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Calculation of the flow around an oscillating cylinder with a coaxial disk in the head at low Reynolds numbers

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Received April 4, 2023
Revised June 5, 2023
Accepted June 20, 2023

The hypothesis of curved models is used to calculate the flow around a cylinder performing damped rotational oscillations in the air flow. The influence of a disk coaxially fixed in the head of the cylinder on the oscillations is studied. The elongation of the cylinder (the ratio of the length of the cylinder to its diameter) is nine. The diameter of the disk is equal to 0.6 of the diameter of the cylinder. It is assumed that the only reason for the damping of on the oscillations is aerodynamic force. The parameters characterizing the attenuation of rotational on the oscillations of the cylinder occurring in the air flow at the Reynolds number Re $= 750$ are determined. It turned out that the presence of a coaxial disk in the head leads to a faster attenuation of on the oscillations. This fact is in qualitative agreement with the results of an experiment to determine rotational derivatives in a wind tunnel at large Reynolds numbers. The hypothesis of quasi-stationary is applied in the calculation of translation vibrations of the cylinder. The parameter characterizing the damping of vibrations of a cylinder with a disk in absolute magnitude slightly exceeds the same parameter for a cylinder without a disk, which corresponds to the results of the experiment.

Keywords: Calculation, translational and rotational oscillations, air flow, cylinder, coaxial disk.

DOI: 10.61011/TP.2023.08.57260.69-23

Introduction

In the aerodynamics of streamlined bodies, such as aircraft wings, the derivatives of the coefficients of aerodynamic forces and moments with respect to the components of angular and translational speed [1] are used to determine the stability of motion. However, for bluff bodies, the possibility of such derivatives use even in the simplest problem of damping rotational or translational oscillations is not obvious, since when the angle of inclination of the body changes, sharp changes in aerodynamic forces and moments occur, caused by the restructuring of extensive separation zones. Even with small deviations from the equilibrium position, there may be no region of linear dependence of the aerodynamic force or moment of force on the angular or translational speed. In this paper, damped oscillations of a cylinder without a disk and a cylinder equipped with a coaxial disk are studied. The purpose of this paper is to describe the oscillations of a body in which the dependence of forces and moments on angular and translational speed is nonlinear and may have discontinuities. The problem is solved numerically using the Krylov-Bogolyubov method. The problem of flow around the rotating cylinders is solved within the framework of the curved body hypothesis. In the problem of translational oscillations of cylinders, the stationarity hypothesis is used. The method of curved models was first used in the work of G.A. Gurzhienko [2]. G.A. Gurzhienko worked on this topic at the suggestion of Professor V.P. Vetchinkin, who wrote the preface to the paper. The principle of fictitious curvature was previously applied in the papers of G. Glauert when taking into account the influence of rotation of a thin airfoil on its lifting force [2]. The method of curved models is based on the idea that the body movement along a curved trajectory, in which translational and rotational components are present, can be replaced by the translational movement of the curved body. In this case, the local angles of attack at the corresponding points of the body retain the same values. If you make a curved model of body, you can measure the aerodynamic coefficients of forces and moments in a wind tunnel. The method of curved models was implemented in the problem of airship motion along a circular arc. G.A. Gurzhienko found that the center line of the curved body shall take the form of a chain line. The coordinates of the curved axis x and z are related by the formula

$$z = R_0 \cos\alpha_0 \left[\mathrm{ch}\left(\frac{x}{R_0 \cos\alpha_0} \right) - 1 \right],$$

where R_0 and α_0 — the radius of the circular arc along which the body moves, and the angle that the body axis makes with the relative gas speed vector. If $x/(R_0 \cos\alpha_0) \ll 1$, then the chain line is close to a parabola and to a circular arc of radius $R_0 \cos\alpha_0/2$. The curved model method was tested on the example of a rectangular wing with an NACA 0012 profile [3]. Wing tests carried out in a rotary machine at the Krylov Central Scientific Research Institute, and experiments performed in a wind tunnel with curved model at TsAGI gave similar results in both linear and nonlinear regions of the dependences of the lift force and pitching moment coefficients on the angle of attack and angular speed. In the article [4] the method of curved models was used in the numerical calculation of two-dimensional flow around the airfoil NACA 0012 performing a steady rotation. Subsequently, the method was extended to the calculation of hypersonic flow around the bodies [5]. In the paper [6] by numerical calculations using the method of curved models the unsteady aerodynamic characteristics of cylindrical models in subsonic and supersonic flow were determined.

1. Formulation of problem. Numerical method

We considered the flow around a circular cylinder, which performs angular oscillations in the air flow according to the harmonic law with small amplitude θ_0 and frequency Ω. Ratio of cylinder length to diameter is $\Lambda = L/D = 9$. In the equilibrium position the axis of the cylinder coincides with the direction of the oncoming flow speed. The influence of a disk located in the head of the cylinder was studied. The disk has a diameter of $d < D$ and is coaxially fixed at a distance of g from the front end of the cylinder. It is known that such a coaxial disk in front of the cylinder significantly reduces drag [7–9]. At the moment of time t, when its axis makes a small angle $\theta = \theta_0 \sin \Omega t$ with the direction of the oncoming flow speed, and the angular speed is $\omega = \dot{\theta} = \Omega\theta_0 \cos \Omega t = \omega_0 \cos \Omega t$. Let us consider the condition of equality of local angles of attack of the rotating straight and curved stationary cylinder, applied to the cylinder axis. Let the rotating cylinder is oriented horizontally. Fig. 1 shows images of the rotating cylinder (left) and the curved stationary cylinder (right).

Let us determine the local angle between the axis of the cylinder rotating with angular speed ω and the speed of the front end relative to the medium. Relative speed v_r is the sum of two vectors. One vector is equal in absolute value to the oncoming flow speed v and is directed horizontally. The second vector is the vector caused by the cylinder rotation about the axis passing through its center. It is directed upward in Fig. 1, and its length is $\omega L/2$. The angle between the cylinder axis and the relative speed of the front end is determined by the formula $\tan \alpha = \omega L/(2v)$. This angle shall coincide with the angle between the relative speed v_r and the axis of the curved cylinder in Fig. 1 on the right. For the curved cylinder the angle is equal to the ratio of the arc length AB to the arc radius R. We assume that the

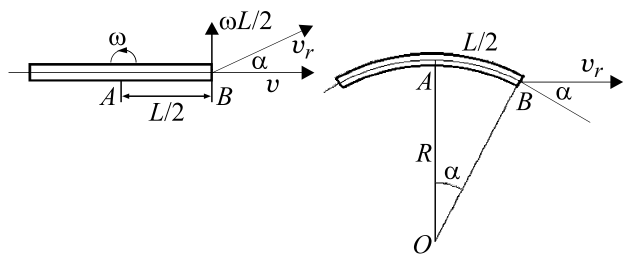

Figure 1. To determine the radius of curvature of the cylinder axis.

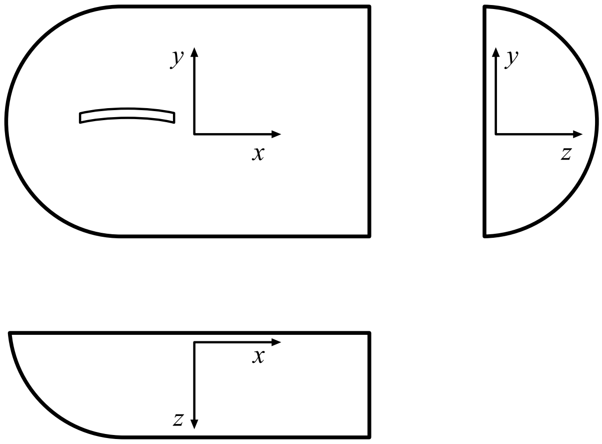

Figure 2. Computational domain with half of cylinder cut in half by a plane of symmetry.

angles are small. We retain only terms of the first order of smallness: $\alpha \approx \tan \alpha$, $v_r = v$. Comparing the expressions for the angle α of the rotating and curved cylinders, we obtain an expression for determining the bending radius R of the axis of the curved cylinder:

$$\frac{\omega L}{2v} = \frac{L}{2R}. \tag{1}$$

The above arguments are valid for any other point on the cylinder axis. From formula (1) it follows that $R = v/\omega$. To find out how the aerodynamic coefficients of the pitching moment of angular velocity change during one period of oscillation, we specified several angles of inclination $\theta_i \leq \theta_0$, and for the corresponding values of angular speed ω_i and the radius of curvature R_i we plotted the computational grids. The symmetry of the problem allowed us to limit ourselves to review of the flow around half of cylinder. The shape of the calculated volume was a combination of a quarter of sphere and a half of cylinder. The computational grids were generated using the freely available Gmsh program [10].

Fig. 2 shows a diagram of the computational domain in three projections. The front boundary of the calculated volume and its upper boundary are located at a distance of 4 m from the center of the cylinder. The rear boundary of the calculated volume is at a distance of 8 m from the center of the cylinder. The length of the cylinder is $L = 0.9$ m, its diameter is $D = 0.1$ m. The coaxial disk in the head of the cylinder had a diameter of $d = 0.6D$ and was located at a distance of $g = 0.5D$ from the front end. The computational grid is hybrid one. The elements adjacent to the wall are hexagons, the thickness of which decreases exponentially as they approach the wall. The rest of the calculated volume is filled with an unstructured grid with elements in the shape of tetrahedrons, prisms and pyramids. The main part of the calculations was carried out with grids containing from 650 to 800 thousand elements. Some of the calculations were carried out with grids containing from 380 to 1300 thousand elements in order to establish the grid independence. The grids in these calculations were refined or coarsened in the region of the boundary layer and in other regions of the calculated volume. Numerical calculations were carried out using the freely distributed software package SU2 [11], version 7.2.1 „Blackbird". The Navier-Stokes equations at the Reynolds number $\mathrm{Re} = 750$ were solved by the finite volume method. The solver for incompressible medium was used. The Courant-Friedrichs-Levy number for the finest grid did not exceed four. At the outer boundary of the calculated volume, the direction and speed of the air flow were specified. The gas density, viscosity and Prandtl number were assumed to be constant. Visualization of the results was carried out using the freely distributed Paraview program. The amplitude of oscillations was taken equal to $\theta_0 = 0.1$ rad. The dimensionless oscillation frequency $\mathrm{Sh} = \Omega L/(2\pi v)$ took two values — 0.286 and 0.057, which corresponds to the oscillation amplitude of the dimensionless angular speed $\omega^* = \omega_0 L/v$ 0.18 and 0.036, respectively. The similarity numbers in the paper are the Reynolds number Re, the elongation of the cylinder L/D, the dimensionless diameter of the disk d/D, the ratio of the gap between the cylinder and the disk to the diameter of the cylinder g/D, amplitude of rotational oscillations, dimensionless amplitude of angular speed oscillations. Thus, the results apply to a wide class of objects for which the indicated similarity numbers coincide. The proportions of the cylinder with the disk $L/D, d/D, g/D$ were chosen from the following considerations: there are experimental results on the damping of rotational oscillations obtained with cylinders and disks of such proportions (at other Reynolds numbers); resiliently fixed cylinders of smaller aspect ratio enter in the flow into oscillations with a constant amplitude; damped oscillations are not typical for cylinders of small aspect ratio; the selected dimensions of the coaxial disk and the distance from the cylinder to the disk provide a significant reduction in drag. Calculation of flow around at high Reynolds numbers is, in particular, of practical interest for modeling the cargo transportation under a helicopter on a hardpoint. In this case, calculations of turbulent flows are required. In the simplest version, the Reynolds-averaged Navier-Stokes equations are used to calculate turbulent flows. For closure, additional equations that model turbulence are used. However, this approach in the case of bluff bodies can lead to incorrect results [12]. For correct modeling, it is necessary to use eddy-resolving approaches that require significant computational resources, so we limited ourselves only to the flow at low Reynolds numbers, following in this example the article [9].

2. Flow around cylinder undergoing rotational oscillations

Let us consider the oscillations of the angular speed of the cylinder with dimensionless amplitude $\omega^* = 0.18$. Fig. 3 shows the distribution of air flow speeds near the cylinder in

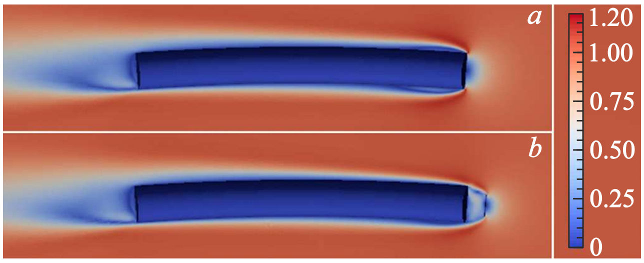

Figure 3. Relative flow speeds v/v_∞ near a cylinder without a disk (a) and a cylinder with a disk (b).

the plane of symmetry of the curved cylinder. The cylinder is in the air flow, the angle of inclination of the cylinder is $\theta = 3.17°$. This angle corresponds to the radius of curvature of the cylinder axis $R = 6$ m.

In the top image there is cylinder without disk, in the bottom there is cylinder with coaxially fixed disk. Comparison of flow around patterns reveals a pattern: the disk significantly reduces the size of the separation zones adjacent to the cylinder near the head part. A decrease in the size of the separation zones when installing the coaxial disk is observed for the entire set of cylinder inclination angles and angular speeds. Changing the size of the separation zones and the flow around the rear part of the cylinder affects the change in pressure distribution over the surface of the cylinder and the change in the moment coefficient. These changes at different flow around angles lead to different consequences. In particular, for the angle of inclination and negative angular speed, corresponding to Fig. 3, the moment coefficient changes insignificantly. Let the cylinder have a moment of inertia I_z and can rotate around the axis OZ passing through the center of mass. The cylinder is resiliently fixed, and its motion, which is oscillations damped under the influence of the aerodynamic torque, is described by the following equation:

$$I_z\ddot{\theta} + k\theta = sL\frac{\rho_0 v^2}{2}m_z(\theta, \omega), \tag{2}$$

where k — elastic suspension stiffness, m_z — aerodynamic moment coefficient, s — cross-sectional area of the cylinder, ρ_0 — air density. Let's introduce designations

$$\mu = sL\frac{\rho_0 v^2}{2I_z}, \quad \Omega^2 = \frac{k}{I_z}.$$

Equation (2) will be written as follows

$$\ddot{\theta} + \Omega^2\theta = \mu m_z(\theta, \omega). \tag{3}$$

In actual elastic suspension, friction is always present, which contributes to the damping of vibrations. However, in a computational experiment it is possible to ignore the suspension resistance and consider aerodynamic forces as the only reason for the damping of oscillations. Parameter $\mu \ll \Omega^2$. This means that the moment of aerodynamic

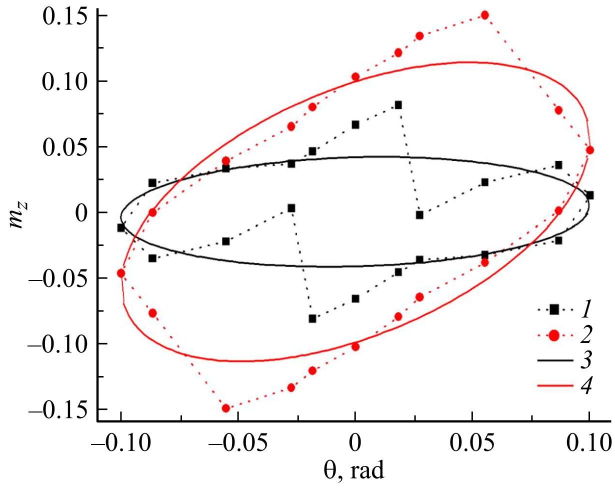

Figure 4. Moment coefficient m_z vs. angle of inclination θ: *1, 3* — cylinder without disk, *2, 4* — cylinder with disk; *1, 2* — calculation, *2, 4* — equivalent ellipses. The amplitude of the dimensionless angular speed is — 0.18.

forces is small compared to the moment of elastic forces. Then the oscillations will be close to harmonic, and a small aerodynamic moment will cause a slow change in the oscillation amplitude θ_0 and a slow change in the phase shift φ:

$$\theta = \theta_0 \cos\psi, \quad \psi = \Omega t + \varphi.$$

The equations for amplitude and phase can be obtained by applying the Krylov-Bogolyubov method to the first approximation [13]:

$$\frac{d\theta_0}{dt} = -\frac{\mu}{2\pi\Omega}\int_0^{2\pi} m_z(\psi)\sin\psi d\psi, \tag{4}$$

$$\frac{d\psi}{dt} = \Omega - \frac{\mu}{2\pi\theta_0\Omega}\int_0^{2\pi} m_z(\psi)\cos\psi d\psi. \tag{5}$$

The integral in equation (4) can be divided into the sum of two integrals with limits from 0 to π and from π to 2π. Then we will change variables $\theta = \theta_0 \cos\psi$. After the simplest transformations we obtain the following equation for the amplitude of rotational oscillations:

$$\frac{d\theta_0}{dt} = -\frac{\mu}{2\pi\Omega\theta_0}\int_{-\theta_0}^{\theta_0} [m_{z1}(\theta) - m_{z2}(\theta)]d\theta = -\frac{\mu}{2\pi\Omega\theta_0}S_\theta, \tag{6}$$

where m_{z2} and m_{z1} — moment coefficients on the intervals $0 < \psi < \pi$ and $\psi < \varphi < 2\pi$ respectively, S_θ — the area of closed contour, which is a graph of the aerodynamic moment coefficient m_z vs. angle of inclination θ. This graph is shown in Fig. 4.

If the oscillations are damped, then the area surrounded by the closed line is proportional to the work expended by aerodynamic forces during one period of oscillation to reduce the total energy of the oscillatory system. In the same way, you can plot moment coefficient vs. dimensionless angular speed of the cylinder $\xi = \omega L/v$. The area S_ω surrounded by the closed loop on this graph is proportional to the change in oscillation frequency:

$$\frac{d\psi}{dt} = \Omega - \frac{\mu v}{2\pi\Omega^2\theta_0^2 L}\int_{-\omega_0 L/v}^{\omega_0 L/v} [m_{z3}(\xi) - m_{z4}(\xi)]d\xi$$

$$= \Omega - \frac{\mu v}{2\pi\Omega^2\theta_0^2 L}S_\omega, \ \ \xi = \frac{L}{R}, \tag{7}$$

where m_{z3} and m_{z4} — moment coefficients at intervals $\pi/2 < \psi < 3\pi/2$ and $3\pi/2 < \varphi < 5\pi/2$, respectively. Typically, the damping of oscillations is described by the rotational derivative of the moment coefficient m_z^ω. It is believed that at small angles θ the linear approximation is valid

$$m_z = m_z^\theta\theta + m_z^\omega\xi. \tag{8}$$

In the case of a bluff body, the range of angles near zero, in which the linear approximation (8) is valid, is either very small or absent, since the dependence is discontinuous. Let us generalize the concept of rotational derivative to the case of nonlinear dependence of the moment coefficient on the angle and angular speed to the ranges of these variables, where this nonlinearity is significant, and introduce the parameters m_z^θ and m_z^ω, which describe the damping oscillations in certain ranges of angles and angular speed. In the general case, these parameters do not coincide with rotational derivatives. Substituting expression (8) into equation (4) and taking into account that $\theta = \theta_0\cos\psi$, $\omega = -\theta_0\Omega\sin\psi$, we obtain

$$\frac{d\theta_0}{dt} = -\frac{\mu}{2\pi\Omega}\int_0^{2\pi}\left(m_z^\theta\theta_0\cos\psi - m_z^\omega\theta_0\Omega\sin\psi\,\frac{L}{v}\right)\sin\psi d\psi$$

$$= \frac{\mu}{2}\,\theta_0 m_z^\omega\,\frac{L}{v}. \tag{9}$$

Expression (8) substitution into equation (5) gives

$$\frac{d\psi}{dt} = \Omega - \frac{\mu}{2\pi\Omega\theta_0}\int_0^{2\pi}\left(m_z^\theta\theta_0\cos\psi\right.$$

$$\left. - m_z^\omega\theta_0\Omega\sin\psi\,\frac{L}{v}\right)\cos\psi d\psi = \Omega - \frac{\mu}{2\Omega}m_z^\theta. \tag{10}$$

Equations (6) and (7) comparison with equations (9) and (10) gives expressions for the parameters m_z^ω and m_z^θ:

$$m_z^\omega = -S_\theta\,\frac{v}{\pi\Omega\theta_0^2 L} = -S_\theta\,\frac{R}{\pi\theta_0 L},$$

$$m_z^\theta = S_\theta\,\frac{v}{\pi\Omega\theta_0^2 L} = S_\omega\,\frac{R}{\pi\theta_0 L}. \tag{11}$$

The parameters m_z^{ω} and m_z^{θ}, determined by formulas (11), characterize the process of oscillations damping, and in the case when the linear dependence (8) is valid, they coincide with the rotational derivatives of moment coefficient. The areas of closed contours in Fig. 4, composed of straight segments connecting the points obtained as a result of the calculation, are equal to the areas of the ellipses plotted in Fig. 4. The area of the ellipse corresponding to cylinder with coaxial disk is greater than the area of the ellipse corresponding to cylinder without disk. This means that the oscillations of the cylinder with the disk will damp faster. This fact is in qualitative agreement with the results obtained during the experiment performed at high Reynolds numbers [14].

In all the considered cases, there is an angle of inclination of the cylinder at which the moment coefficient changes sharply. This is most clearly expressed for slow oscillations, in which the dimensionless amplitude of the angular speed is $\omega^* = 0.036$. Fig. 5 shows the patterns of flow around the cylinder with disk at two close inclination angles -1.05 and $-0.5°$. The radii of curvature of the cylinder axis are 25.4 and 25.1 m, respectively. In two images in Fig. 5 the flow around patterns differ significantly. When the angle of inclination changes from -0.5 to $-1.05°$, the area of reduced pressure in the rear section moves from the bottom surface of the cylinder to the top. In this case, the moment coefficient mz changes sign and increases sharply from -0.064 to 0.025. Dependence of the moment coefficient m_z on the angle of inclination θ of the cylinder with disk, which for fast oscillations was a shape close to ellipse, for slow oscillations turns into a zigzag curve. It is shown in Fig. 6. The areas of closed curves related to cylinders without disk and with coaxial disk, as in the case of fast oscillations, differ. The evaluation of areas of closed contours allows us to conclude that the coaxial disk accelerates the damping of the rotational oscillations of the cylinder. The Table summarizes the values of the calculated parameters.

The hypothesis of curved bodies is an approximate one for the unsteady rotational motion of bodies. In particular, this paper does not take into account that the flow perturbations created by the head part of the cylinder reach other fragments of the cylinder not instantly, but over a period of time. If the angle of inclination of the body changes, then the time of perturbations arrival in the case of streamlined bodies is taken into account by introducing another rotational derivative m_z^{θ}. In oscillatory motions of the body the damping effect is described by the sum of two rotational derivatives $m_z^{\omega} + m_z^{\theta}$. For the aircraft the rotational derivative m_z^{θ} is from 40 to 60% of m_z^{ω} [15]. Thus, in this paper the damping effect is underestimated, and the results obtained are have qualitative nature.

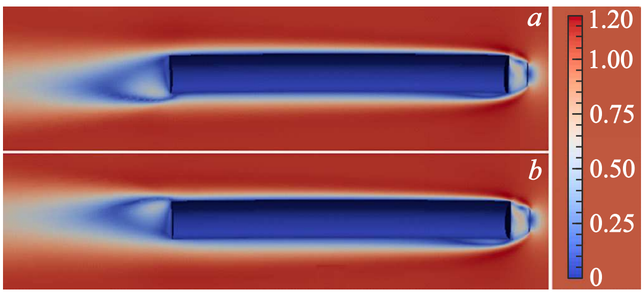

Figure 5. Relative flow speeds v/v_∞ for two near angles of inclination of the cylinder with disk: *a* — angle of inclination 0.5, *b* — angle of inclination 1.05°.

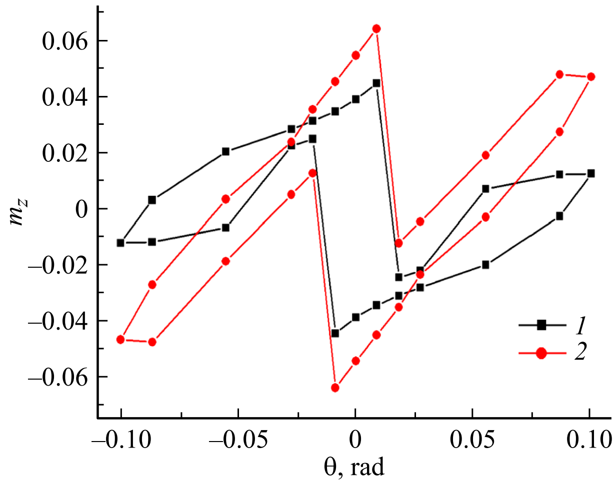

Figure 6. Moment coefficient m_z vs. angle of inclination θ: *1* — cylinder without disk, *2* — cylinder with disk. The amplitude of the dimensionless angular speed is —0.036.

Coefficients m_z^{ω}, m_z^{θ}

Coefficients	Fast oscillations	Slow oscillations
m_z^{ω} without disk	-0.23	-0.42
m_z^{ω} with disk	-0.55	-0.56
m_z^{θ} without disk	0.046	0.015
m_z^{θ} with disk	0.56	0.35

3. Flow around cylinder undergoing translational oscillations

To model translational oscillations of poorly flowed around bodies, a quasi-stationary approximation is often used, in which it is assumed that the aerodynamic normal force acting on the body depends on the instantaneous angles of attack only [16]. Normal force coefficients can be determined in a wind tunnel or calculated numerically. Let damped translational oscillations of the cylinder occur along the axis OY, directed perpendicular to the oncoming flow speed vector v and the cylinder axis, y — coordinate of center of mass of the cylinder. During translational oscillations, the tangent of the local angle of attack a is the ratio of the vertical speed of the body, taken with the opposite sign, to the speed of the oncoming flow. For a

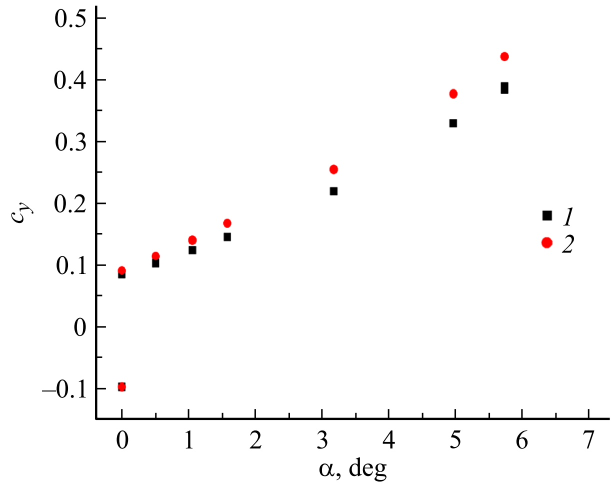

Figure 7. Normal force coefficient c_y vs. angle of attack α: *1* — cylinder without disk, *2* — cylinder with disk.

small angle of attack

$$\alpha \approx \tan\alpha = -\frac{\dot{y}}{v}.$$

If the normal force coefficient is related linearly to the dimensionless speed of motion of the body along the axis OY $\frac{\dot{y}}{v}$, then within the framework of the quasi-stationary approximation the vibrational derivative $c_y^{\dot{y}}$ is equal to the rotational derivative c_y^{α}, taken with the opposite sign:

$$c_y = c_y^{\dot{y}}\frac{\dot{y}}{v} = -c_y^{\alpha}\alpha.$$

Values of normal force coefficient c_y were numerically determined depending on the angle of attack α for uncurved cylinder with and without disk. Results are shown in Fig. 7. The points on the graph deviate significantly from linear relationships passing through the origin. Deviations are maximum in the vicinity of the origin. Therefore, we estimated the effective coefficients describing the damping of translational oscillations using the method outlined above for rotational oscillations. Let harmonic translational oscillations $y = y_0\cos\psi_y$, where $\psi_y = \Omega_y t + \phi_y$, occur with amplitude at which the maximum instantaneous angle of attack is $\alpha_0 = 0.05$ rad. Then, using the Krylov-Bogolyubov method, we find that the effective rotational derivative is determined by the expression

$$c_y^{\alpha} = \frac{v}{y_0\Omega_y\pi}S_y = \frac{1}{\alpha_0\pi}S_y,$$

where S_y — the area of the closed contour, which forms the dependence of the normal force coefficient c_y on the dimensionless coordinate $\eta = y/y_0$. coefficients c_y^{α} are equal to 2.41 for the cylinder without disk, and 2.76 for the cylinder with disk. The disk presence increases the rotational derivative c_y^{α}. This fact is in qualitative agreement with the results of the experimental determination of the rotational derivative at high Reynolds numbers [17]. The stationarity hypothesis (quasi-stationary hypothesis) for describing translational oscillations of poorly flow around bodies is also approximate, but there is evidence that it gives results that coincide with experimental ones [16].

Conclusion

As a result of the calculations coefficients were determined that characterize the damping in the flow of rotational and translational oscillations of resiliently fixed cylinder with coaxially fixed disk and without disk in the head part. These coefficients are a generalization of aerodynamic derivatives that characterize the influence of aerodynamic forces on the damping of oscillations of streamlined bodies. The calculations used the curved model hypothesis and the quasi-stationary hypothesis. The method is proposed in which the numerical solution of the Navier-Stokes equations is carried out jointly with the use of the Krylov-Bogolyubov method. Fast and slow oscillations were considered. It turned out that the coefficients describing the damping of rotational oscillations depend on the amplitude of oscillations of the angular speed of the cylinder. The coefficients describing the damping of translational oscillations depend on the amplitude of translational oscillations. The presence of coaxial disk promotes faster damping of oscillations.

Acknowledgments

Equipment of the Computing Center at the St. Petersburg University Research Park was used in the studies.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by I.Mazurov