with a saturable absorber, the interaction of the laser pulses with the absorbing and amplifying media is incoherent and the duration of the generated pulses is always limited by the polarization relaxation time T_2 of the absorber and amplifier.

Keywords: passive mode locking, coherent mode locking, self-induced transparency.

DOI: 10.61011/EOS.2023.07.57131.4881-23

Introduction

In two-section lasers containing a section with amplifying and absorbing media, a passive mode locking (PML) behaviour may occur, in which ultrashort pulses are generated with a repetition frequency close to the bypass frequency of a bare resonator [1–3]. Ultrashort laser pulses with durations in the femtosecond and picosecond ranges find numerous applications in the control of ultrafast processes in matter, image processing systems, medicine and other applications [4,5]. In conventional laser systems with a PML (laser with a saturable absorber), the appearance of short pulses occurs due to incoherent saturation of the gain in the amplifying medium and saturation of absorption in the absorbing media [1–5]. Meanwhile, the interaction of laser pulses with the medium is incoherent and the duration of the laser pulses τ cannot be shorter than the relaxation time of the laser media polarization T_2.

The study of methods for producing ultrashort laser pulses already at the beginning of the laser era led to the discovery of the phenomenon of self-induced transparency (SIT) by McCall and Hahn [6–8]. This phenomenon occurs when the interaction of laser radiation with a resonantly absorbing medium is coherent, i.e. the pulse duration is shorter than the relaxation time of the medium polarization T_2. The SIT phenomenon is that a short pulse with a certain initial amplitude and duration can propagate in a resonant environment without loss. An important characteristic of laser pulses in the theory of coherent interaction of radiation with matter is the pulse area $\Theta(z,t)$, defined [6–8] as the integral of the slow pulse envelope $A(z,t)$:

$$\Theta(z,t) = \frac{d_{12}}{\hbar} \int_{-\infty}^{\infty} A(z,t)dt. \tag{1}$$

Here d_{12} —transition dipole moment, $\hbar$ — reduced Planck constant, z — longitudinal coordinate, t — time. The SIT pulse area is 2π. Self-induced transparency — is a rather spectacular phenomenon in nonlinear optics; for a long time it aroused purely academic interest and the opportunity of its practical application for the generation of short pulses was not considered (see the history of the issue in the review [9]).

Subsequently, in papers [10–22] the opportunity of using the SIT phenomenon to obtain the PML behaviour in two-section lasers was theoretically shown. With a twofold difference in the transition dipole moments in the absorbing and amplifying media, 2π SIT pulse is formed in the absorbing medium, and π pulse is formed in the amplifying medium, removing the inversion in the medium, as a result of which the medium becomes absorbing. This PML behaviour, which arises due to the SIT phenomenon in the absorbing medium, is called the coherent mode-locking behaviour (CML, „coherent mode-locking" or „self-induced transparency mode-locking") [10–22]. It was discovered experimentally quite recently in a laser with a coherent absorber [23–26]. Using CML, it is possible to obtain short pulses with a duration much shorter than the polarization relaxation time $T_2$of the amplifying and absorbing media. This behaviour, as shown by the results of theoretical studies, in principle allows to obtain short pulses with a duration of several oscillation cycles, up to one cycle in a laser with an ultrashort resonator [19].

In conventional lasers with PML, which have a saturable absorber, the interaction of pulses with the amplifying and absorbing media is incoherent. And when theoretically analyzing PML behaviours, for example, in semiconductor lasers, the medium polarization is adiabatically excluded from the model equations [27–34]. This leads to the exclusion from consideration of coherent effects in the interaction of generated pulses with an amplifier and absorber. However, as of this date it has become possible to obtain short pulses with a duration of hundreds of femtoseconds, for example, in fiber lasers [35–37] and in semiconductor quantum dot lasers [38–40]. This pulse duration can already be comparable to the polarization relaxation time T_2 in such systems. Therefore, at such pulse durations, in our opinion, it is required to take coherent effects into account when theoretically analyzing PML behaviours in these laser systems. Moreover, obtaining femtosecond pulses (for example, in a titanium-sapphire laser [41]) as of this date requires much more complex installations and additional compression systems [42]. At the same time, the CML behaviour seems attractive for producing single-cycle pulses in compact laser sources [13,14,19].

To our knowledge, for the first time, the opportunity of reducing the duration of pulses in the CML behaviour with an increase in the ratio of the dipole moments of the absorbing and amplifying media was briefly brought to the attention in papers [15–17]. The issues of comparing lasing in the CML behaviour and in the conventional PML behaviour with a saturable absorber for given parameters of the amplifying medium and resonator remained unconsidered.

In this paper, based on a numerical solution of the Maxwell-Bloch system of equations, a comparison is implemented of the opportunities of reducing the generation duration in a laser with a ring resonator in the CML and PML behaviours through the use of a saturable absorber. A comparison has been implemented of the efficiency of generating short pulses in the CML and PML behaviours with a saturable absorber, which shows the advantage of the CML.

Physical mechanism for reducing the pulse duration in a laser due to the CML behaviour

Preface to the ground content, it seems appropriate to once again consider in detail the physical idea on which the reduction of pulse duration in the CML behaviour is based [15–17]. With coherent propagation of ultrashort pulses with resonantly amplifying and absorbing media, the pulse area (1) satisfies the McCall and Hahn area theorem [6–8]. This theorem uses an equation describing the evolution along the pulse area coordinate in a resonance medium with heterogeneous broadening of the resonance transition line:

$$\frac{d}{dz}\,\Theta(z) = |\alpha| \sin\Theta(z), \tag{2}$$

α — linear absorption (gain) coefficient per unit length.

In the papers [16,18,21] a diagrammatic approach was proposed for the theoretical description of the CML behaviour, based on solutions of equation (2), in a two-section laser with a ring resonator. According to this approach, the evolution of the pulse area in the amplifier and absorber is described based on the area theorem. As the results of theoretical studies show, when using this approach, stable limit cycles exist in the system, when an 2π SIT pulse is formed in the absorber, and — π pulse is formed in the amplifier.

The approach allows us to qualitatively predict the dynamics of the system. For example, the branches of the solution to equation (2) of the area theorem for two different initial values of the pulse area $\Theta_0 = 1.1\pi$ and $\Theta_0 = 2.9\pi$ are shown in Fig. 1 (lower curve and upper curve, respectively). According to Fig. 1, if the area of the initial pulse is slightly larger than π or slightly smaller 3π, then the area of the pulse tends to a stationary value 2π as it propagates. This leads to the formation of the SIT behaviour in the absorbing medium. Meanwhile, as the results of numerical calculations show, in the first case (lower branch in Fig. 1) the propagation of a pulse is accompanied by an increase in its duration, since its area increases [6–8].

However, in the second case (upper branch in Fig. 1), the pulse area decreases, which leads to a reduction in its duration [6–8]. This circumstance, the opportunity of reducing the duration of a pulse with an initial area slightly smaller than 3π due to the SIT phenomenon, was discovered experimentally in the paper [43] and theoretically in a laser with CML [15–17].

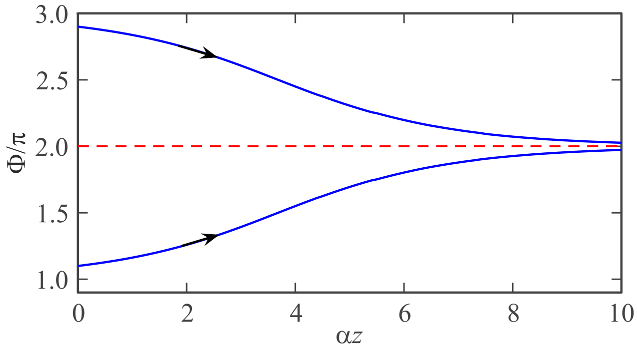

Figure 1. Branches of the solution of the area theorem of equation (2) for two different initial values of the pulse area $\Theta_0 = 1.1\pi$ (lower curve) and $\Theta_0 = 2.9\pi$ (upper curve).

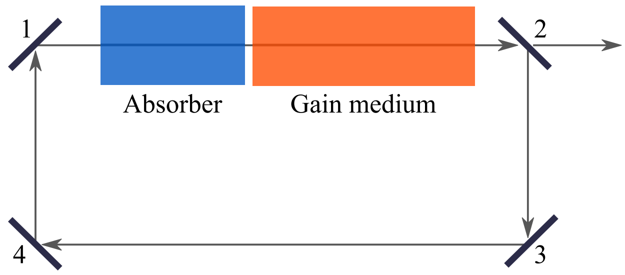

Figure 2. Modeled system. A two-section laser with a ring resonator containing amplifying and absorbing media, in which unidirectional propagation is realized (elements providing this mode are not shown). *1–4* — resonator mirrors.

These considerations can be used to reduce the duration of pulses in lasers with CML, if we take the ratio of the dipole moments of the absorbing and amplifying media to be equal to 3 (or close to this value) [15–17]. Indeed, in this case, when a pulse propagates in an absorbing medium in the SIT behaviour, its area will tend to 2π and its duration will decrease. In an amplifying medium, the area of this pulse will tend to π, and its duration will also decrease. Thus, the combined action of the amplifying and absorbing medium, as will be shown below, can lead to the generation of femtosecond pulses based on the CML behaviour.

The variations of the dipole moment mentioned above seem difficult to realize experimentally. However, in practice, a change in the dipole moment of the absorber is not at all necessary. As was noted in the paper [10], the change in the pulse area is important. This, indeed, can be realized by changing the dipole moment or changing the beam cross section in the absorber, which follows from formula (1).

The last important thing to note. The studied diagrams describe the behavior of the envelope area and do not show the ground feature of the CML. It consists in reducing the pulse duration with increasing generation power. Pulse energy, i.e. the field amplitude increases. But the area remains constant. This means that the duration is reduced. This fact was demonstrated experimentally in papers [25,26].

Theoretical model

Numerical modeling was carried out for a two-section laser containing amplifying and absorbing media placed in a ring resonator in which a unidirectional lasing behaviour is implemented (Fig. 2). Mirrors *1, 3, 4* completely reflect radiation, and mirror *2* has a reflectance different from unity. The amplifying and absorbing media were modeled in a two-level approximation. The transition broadening was assumed to be homogeneous. Numerical calculations were carried out based on solving the Maxwell-Bloch systems of equations for the slowly varying amplitude of the electric field $A(z,t)$, the slowly varying amplitude of the imaginary part of the off-diagonal element of the density matrix of the two-level medium $P_s(z,t)$ and the population difference of the two-level medium (inversion) $n(z,t) = \rho_{11} - \rho_{22}$. This system of equations is as follows [8–21]:

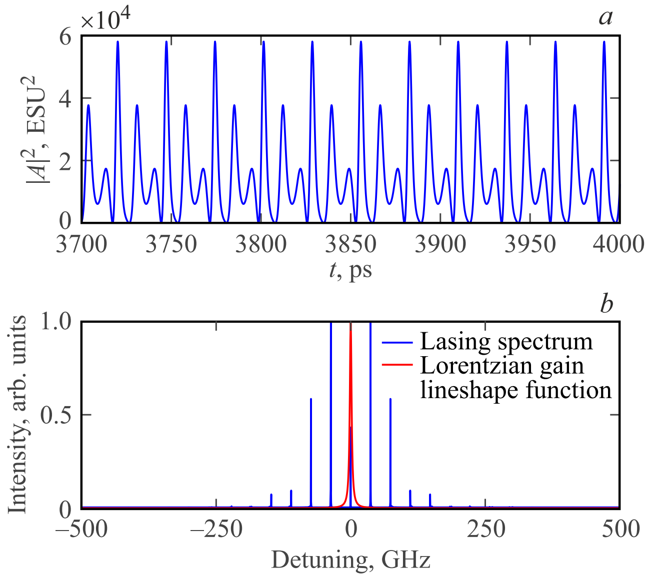

Figure 3. *a* — time dependence of the squared amplitude of the electric field (generation intensity) on time at the exit from the resonator; *b* — generation spectrum (blue curve), Lorentz contour of the active medium gain line (red curve).

$$\frac{\partial}{\partial t} A_{a,g}(z,t) + \frac{\partial}{\partial z} A_{a,g}(z,t) = 4\pi\omega_0 d_{12_{a,g}} N_{0_{a,g}} P_{s_{a,g}}(z,t), \tag{3}$$

$$\frac{d}{dt} P_{s_{a,g}} = -\frac{P_{s_{a,g}}(z,t)}{T_{2_{a,g}}} + \frac{d_{12_{a,g}}}{2\hbar} n_{a,g}(z,t) A_{a,g}(z,t), \tag{4}$$

$$\frac{d}{dt} n_{a,g}(z,t) = -\frac{n_{a,g}(z,t) - n_{0_{a,g}}}{T_{1_{a,g}}} - \frac{2d_{12_{a,g}}}{\hbar} A_{a,g}(z,t) P_{s_{a,g}}(z,t). \tag{5}$$

The system of equations (3)–(5) is written in the approximations of slowly varying amplitudes and a rotating wave. Index a refers to the absorbing medium, index g — to the amplifying one. In this system of equations, $T_{1_{a,g}}$ — relaxation time of the population difference, $T_{2_{a,g}}$ — polarization relaxation time, $d_{12_{a,g}}$ — transition dipole moment,$n_{0_{a,g}}$ — equilibrium value of the population difference ($n_{0_a} = 1$, $n_{0_g} = -1$), $N_{0_{a,g}}$ — particle concentration, ω_0 — resonance transition frequency in the amplifier and absorber. Parameters used in the numerical calculation are given in the table. The same values are taken for the amplifier and absorber, with the exception of the length of the medium (the length of the absorbing medium $L_a = 0.2$ cm, amplifying —$L_g = 0.6$ cm) and the transition dipole moment (the transition dipole moment of the absorber d_{12_a} is 2 times greater than the dipole moment amplifier torque d_{12_g}. The total length of the resonator is

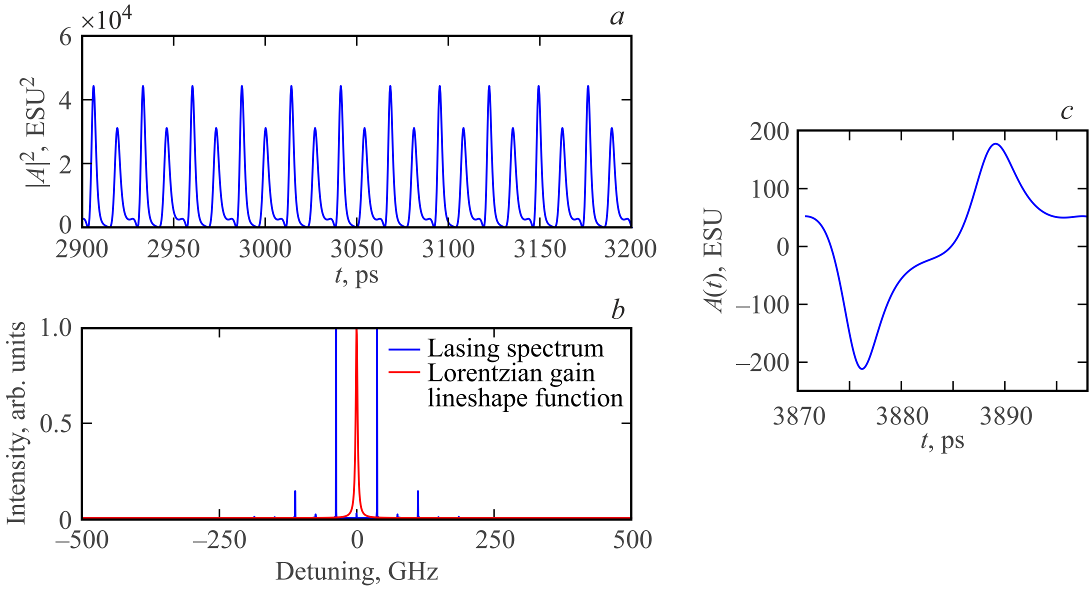

Figure 4. *a* — time dependence of the squared amplitude of the electric field (generation intensity) on time at the exit from the resonator; *b* — generation spectrum (blue curve), Lorentz contour of the active medium gain line (red curve); *c* — single lasing pulse when the dipole moments in the absorber and amplifier are equal to $d_{12_a} = d_{12_g} = 5$D. Other parameters are shown in table.

Parameters used in the numerical calculation

Absorbing medium length, cm	L_a	0.2
Amplifying medium length, cm	L_g	0.6
Total resonator length, cm	L	0.8
Amplitude factor mirror reflections	R	0.8
Resonance transition wavelength amplifier/absorber, nm	λ_0	700
Dipole transition moment amplifier, D	d_{12_g}	5
Dipole transition moment absorber, D	d_{12_a}	10
Inversion relaxation time in amplifying/absorbing medium, ns	$T_{1_{g,a}}$	0.16
Polarization relaxation time in an amplifier/absorber, ps	$T_{2_{a,g}}$	40
Concentration of amplifying and absorbing particles, cm^{-3}	$N_{0_{a,g}}$	$12.5 \cdot 10^{14}$

$L = 0.8$ cm, and then the resonator bypass period T is equal to $L/c = 26.6$ ps.

Results of numerical simulation of the CML behaviour in a single-section laser

For comparison, we first performed a numerical simulation of the dynamics of the CML in a single-section laser in the absence of an absorber. The results of calculating the time dependence of the generation intensity and its spectrum are presented in Fig. 3. It shows that in the steady state, there are periodically repeating multi-pulse structures. The next section shows that to suppress the multi-pulse behaviour in the situation under consideration, it is required to introduce an absorber section.

Results of numerical simulation of generation in a two-section laser

The results of calculating the lasing intensity and its spectrum in a two-section laser are presented in Fig. 4, *a, b* respectively. Numerical calculations were carried out for the case when the dipole moments of the amplifying and absorbing media are the same, $d_{12_a} = d_{12_g} = 5$D. Other parameters are shown in the table. In this case, there is generation of pulses of zero area (0π pulses) the laser. This is also evident from the generation spectrum in Fig. 4, *b*, in which there is practically no component at the resonance transition frequency. The shape of a single 0π pulse is shown in Fig. 4, *c*. It consists of two envelope half-waves of opposite polarity. Let us note that in earlier paper [20], when the dipole moments of the amplifier and absorber were equal, there was generation behaviour of π pulses in both media. Moreover, the calculation parameters differed slightly (the dimensions of the media and the length of the resonator changed). This indicates the sensitivity of the behaviour to variations in the characteristics of the media and the resonator.

Experimentally, there was mode locking behaviour with the formation of 0π pulses under the action of an absorber in a dye laser with a coherently absorbing cell made of molecular iodine vapor [23,24].

When the ratio of the transition dipole moments is from 1 to 3, there is a reduction in the pulse duration in

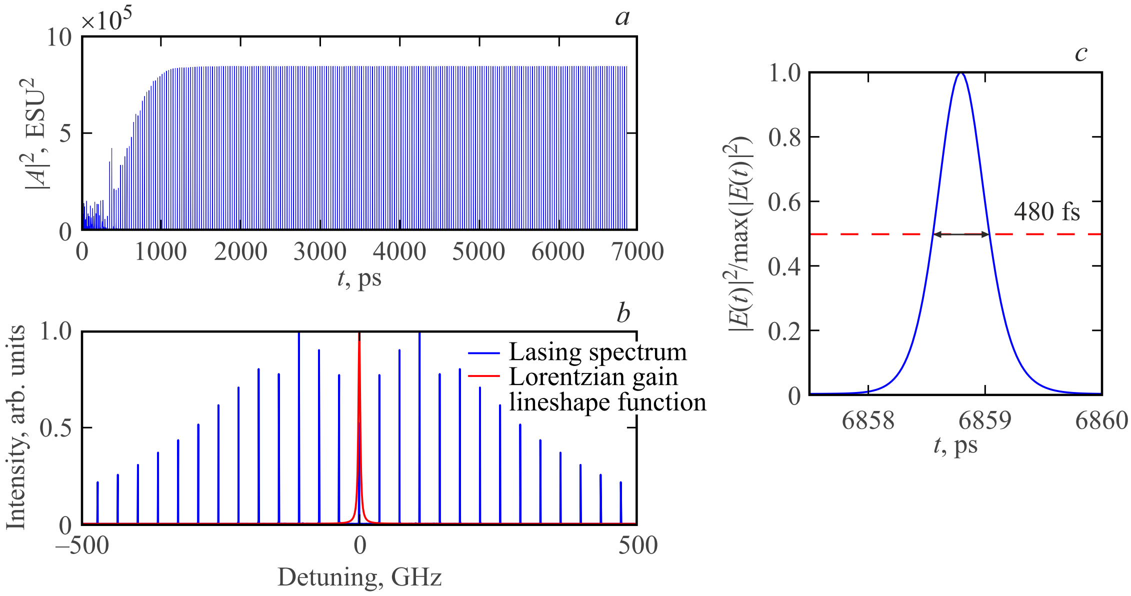

Figure 5. *a* — time dependence of the squared amplitude of the electric field (generation intensity) on time at the exit from the resonator; *b* — generation spectrum (blue curve), Lorentz contour of the gain line of the active medium (red curve); *c* — single lasing pulse when the ratio of dipole moments in the absorber and amplifier is 2.

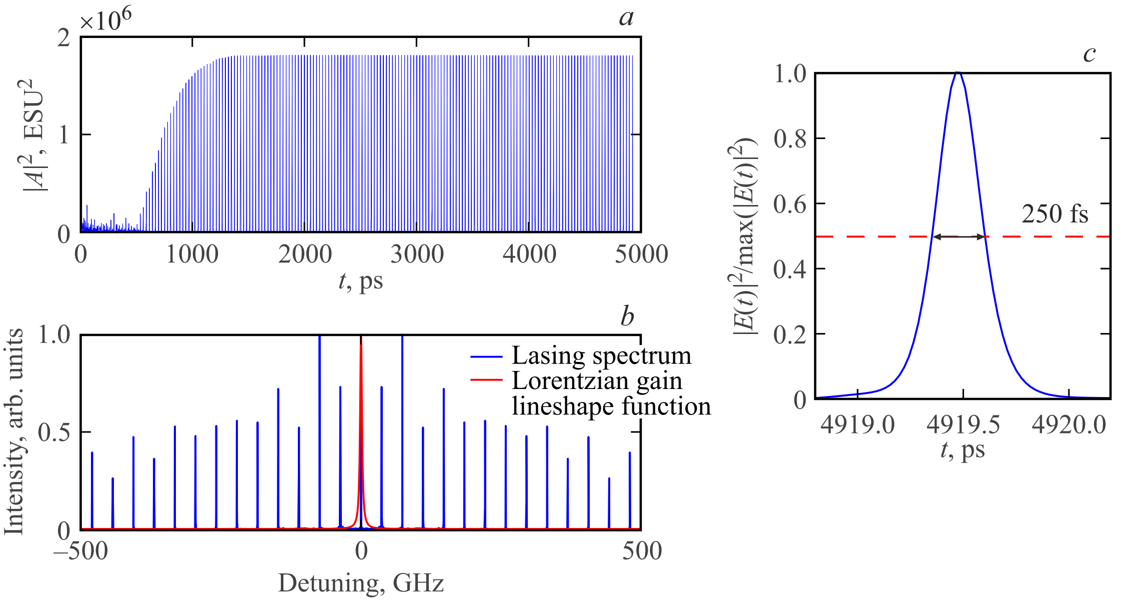

Figure 6. *a* — time dependence of the squared amplitude of the electric field (generation intensity) on time at the exit from the resonator; *b* — generation spectrum (blue curve), Lorentz contour of the gain line of the active medium (red curve); *c* — single lasing pulse when the ratio of dipole moments in the absorber and amplifier is 2.5.

accordance with the mechanism described above, which leads to the production of pulses with a duration of hundreds of femtoseconds [15–17]. Examples of the time dynamics of the lasing intensity, its spectrum and the shape of a single lasing pulse for dipole moment ratios of 2, 2.5 and 3 are presented in Fig. 5–7, respectively. As can be seen from Fig. 7, *c*, the pulse duration can reach a value of 200 fs.

Let us note also that the CML behaviour shown in Fig. 3–7 is self-starting, i.e. to create the CML behaviour, a seed external pulse is not required, in contrast to the results of earlier works [10–14]. In numerical calculations, the CML arose from a very weak initial field, uniformly distributed along the length of the resonator. The length of the resonator in the examples considered was $L = 8$ mm, which corresponds to a pulse repetition

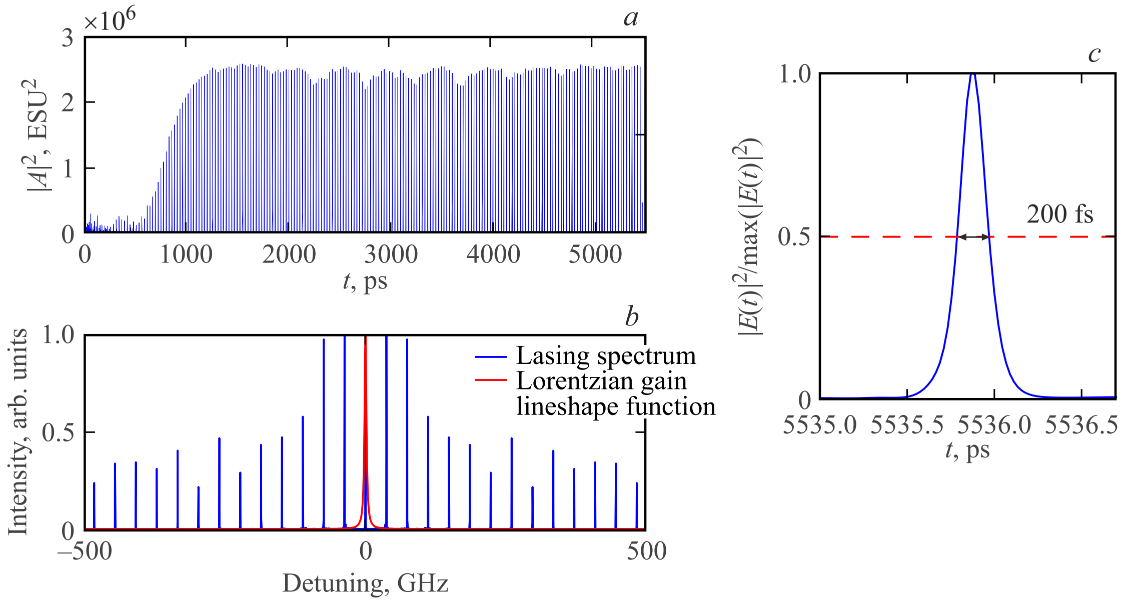

Figure 7. *a* — time dependence of the squared amplitude of the electric field (generation intensity) on time at the exit from the resonator; *b* — generation spectrum (blue curve), Lorentz contour of the gain line of the active medium (red curve); *c* — single lasing pulse when the ratio of dipole moments in the absorber and amplifier is 3.

rate of the order of $f_0 = 37.5\,\mathrm{GHz}$. Similar pulse repetition rates are easily realized in two-section quantum dot lasers [38–40].

Comparison of laser lasing parameters in coherent and incoherent mode locking behaviours when the absorber operates in saturation behaviour

To implement the standard incoherent mode locking behaviour with a saturable absorber, it is required that the relaxation times in the absorber to be comparable to the desired pulse duration [1–5]. Meanwhile, the time T_2 should be less than the pulse duration. Leaving the parameters of the amplifier unchanged (table), changing the relaxation times of the absorber in the indicated manner ($T_1 = 6\,\mathrm{ps}$, $T_2 = 0.1\,\mathrm{ps}$), making the concentration of particles in the absorber $N_0 = 8.5 \cdot 10^{14}\,\mathrm{cm}^{-3}$, without changing the dipole moment (table), the calculation showed the mode locking behaviour for account of the action of the absorber in the incoherent behaviour. An example of generation pulses, their spectrum, and the emission spectra of the absorber and amplifier are given in Fig. 8, *a, b*, respectively.

From Fig. 8, *b* it is clear that the laser emission spectrum (blue curves) is entirely located inside the absorption line contour (green curve), since the absorber operates in an incoherent behaviour.

The shapes of single generation pulses in this case (Fig. 8) and for the case of CML, when the parameters from the table were used (the calculation results shown in Fig. 5 correspond to this situation) are shown in Fig. 9, *a, b* respectively. From this figure it is clear that the pulse duration during incoherent mode locking (Fig. 9, *a*) increased by 4.2 times, and the peak amplitude decreased

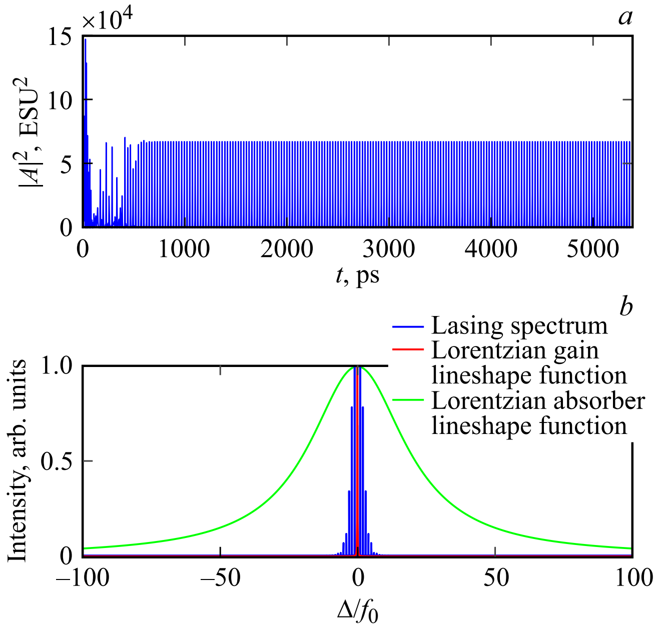

Figure 8. *a* — time dependence of the squared amplitude of the electric field (generation intensity) on time at the exit from the resonator; *b* — generation spectrum (blue curve), Lorentz contour of the active medium gain line (red curve) and Lorentz contour of the absorption line of the absorber (green curve). Absorbing medium parameters: $T_1 = 6\,\mathrm{ps}$, $T_2 = 0.1\,\mathrm{ps}$, $N_0 = 8.5 \cdot 10^{14}\,\mathrm{cm}^{-3}$, the rest of the parameters are given in the table.

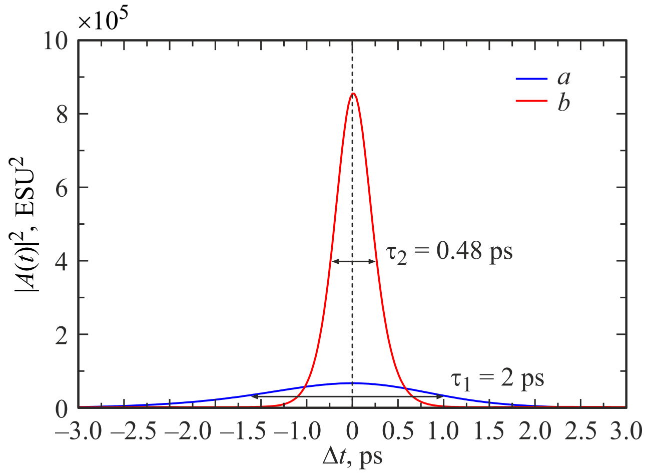

Figure 9. The shape of the generation pulse under (*a*) incoherent mode locking for the situation in Fig. 8 (blue curve); *b* — coherent mode locking (red curve) for the situation in Fig. 5, the calculation parameters are indicated in the table.

by 3.5 times. The total pulse energy decreased by 2.7 times (almost three times).

The given example illustrates the advantages of the CML behaviour in relation to the standard PML with a saturable absorber. It allows to obtain shorter pulses with more energy. The reason for this is understandable, since in the saturation mode part of the radiation energy at the absorption interval is irretrievably lost. And with CML, the pulse propagates in the absorber in the SIT regime with virtually no losses. Calculations also showed a narrower range of changes in the absorber parameters and their influence on the pulse duration compared to the case of CML.

Conclusion

The physical mechanisms of the formation of short pulses in the CML behaviour are discussed in detail and there is an illustration of the influence of the coherent absorber parameters. The CML behaviour is compared with the mode-locking behaviour formed by a saturable absorber. It has been shown that the latter is less effective — the duration of the pulses becomes longer and their energy less. Calculations also showed that there are fewer opportunities to influence the range of changes in pulse duration with variations in the parameters of the media. Numerical calculations also showed the stabilizing role of the absorber to obtain a stable CML behaviour.

The ideas expressed above can form the basis of a convenient method for generating femtosecond pulses in compact lasers with a GHz repetition rate, for example, in quantum dot laser systems, the value of the relaxation time T_2 in which can be increased when they are cooled [44]. The CML behaviour allows to reduce the duration of generation pulses in such systems by several times compared to those achieved in them using a saturable absorber.

Funding

Work by R.M. Arkhipov, A.V. Pakhomov and O.O. Diachkova was supported by Foundation for the Advancement of Theoretical Physics and Mathematics „BASIS“. The research of M.V. Arkhipov and N.N. Rosanov is supported by the State Assignment of Ioffe Institute, topic 00402019-0017.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by E.Potapova