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Runaway electrons in a gas diode with a wedge-shaped cathode

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The features of electron runaway in a gas diode with a wedge-shaped cathode providing a sharply inhomogeneous distribution of the electric field in the interelectrode gap are studied. It is shown that the character and conditions of runaway are qualitatively different for wedges with relatively large and small opening angles, i.e., in fact, for different degrees of field inhomogeneity. In the first case, the transition to the runaway mode is determined by the behavior of electrons in the immediate vicinity of their starting point, the vertex of the wedge-shaped cathode. For a wedge close in shape to a blade (opening angle less than 30° degrees), the relative contribution of the braking force for electrons in the gas increases with distance from the cathode, and their behavior at the periphery, near the anode, begins to play a key role in the analysis of runaway conditions. The influence of an external magnetic field on the geometry of the ionized region near the wedge vertex, starting from which the electrons become runaways, is also discussed.

Keywords: Runaway electrons, subnanosecond gas breakdown, sharply inhomogeneous electric field, guiding magnetic field.

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Introduction

Free electrons in a gas or plasma are capable of continuous acceleration when a sufficiently strong external electric field is applied [1–3]. The presence of such — runaway — electrons (RAEs) in gas discharges has been established experimentally: see, for example, [4–16]. Under laboratory conditions, they gain energies of tens to hundreds of kiloelectron volt [10,12,13,17–20] and even higher [21]. The speed of RAEs becomes comparable to the speed of light, as a result of which they cross the gas gap with a length of units to tens of millimeters for tens to hundreds of picoseconds. During these times, the RAEs pre-ionize the gap, thereby initiating its breakdown in the sub-nanosecond time range [19,22–26] (see also [6,16,27–33]). Note that the RAEs energy can exceed the energy corresponding to the voltage applied to the gap. The registration of such „anomalous" electrons was first reported in [8]. This phenomenon has been investigated in detail experimentally and theoretically for cathodes of different geometries, e.g., in [18,19,22,24,34].

In conditions of a homogeneous electric field, the mass transition of free electrons into the runaway mode occurs when its strength E exceeds some threshold value E_c, depending on the gas grade and its density (pressure) [3,6,28,35]. What is meant here is a situation where electrons can become runaway electrons regardless of their initial energy. In particular, low-energy thermal electrons run away, which is why this mode is often called „cold" or „thermal" runaway — see, for example, [36]. According to [28,29], $E_c \approx 270\,\text{kV/cm}$ can be taken for atmospheric air at a pressure of 760 Torr and a temperature of 300 K (other estimates for the critical field in the range of $220-450\,\text{kV/cm}$ are given in [35–39]). The threshold character of electron runaway is due to the non-monotonicity of the dependence of the friction (braking) force acting on electrons in the gas on their kinetic energy ε. For low energies, the frictional force increases with increasing ε, reaching some maximum $F_{\max}$ at energies ε_c on the order of 100 eV. For air, the maximum is at the energy $\varepsilon_c \approx 110\,\text{eV}$ [35,40] (note that other values, such as 150 eV in [28,29], are given in the literature). For energies greater ε_c, but not reaching relativistic values in units of mega-electronvolts, the frictional force decreases with increasing ε because of the falling cross section of the interaction of fast electrons with gas particles. As a consequence, if an external force exceeding in absolute value $F_{\max}$ acts on the electron, it will accelerate unboundedly — run away. The criterion for „cold" runaway of electrons in a homogeneous field is $E > E_c \equiv F_{\max}/e$ (here e is elementary charge) or $U > E_c D$ in terms of the potential difference U applied to a gap of length D.

In laboratory experiments with RAEs, the electric field distribution is often sharply inhomogeneous — its strength in the gap varies by more than an order of magnitude. This is due to the use of pointed cathodes [20,24,41–46], providing local field enhancement at the tip to values necessary for the initiation of field-emission processes and, as a consequence, the appearance of primary free electrons in the gas. In addition, from a practical point of view, it is much easier to ensure the realization of conditions for the transition of electrons to the runaway mode in a relatively small near-cathode region than in the entire gap, when much higher voltages would be required. An electron that has gained sufficiently high energy in the field-enhanced area near the pointed cathode can continue to run away in the peripheral area with a relatively low field due to the rather rapid, according to the law $\varepsilon^{-1}\ln\varepsilon$ [47], decrease in friction force with increasing ε. Thus, for example, in a homogeneous field for an air gap with a length of 20 mm for „cold" runaway of electrons, a sufficiently high value (from the practical point of view) of the applied potential difference, at least $E_c D = 540$ kV, is required. In the inhomogeneous field caused by the use of a tubular edge cathode with an edge radius of 200 μm, at the same gap length, RAEs were recorded at much lower voltage values — 84 kV [41]. This corresponds to an average field in the interval 42 kV/cm, which is obviously less than the threshold value E_c. The field at the cathode edge was estimated to be 500–600 kV/cm, i.e., larger than E_c. We note that the $E > E_c$ condition at the cathode edge is only a necessary, but not a sufficient, runaway condition. In the opposite case, RAE generation could occur at any small (but finite) voltages applied to the gap in the case of using a cathode with a sufficiently small radius of curvature of the tip to fulfill this condition, which, of course, does not make physical sense. A series of works [44,48,49] purposefully investigated the conditions of electron runaway in the air gap „tubular edge cathode — flat anode" as a function of the radius of rounding of the cathode edge. On the basis of analytical and numerical studies of the RAE dynamics, it was concluded that in the conditions of a sharply inhomogeneous field, the condition of voltage U exceeding a certain threshold U_c, depending both on the gas parameters and on the gap geometry, is stronger than the $E > E_c$ condition. Indeed, in experiments [44] with an air gap of 7.5 mm, RAEs were observed only at $U > 40 - 44$ kV, despite the field strength significantly exceeding the value E_c at the cathode edge sharpened to extremely small values of the rounding radius 5–50 μm.

In the present paper, we consider the dynamics and conditions of runaway of electrons starting from the edge of a wedge-shaped cathode with an arbitrary opening angle. This will allow us to study how the character of electron runaway changes when the degree of inhomogeneity of the electric field distribution varies. The field strength decreases with distance r from the wedge edge according to a power law, $E \propto r^{\gamma-1}$, where the exponent γ depends on the opening angle and belongs to the range $1/2 \le \gamma \le 1$ (see Section 1). The upper limit of this interval ($\gamma = 1$) corresponds to the trivial case of a uniform field, $E =$ const, realized for a wedge with an angle of 180°, i.e., for a flat cathode. The lower limit ($\gamma = 1/2$) — the case of a sharply inhomogeneous field $E \propto 1/\sqrt{r}$, realized for the blade cathode (wedge with zero opening angle) and the related tubular sharp-edge cathode [49,50].

In result of our consideration, it will be shown that the behavior of RAEs in the electric field of a wedge-shaped cathode with relatively large (exceeding $\sim 30°$) opening angles, is generally similar to the behavior of electrons in a uniform field. The possibility of their transition to the runaway mode and subsequent acceleration in the whole gas gap is entirely determined by the local distribution of the electric field in the area from which they start. For a sharp wedge whose opening angle is smaller than $\sim 30°$ (note that the value of the angle depends logarithmically weakly on the system size, see section 5), local processes near the cathode edge cease to play a determining role, and the runaway conditions take on a nonlocal character. An electron that has passed to the runaway mode in the near-cathode region may begin to slow down and eventually become thermal at the periphery, in the area of the weak electric field. In such a situation, when analyzing the possibility of continuous acceleration of the electron in the entire gas gap, the balance of forces acting on it near the anode begins to play a key role. The nonlocality of the processes under consideration is manifested in the fact that the braking force acting on the electron near the anode depends on the kinetic energy with which it reaches it, and to determine this energy it is necessary to take into account the entire prehistory of its motion.

Also in connection with recent experiments [20,51] on the control of RAE flows in air gaps by means of an external guiding magnetic field, the paper discusses its influence on the geometry of the electron runaway area near the top of the wedge (the ionized region, starting from which the electrons will run away). It is demonstrated that this area begins to deform (shrink) noticeably at values of magnetic induction in units of tesla.

1. Problem statement, results of numerical calculations of runaway conditions

We investigate the peculiarities of electron runaway in a gas diode under inhomogeneous electric field conditions caused by the use of a wedge-shaped cathode. Consider a cathode in the form of an ideal (with zero edge rounding radius) wedge with an opening angle β (Fig. 1). The origin coincides with the top of the wedge; the x-axis lies in the plane of symmetry of the system, and the y-axis is perpendicular to it. Since RAEs are generated at the initial stage of breakdown development, and their number at the conditions threshold for runaway is minimal, it can be considered that they cross the gap when the electric field distribution is not yet distorted by the bulk electric charge. Then the electric field potential (φ) will satisfy the Laplace equation, which is conveniently written using polar

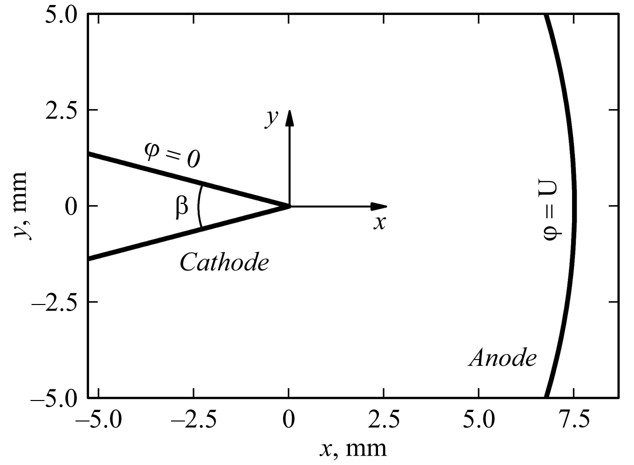

Figure 1. Geometry of the interelectrode gap for $\beta = 30^\circ$ and $D = 7.5$ mm

coordinates with origin at the top of the wedge:

$$\frac{\partial^2\varphi}{\partial r^2} + r^{-1}\frac{\partial\varphi}{\partial r} + r^{-2}\frac{\partial^2\varphi}{\partial\theta^2} = 0.$$

Here $r = \sqrt{x^2 + y^2}$ is the distance from the point, and $\theta = \arctan(y/x)$ is the angle read from the x axis. The solution for φ is found by separation of variables — by substituting $\varphi - \varphi_0 = A(r)B(\theta)$, where A and B are unknown functions, and φ_0 is the cathode potential, which without loss of generality can be assumed to be zero (the anode potential will be positive). Taking into account the symmetry of the problem with respect to the plane $y = 0$, we get:

$$\varphi(r, \theta) = U(r/D)^\gamma \cos(\gamma\theta), \tag{1}$$

where D is interelectrode distance, $U > 0$ is constant potential difference applied to the interelectrode gap, γ is some positive (otherwise the potential will not go to zero at the top of the wedge $r = 0$) constant. Thus, a power dependence of the potential on the distance to the wedge edge $\varphi \propto r^\gamma$ is realized, in which the degree of inhomogeneity of the field distribution in the gap is characterized by the exponent γ.

Let us now require the equipotentiality condition for the faces of the wedge: $\varphi|_{\theta=\pm\pi\mp\beta/2} = 0$. Substituting here the expression (1) leads to the simple trigonometric equation $\cos(\pi\gamma - \beta\gamma/2) = 0$, the solution of which gives the following relationship between the exponent γ and the wedge opening angle β:

$$\gamma = \frac{\pi}{2\pi - \beta}. \tag{2}$$

For the allowable range $0 \le \beta \le \pi$ of wedge opening angles, it gives the following range of values: $1/2 \le \gamma \le 1$ (graphically, the relationship of γ and β is shown in Fig. 2. The upper limit of this range ($\gamma = 1$) corresponds to an expanded angle of 180°. In this case, the cathode is flat and the electric field is homogeneous. The lower bound ($\gamma = 1/2$) corresponds to an infinitely thin wedge, $\beta = 0^\circ$, i.e., a blade cathode. The electric field decreases with distance from the blade edge according to the root law $E \propto 1/\sqrt{r}$.

From general considerations, it is clear that the direction x is the most favorable for the runaway of electrons. Therefore, to investigate the conditions of runaway — determining the minimum value of the voltage at which the electrons starting from the top of the wedge runaway — it is sufficient to consider the one-dimensional problem of the motion of free electrons along the symmetry axis x (in this case $\theta = 0$ and $r \equiv x$). According to (1), the electric field potential φ and the absolute value of electric field strength E are given by the expressions

$$\varphi(x) = \frac{Ux^\gamma}{D^\gamma}, \quad E(x) = \left|\frac{d\varphi}{dx}\right| = \frac{\gamma U x^{\gamma-1}}{D^\gamma} \tag{3}$$

i.e. $\varphi \propto x^\gamma$ and $E \propto x^{\gamma-1}$. In force the negativity of the degree exponent $\gamma - 1$ for E (except for the trivial case of $\gamma = 1$), the field is enhanced in the region of small x (formally, $E \to \infty$ at $x \to 0$), which is necessary both to initiate the field emission of primary free electrons and for their transition to the runaway mode.

The equation of one-dimensional motion of an electron starting from the cathode with zero velocity is conveniently written in terms of its kinetic energy $\varepsilon(x)$ [6,28]:

$$\frac{d\varepsilon}{dx} = f(x, \varepsilon), \quad \varepsilon(0) = 0, \tag{4}$$

where $f = eE(x) - F(\varepsilon)$ is the total force acting on the electron. The first term (eE) corresponds to the force acting on the particle from the electric field. The second term (F) is the friction (braking) force acting on the electron in the gas, to determine which we will use the non-relativistic Bethe [35,47] formula, represented in compact form

$$F(\varepsilon) = \frac{eE_c\varepsilon_c}{\varepsilon} \ln \frac{2.718\varepsilon}{\varepsilon_c} \tag{5}$$

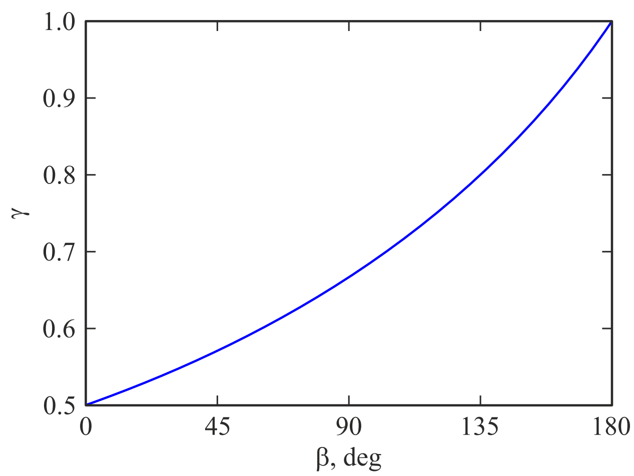

Figure 2. Dependence of the field inhomogeneity exponent γ on the opening angle of the wedge-shaped cathode β.

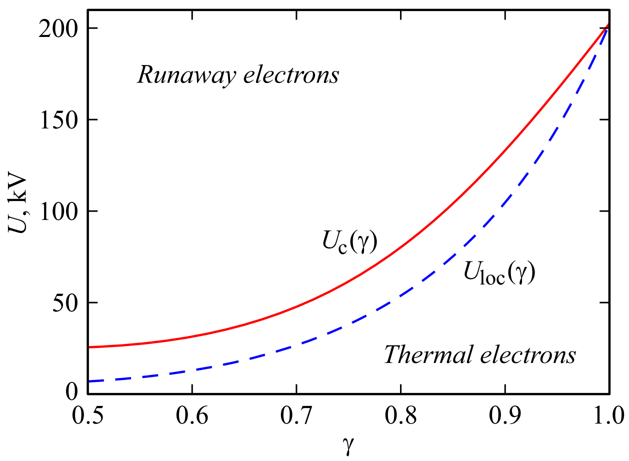

Figure 3. The threshold voltage for electron runaway as a function of the field inhomogeneity exponent γ (gas — atmospheric air at pressure 760 Torr and temperature 300 K, $D = 7.5$ mm): U_c — numerical calculation, U_{loc} — local criterion (7).

when using the values E_c and ε_c (here 2.718 is the base of the natural logarithm). Note that in numerical calculations in the range of relatively low energies $\varepsilon < 0.72\varepsilon_c \approx 79.2$ eV we will use the approximation $F \propto \sqrt{\varepsilon}$ instead of formula (5). It corresponds to an electron drift with fixed mobility. In the high energy band, the applicability of formula (5) is limited by the condition $\varepsilon < mc^2 \approx 510$ keV, where m is the rest mass of the electron, c is the speed of light. It is clear that the description of electron motion within the framework of the compact equations (4) and (5) is a considerable simplification; up-to-date data on electron-medium interaction cross sections can be found, for example, in the NIST database.

The RAE is assumed to be continuously accelerated throughout the interelectrode gap. Then the runaway condition is

$$f > 0, \quad 0 \le x \le D, \tag{6}$$

and the threshold value of the applied potential difference U for runaway will be its minimum value U_c, at which (6) is satisfied.

In Fig. 3, the red solid line (in the online version) shows the results of a numerical study of threshold runaway conditions for $D = 7.5$ mm (this gap length corresponds to experiments [44]), $\varepsilon_c = 110$ eV, and $E_c = 270$ kV/cm (these values correspond to atmospheric air at a pressure of 760 Torr and a temperature of 300 K [28,29,40]). Equation (4) was solved for field inhomogeneity exponents γ over the entire allowable range of values of $1/2 \le \gamma \le 1$. In result of varying the voltage U, a threshold U_c, below which the electron stopped running away, was calculated. For $\gamma = 1$(the case of a homogeneous field), as one would expect, U_c is maximized and equal to $E_cD = 202.5$ kV. The local electric field enhancement near the tip at $\gamma < 1$ facilitates the transition of the free electron into the runaway mode and consequently leads to a decrease in U_c. As the degree of field inhomogeneity increases (i.e., as the value of the exponent γ decreases), the value U_c monotonically decreases, reaching a minimum ≈ 25.8 kV at $\gamma = 1/2$.

2. Local runaway criterion

We will call local a criterion, for which the key for the transition of the electron into the runaway mode is its dynamics in the region from which it starts. In our case, this is the neighborhood of the apex of the wedge-shaped cathode. It is assumed that if the electron has not become thermal in this region, it will run away in the rest of the gap as well. Thus, in a weakly inhomogeneous field ($dE/dx \ll U/D^2$) the runaway condition of an electron starting from some point x_0, is the inequality $E(x_0) > E_c$, i.e., the runaway condition is clearly of local character — all determines the field distribution at the starting point. In the case of a sharply inhomogeneous field of interest to us, such a criterion loses significance: the value of $E(0)$ formally goes to infinity at $\gamma < 1$.

A more adequate, yet still local, criterion may be proposed, based on the requirement that the $E > E_c$ condition must be satisfied at the point x_c, at which the electron gains energy ε_c and, according to formula (5), the frictional force is maximum. The position of this point in the vacuum approximation (i.e., neglecting energy losses in collisions with gas molecules) is determined from the equation $\varepsilon_c = e\varphi(x_c)$. The runaway threshold — magnitude U_{loc} in terms of the voltage applied to the gap — is then found from the condition $E(x_c) = E_c$. Given the field and potential distributions (3), we obtain a system of equations with two unknowns U_{loc} and x_c:

$$\varepsilon_c = \frac{eU_{loc}x_c^{\gamma}}{D^{\gamma}}, \qquad \frac{\gamma U_{loc}x_c^{\gamma-1}}{D^{\gamma}} = E_c.$$

We find it from it

$$U_{loc} = \left(\frac{\varepsilon_c}{e}\right)^{1-\gamma}\left(\frac{E_cD}{\gamma}\right)^{\gamma}, \qquad x_c = \frac{\gamma\varepsilon_c}{eE_c}. \tag{7}$$

The runaway criterion

$$U > U_{loc}(\gamma) \tag{8}$$

can be considered local since under the considered conditions at $1/2 \le \gamma \le 1$ will be $x_c \approx 2-4\,\mu$m, i.e., the point $x = x_c$ is in close proximity to the cathode.

The dependence of U_{loc} on γ corresponding to (7) is shown in Fig. 3 by the blue dashed line (in the online version). It can be seen that the local runaway criterion (7), (8) is exact in the trivial case of the homogeneous field $\gamma = 1$, when $U_{loc} = U_c = E_cD = 202.5$ kV. It gives acceptable accuracy (i.e. $U_{loc} \approx U_c$) for values of the exponent γ approaching unity, i.e., for the case of a weakly inhomogeneous distribution of the electric field in the gap (a wedge with a large opening angle β). At the same time, it is obvious from the figure that the criterion (7), (8) is inapplicable for values γ close to 1/2 (wedge with small β).

Thus, $U_{\text{loc}}(0.5) \approx 6.7$ kV, which is almost four times smaller than the numerically calculated value of $U_c(0.5) \approx 25.8$ kV. This means that in the case of a sharply inhomogeneous field, the runaway criterion is not local, and the processes both near the cathode and at the periphery will determine the runaway threshold. We will discuss the reasons for this behavior of electrons in Sections 3 and 4.

3. Reasons for the limitations in applicability of the local runaway criterion

Let us discuss the motion of an electron in space not confined by the anode, i. e., when its energy can grow unlimitedly with time. Let us compare the forces acting on the electron eE and F at $x \to \infty$. Suppose here $eE \gg F$, i.e. the electron at the periphery moves as in vacuum, and the energy gained by it will be determined by the passed potential difference: $\varepsilon(x) \approx e\varphi(x)$. In this case, $\varepsilon \propto x^{\gamma}$, which allows us to estimate the friction force (5) as $F \propto x^{-\gamma} \ln x$. Comparing this force with the electric one $eE \propto x^{\gamma-1}$, we find that our assumption of the dominance of eE over F is valid at $1/2 < \gamma \leq 1$ and is violated by the presence in (5) of a logarithmic multiplier at the single point $\gamma = 1/2$, corresponding to the zero opening angle of the wedge (blade cathode). This means that if an electron has gone into runaway mode near the cathode, it will continue to run away at the periphery at $1/2 < \gamma \leq 1$. At $\gamma = 1/2$, in the $x \to \infty$ limit, the frictional force F will dominate the electric force eE. The electron will start to lose energy and will inevitably become thermal.

Thus, the electron dynamics is qualitatively different for the cases of $\gamma = 1/2$ and $\gamma > 1/2$. However, these differences are evident in the limit $x \to \infty$. In real situation, when the trajectory of the electron is limited by the gap $0 \leq x \leq D$, a smooth transition between these cases should be expected when the value of the exponent γ decreases, and the answer to the question of the conditions and character of the runaway of electrons requires a more detailed analysis.

4. Two modes of electron runaway

To illustrate the differences in RAE dynamics at different γ, we give characteristic distributions of the total force acting on the electron f in the interelectrode gap $0 \leq x \leq D$ when the voltage U minimally exceeds the threshold U_c. It is convenient to introduce a dimensionless reduced (normalized by eE) force

$$f_n \equiv f/(eE) = 1 - F/(eE),$$

which for RAEs is always in the interval $0 < f_n < 1$. The Fig. 4 shows the dependencies of $f_n(x)$ for $\gamma = 0.5$, 0.7, 0.9. They confirm our assumption about the different nature of RAE behavior for different parts of the allowable range γ. For large γ (i.e. $\gamma = 0.7$ and 0.9 in Fig. 4) inside the gap there is a pronounced minimum of the reduced force f_n, where its value approaches zero. When the voltage decreases below the threshold U_c, the force would become negative; in such a situation, the electron would begin to lose energy, i.e., it would no longer be runaway. Above the threshold, the value f_n is positive at the minimum. The electron continuously gains energy in the gap and also formally in the region behind the anode $x > D$.

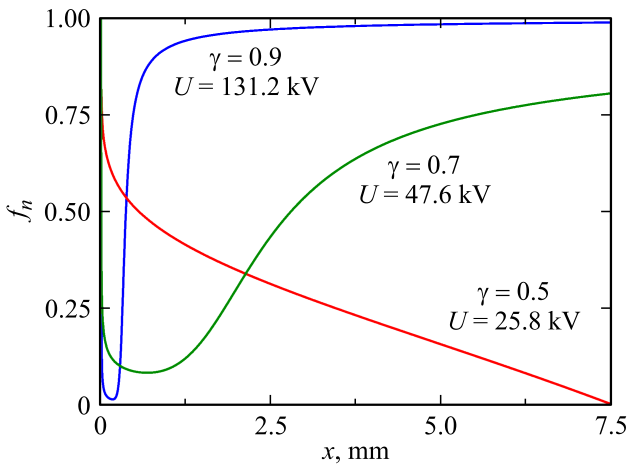

Figure 4. Distributions of the reduced force acting on the RAE f_n in the interelectrode gap for $\gamma = 0.5$, 0.7, 0.9 at near-threshold voltage values $U \approx 25.8$, 47.6, 131.2 kV respectively (gas — atmospheric air at pressure 760 Torr and temperature 300 K, $D = 7.5$ mm).

For relatively small γ (i.e. $\gamma = 0.5$ in Fig. 4) a qualitatively different picture is observed. The induced force decreases monotonically over the entire interval $0 \leq x \leq D$, reaching a minimum — zero — value at the anode $x = D$. This means that the electron gains maximum energy at the anode, and behind it formally begins to lose it.

Thus, two modes of electron runaway can be distinguished. The first mode — when the runaway threshold is due to the presence of a force minimum f_n inside the gap (cases of $\gamma = 0.7$, 0.9 in Fig. 4). When this narrow spot is overcome, the electron will continue to run away in the rest of the gap. The second mode — when the minimum (for the runaway threshold — zero) value of the reduced force is at the anode (case $\gamma = 0.5$ in Fig. 4). In first mode, the electron runs away at any $x \geq 0$; in the second mode — only inside the interelectrode gap $0 \leq x \leq D$. Below, we analytically formulate and analyze the runaway conditions for both regimes.

5. Non-local runaway conditions

The above analysis indicates that the local runaway criterion $U > U_{\text{loc}}$, based on the analysis of the behavior of the electron at the place of its start — the vicinity of the apex of the wedge-shaped cathode, — does not work for close to 1/2 values of the exponent γ, i.e., at small opening angles of the wedge (Fig. 2). This is due to the fact that, as can be seen from Fig. 4, the relative contribution of the friction force to the total force acting on the electron increases as it moves toward the anode. In this situation, the runaway threshold will not be determined by its dynamics near the cathode, as was assumed in the formulation of the local runaway criterion (7), (8).

We derive in this section the necessary nonlocal (i.e., taking into account the motion of the electron in the whole gap) runaway criterion. To avoid confusion, we note that the analyzed criterion has no relation to the nonlocal condition of the absence of the Townsend multiplication of electrons $\alpha_i D \leq 1$ (here α_i is multiplication factor), which the authors [52] proposed to use as a runaway criterion. We will use the method of successive approximations to derive the criterion.

In first, vacuum approximation, we neglect the friction force F. The energy of the electron will then be determined by the potential difference it has traveled through: $\varepsilon_{\text{vac}}(x) = e\varphi(x)$. In the next approximation we will take into account the influence of the friction force, but for its calculation we will use the energy of the electron found at the previous step, i.e. ε_{vac}. The total force acting on the electron is approximated as $f(x, \varepsilon_{\text{vac}}(x))$. It is clear that the inequality $\varepsilon_{\text{vac}}(x) \geq \varepsilon(x)$ is always true. According to Bethe (5) formula, the friction force decreases monotonically with increasing ε at $\varepsilon > \varepsilon_c$, then $F(\varepsilon_{\text{vac}}(x)) < F(\varepsilon(x))$ and $f(x, \varepsilon_{\text{vac}}(x)) > f(x, \varepsilon(x))$. This means, that in the approximation used we overestimate the total force acting on the electron. Consequently, the runaway criterion derived from it will underestimate the voltage threshold; it will be necessary but not sufficient.

Thus, by analogy with (6), the necessary nonlocal runaway condition is

$$f(x, \varepsilon_{\text{vac}}(x)) = eE(x) - F(e\varphi(x)) > 0, \quad 0 \leq x \leq D \quad (9)$$

and the threshold value of the applied potential difference U for runaway will be the minimum value U_{nl}, at which inequality (9) is satisfied. Substituting expressions (3) into (9), we obtain after simple transformations

$$\frac{\gamma e U^2 x^{2\gamma-1}}{\varepsilon_c E_c D^{2\gamma}} - \ln\left(\frac{2.718 e U x^\gamma}{\varepsilon_c D^\gamma}\right) > 0.$$

To analyze this expression, it is important that its left-hand side has a minimum at some $x = x_{\min}$. It is found from the condition that the derivative of the left-hand side by x goes to zero, i.e.

$$\frac{\gamma(2\gamma-1) e U^2 x_{\min}^{2\gamma-2}}{\varepsilon_c E_c D^{2\gamma}} - \frac{\gamma}{x_{\min}} = 0.$$

From here we find that:

$$x_{\min} = \left[\frac{\varepsilon_c E_c D^{2\gamma}}{(2\gamma-1) e U^2}\right]^{\frac{1}{2\gamma-1}}. \quad (10)$$

In the trivial case of a uniform electric field ($\gamma = 1$, $\beta = 180^\circ$), when the exact value of the threshold voltage is $E_c D$, formula (10) gives $x_{\min} = \varepsilon_c/(eE_c) \approx 4\,\mu$m, i.e., the minimum is in the vicinity of the cathode. This is exactly the same as the analogous case of $\gamma = 1$ for the local runaway criterion — see (7).

In the opposite limit of $\gamma \to 1/2$, it follows from (10) that $x_{\min} \to \infty$ (it is important for this conclusion that U is finite). This means that for values of γ close to 1/2, the minimum falls outside the range $0 \leq x \leq D$, in which condition (9) must be satisfied.

From this analysis, we can conclude that two types of runaway criterion should be distinguished depending on the value of γ. For the first type, the minimum (10) falls within the interelectrode gap $0 < x_{\min} < D$, while for the second type it is outside it, $x_{\min} \geq D$.

In the first case, after passing a „narrow" place in which the magnitude of the friction force F approaches the magnitude of the electric force eE, the electron will run away in the remaining part of the gap. Then to formulate the runaway condition, it suffices to consider its behavior at a single point $x_{\min}$. The electron will run away if the force f is positive in it. The threshold value for runaway strength U_{nl1} (to be precise — its estimate from below) corresponds to the force turning to zero:

$$f(x_{\min}, \varepsilon_{\text{vac}}(x_{\min})) = 0, \quad (11)$$

or, similarly, a pair of conditions $f = 0$ and $df/dx = 0$, simultaneously defining values $x_{\min}$ and U_{nl1}. Substituting (10) into (11) and solving the resulting expression with respect to U, we obtain the desired nonlocal runaway criterion of the first type:

$$U > U_{\text{nl1}}(\gamma), \quad U_{\text{nl1}} = \left(\frac{\varepsilon_c}{2.718e}\right)^{1-\gamma}\left(\frac{E_c D}{2\gamma-1}\right)^\gamma. \quad (12)$$

Using (10) and (12), we find the position of the minimum force f for the runaway threshold

$$x_{\min} = \frac{(2\gamma-1)\varepsilon_c}{eE_c} \exp\left(\frac{2-2\gamma}{2\gamma-1}\right).$$

With the help of this expression, it is easy to find the threshold value γ_c of the exponent γ, below which the first runaway criterion (12) is not applicable. The threshold corresponds to the situation where the position of the force minimum falls on the anode, i.e. $x_{\min} = D$ (more generally, the threshold γ_c is determined from the conditions $f|_{x=D} = 0$ and $df/dx|_{x=D} = 0$). We obtain the following transcendental equation for γ_c:

$$(2\gamma_c - 1)\exp\left(\frac{2-2\gamma_c}{2\gamma_c-1}\right) = \frac{eE_c D}{\varepsilon_c}. \quad (13)$$

It shows that γ_c depends logarithmically weakly on the dimensionless complex $eE_c D/\varepsilon_c$ or, given that the critical field E_c is proportional to the gas pressure p [6,35,37], the product pD. Thus, when the value of $eE_c D/\varepsilon_c$ is varied over a wide range from 800 to 8000 (with the interelectrode distance D varying from 3 mm to 3 cm), the value γ_c changes (decreases) only slightly, from 0.55 to 0.54. According to (2), the corresponding threshold value of the wedge opening angle changes from 33° to 27°. For the example considered in this paper ($D = 7.5$ mm, $E_c = 270$ kV/cm, $\varepsilon_c = 110$ eV) we have for the right-hand side of (13): $eE_cD/\varepsilon_c \approx 1840$. The solution of (13) is then $\gamma_c \approx 0.546$. The following threshold angle of the wedge corresponds to this value of the exponent: $\beta_c \approx 30°$ (the cathode configuration shown in Fig. 1 corresponds to this angle value). Given the weakness of the dependence of γ_c and β_c on system size, the indicated values (0.546 and 30°, respectively) can be considered universal. Thus, the criterion (12) is applicable for $\gamma_c \le \gamma \le 1$, i.e., for the case of a weakly inhomogeneous electric field. This range of values of the exponent γ corresponds to a wedge-shaped cathode with relatively large opening angles $30° < \beta \le 180°$.

In the second case, the minimum (10) is either outside the interelectrode gap, $x_{\min} > D$, or absent altogether (special case $\gamma = 1/2$). The reduced power f_n then decreases monotonically in the gap with distance from the cathode and is minimal at the anode $x = D$. Then, for the runaway of the electron, it is sufficient for the force to be positive at the anode, i.e. $f|_{x=D} > 0$. The threshold voltage U_{nl2} corresponds to the force turning to zero:

$$f(D, \varepsilon_{vac}(D)) = 0.$$

We obtain from here a non-local runaway criterion of the second type:

$$U > U_{nl2}(\gamma), \qquad (14)$$

where threshold U_{nl2} is defined by the transcendental equation

$$\frac{\gamma e U_{nl2}^2}{\varepsilon_c E_c D} = \ln\left(\frac{2.718 e U_{nl2}}{\varepsilon_c}\right). \qquad (15)$$

In a sufficiently wide range of parameters, the approximation can be used to determine U_{nl2}:

$$U_{nl2} \approx \left(\frac{\varepsilon_c E_c D}{\gamma e}\left(1.95 + 0.5\ln\frac{eE_cD}{\gamma\varepsilon_c}\right)\right)^{1/2}.$$

The criterion (14), (15) is applicable for $0.5 \le \gamma \le \gamma_c \approx 0.546$, i.e., for the case of a sharply inhomogeneous electric field. This range of values of the exponent γ corresponds to wedge-shaped cathodes with small opening angles $0° \le \beta \le 30°$.

So, the necessary nonlocal condition for runaway is

$$U > U_{nl}(\gamma), \quad U_{nl}(\gamma) = \begin{cases} U_{nl2}(\gamma), & 1/2 \le \gamma \le \gamma_c, \\ U_{nl1}(\gamma), & \gamma_c < \gamma \le 1, \end{cases} \qquad (16)$$

where the values U_{nl1} and U_{nl2} are determined by expressions (12) and (15). The dependence of the threshold voltage U_{nl} on the exponent γ corresponding to (16) is shown in Fig. 5 by the solid green line (in the online version). The $U_c(\gamma)$ (red dashed line (in the online version)) and $U_{loc}(\gamma)$ (blue dotted line (in the online version)) dependencies demonstrated earlier in Fig. 3 are shown for comparison. It can be seen that taking nonlocal effects into account allowed us to noticeably increase the accuracy of the analytical runaway criterion in the range of small, approaching 1/2, values of the exponent γ compared to the local criterion. In particular, $U_{nl}(0.5) \approx 16.4$ kV, which is much closer to the numerically calculated value of $U_c \approx 25.8$ kV than the local criterion, $U_{loc}(0.5) \approx 6.7$ kV (see insert in Fig. 5, where the range of $0.5 \le \gamma \le 0.6$ is given with magnification).

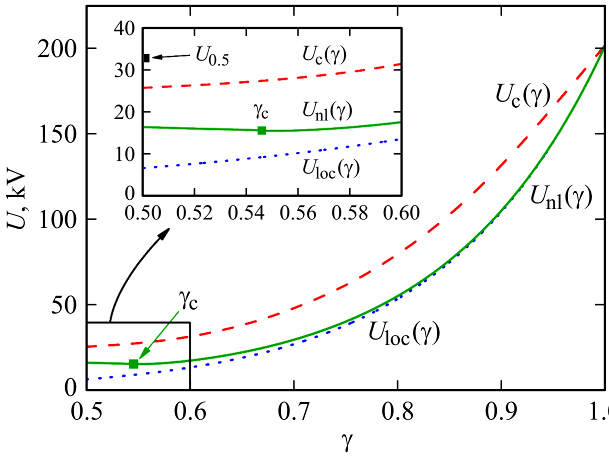

Figure 5. The solid green line (in the online version) — the threshold voltage U_{nl} for electron runaway as a function of the value γ according to the nonlocal criterion (16) (gas — atmospheric air at pressure 760 Torr and temperature 300 K, $D = 7.5$ mm); the green dot marks the $\gamma = \gamma_c \approx 0.546$ boundary of applicability of formulas (12) and (15), which determine the values U_{nl1} and U_{nl2}. The results of the numerical calculation of the threshold voltage $U_c(\gamma)$ (red dashed line (in the online version)) and corresponding to the local runaway criterion (7), (8) dependence $U_{loc}(\gamma)$ (blue dashed line (in the online version)) are shown for comparison. The inset — enlarged area of the relatively small γ, the black dot shows the analytical evaluation of the $U_{0.5} \approx 32.7$ kV [44,48] voltage threshold for $\gamma = 0.5$.

In significant part of the $\gamma_c < \gamma \le 1$ area, as can be seen from Fig. 5, the nonlocal runaway criterion is quite weakly different from the local criterion, i.e. $U_{nl} \approx U_{loc}$. This can be attributed to the smallness of the ratio $x_{\min}/D$ when the ratio is sufficiently large γ. So, for example, when $0.59 \le \gamma \le 1$ it will be $x_{\min}/D < 0.01$. A sharp increase in $x_{\min}$ to a value D occurs only in the vicinity of the boundary value of the exponent $\gamma = \gamma_c$. This means that despite the non-local approach used in finding the $U_{nl1}(\gamma)$ dependence, the runaway criterion itself at $0.59 \le \gamma \le 1$ is actually local in nature. The transition of the free electron into the runaway mode is determined by processes in the immediate vicinity of the cathode, as for the coarser criterion (7), (8).

We will discuss separately the runaway conditions for $\gamma = 1/2$. This isolated case, corresponding to the consideration of the blade cathode, was considered by us in [44,48,49]. The progress in its analysis was due to the revealed possibility of solving the free electron equation of motion exactly for large ε, when one can neglect the weak logarithmic dependence on energy in the Bethe (5) formula and consider $F \propto 1/\varepsilon$. The threshold voltage $U_{0.5}$ for this case, according to [44,48], can be estimated from the transcendental equation

$$\frac{eU_{0.5}^2}{8\varepsilon_c E_c D} = \ln\left(\frac{2.718eU_{0.5}}{2\varepsilon_c}\right), \tag{17}$$

differing from (15) by coefficients. For the values of D, E_c, ε_c used in this study, formula (17) gives $U_{0.5} \approx 32.7$ kV (black square in the inset of Fig. 5), which slightly exceeds the numerically calculated value of ≈ 25.8 kV. The reason for this discrepancy is that in [44,48] the asymptotic behavior of the electron at $x \to \infty$, i.e., actually behind the anode, was analyzed. In numerous calculations of Sec. 1 we have, more correctly, considered the runaway of an electron in a given gap $0 \leq x \leq D$.

6. Runaway area; effect of magnetic field

Above we analyzed the threshold conditions of electron runaway and therefore limited ourselves to the consideration of particles starting from the top of the wedge: their transition to the runaway mode is the most probable. Consider now the case where, for a given γ, the applied voltage exceeds the runaway threshold, $U > U_c$. In this situation, electrons starting not only directly from the top of the wedge, but also from some region of space surrounding the top will be able to run away. Let us consider the geometry of such a region, which we will call the runaway area for the sake of brevity, and in connection with recent works [20,51] its deformation under the influence of an external homogeneous magnetic field B, directed along the axis x.

Let us first define the configuration of the region of space near the wedge apex in which the absolute value of the electric field strength exceeds the runaway threshold, i.e. $E = E_c$. Its boundary is given by the equality $E > E_c$. From Eq. (1), we find

$$E = \sqrt{\left(\frac{\partial\varphi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\varphi}{\partial\theta}\right)^2} = \frac{\gamma U r^{\gamma-1}}{D^\gamma}.$$

It follows that in the plane $\{x, y\}$ the boundary is a circle of radius

$$r_c = \left(\frac{\gamma U}{E_c D^\gamma}\right)^{\frac{1}{1-\gamma}}.$$

Since the $E > E_c$ condition is a necessary local condition for electron runaway, the sought runaway area falls inside the circle $r < r_c$, i.e. r_c gives an upper estimate of its scale.

Let us consider the most important case for applications $\gamma = 1/2$, when the wedge opening angle is zero and the cathode is a blade (Fig. 6). In this case,

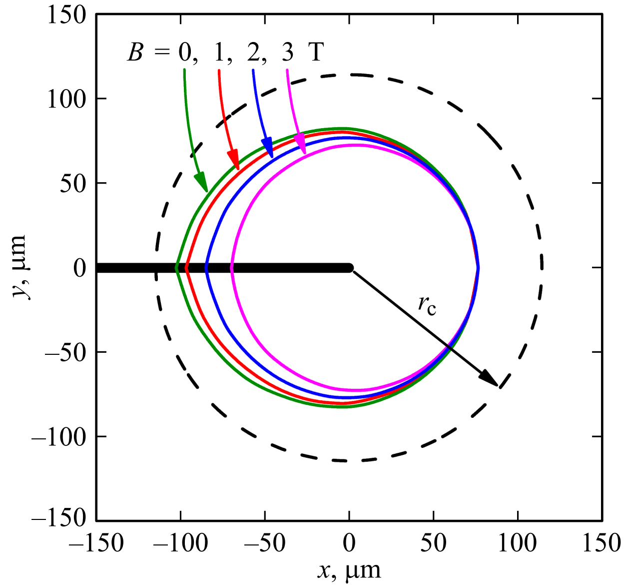

Figure 6. Runaway areas in the vicinity of the blade cathode apex for $B = 0, 1, 2, 3$ T ($U = 50$ kV, $D = 7.5$ mm, gas — atmospheric air at pressure 760 Torr and temperature 300 K). The dashed line shows the area (circle of radius $r_c \approx 114$ μm) in which the electric field strength exceeds the threshold E_c.

$$r_c = \frac{U^2}{4E_c^2 D}. \tag{18}$$

Let us take for definiteness the voltage value of $U = 50$ kV, which is about twice the runaway threshold (recall that for $\gamma = 1/2$ it is about 25.8 kV). We obtain $r_c \approx 114$ μm. Note that this estimate agrees with the calculated in [19,24] dimensions of the near-cathode region in which free electrons multiply as a result of impact ionization of gas molecules and their subsequent transition to the runaway mode.

In Fig. 6, the solid colored lines show the geometry of the runaway areas for $B = 0, 1, 2, 3$ T. They were determined on the basis of numerical calculations of the three-dimensional dynamics of free electrons launched from different points in the vicinity of the blade tip and moving under the action of the Lorentz force and the friction (braking) force in the gas. These areas fall, as expected, inside the circle $r = r_c$ (black dashed line), where the necessary local runaway condition $E > E_c$ is satisfied. It can also be seen that the runaway areas deform (decrease) with increasing B due to the magnetization of electrons, which prevents them from gaining energy under conditions of crossed electric and magnetic fields. This situation is realized, for example, when electrons start from the side edges of the cathode.

Let us estimate the characteristic values of magnetic induction at which the magnetic field begins to have a significant effect on the RAE. In addition to the scale r_c, in the presence of a magnetic field there is an additional scale — the Larmor radius (gyroradius) $r_g = mu/(eB)$, where $u = \sqrt{2\varepsilon/m}$ is the characteristic velocity of the electron (we consider it nonrelativistic), m is its mass. It is clear that in a rigorous analysis of the motion of the electron the transverse and longitudinal with respect to the direction of the vector B components of velocity should be separated; however, for qualitative evaluations it is sufficient to limit ourselves to the use of the absolute value of velocity.

Obviously, a relatively weak magnetic field, for which the Larmor radius r_g is much larger than r_c, will have little effect on the configuration of the runaway area. It starts to deform noticeably if the Larmor radius r_g is smaller r_c. The threshold value B_c of the magnetic induction is determined from the condition $r_g = r_c$, which, taking into account the definition of the Larmor radius and the formula (18) for r_c gives

$$\frac{\sqrt{2m\varepsilon}}{eB_c} = \frac{U^2}{4E_c^2 D}.$$

The electron energy in the runaway area is estimated in the vacuum approximation from the scale r_c: $\varepsilon \approx eU\sqrt{r_c/D}$. Finally, we find

$$B_c \approx \frac{4m^{1/2}E_c^{3/2}D^{1/2}}{e^{1/2}U}.$$

For the parameters we consider, $B_c \approx 2.3\,\mathrm{T}$, which is consistent with the results of numerical calculations of the geometry of the runaway areas demonstrated in Fig. 6. Significant changes in the shape of the runaway areas occur at values of magnetic induction close to B_c. Importantly, B_c falls within the range of $1-5\,\mathrm{T}$ values used to control RAE flows in [20,51]. This suggests the necessity to take into account the deformation of the electron runaway area under the influence of the magnetic field under experimental conditions.

Conclusion

In work we analyzed the conditions of electron runaway in a gas diode with a cathode representing an ideal (with zero radius of curvature of the edge) wedge with an opening angle in the range $0 \le \beta \le 180°$. At such a cathode configuration, the electric field distribution in the interelectrode gap is essentially inhomogeneous. The field strength satisfies the scaling of $E \propto r^{\gamma-1}$ with $0.5 \le \gamma < 1$, which provides that E formally turns to infinity at $r \to 0$.

Using the example of an air gap with a length of 7.5 mm, it was shown that the character of electron runaway is qualitatively different for cathodes with relatively large ($\beta_c \le \beta < 180°$; $\beta_c \approx 30°$) and small ($0 \le \beta \le \beta_c$) opening angles. In the first case, when the electric field distribution can be conventionally considered weakly inhomogeneous, the transition of electrons to the runaway mode is determined by their behavior near the top of the wedge, from where they start. The relative contribution of the friction force to the total force acting on the RAE reaches a maximum here. This situation is in general similar to the one arising for a homogeneous field. In the second case — the case of a sharply inhomogeneous field — the relative contribution of the friction force increases with distance from the cathode, and the narrow place determining the possibility of electron runaway is no longer the near-cathode, but the near-anode part of the gap.

The identified differences in the behavior of electrons at different degrees of electric field inhomogeneity also lead to different runaway conditions, (12) and (14), (15), applicable for angles $\beta_c \le \beta \le \pi$ and respectively $0 \le \beta \le \beta_c$. Comparison of these conditions with the results of numerical calculations showed qualitative agreement, see the graphs U_{nl} and U_c in Fig. 5. Note that the threshold value of the opening angle β_c depends very weakly (logarithmically) on the dimensionless complex eE_cD/ε_c. Thus, when eE_cD/ε_c is increased by an order of magnitude from 800 to 8000, the angle changes relatively little: it decreases from 33° to 27°. This allows us to consider the evaluation of $\beta_c \approx 30°$, corresponding to the $eE_cD/\varepsilon_c \approx 1840$ case we have discussed in detail, to be fairly universal.

The effect of an external magnetic field on the geometry of the runaway area near the top of the wedge was also investigated in this work. It was shown that it begins to deform (shrink) noticeably at values of magnetic induction in units of tesla when the Larmor radius for the RAE becomes comparable to the scale of the region of the subcritical ($E > E_c$) field near the tip. Just such fields ($B = 1-5\,\mathrm{T}$) were used to control the RAE flows in recent experiments [20,51]. The noted compression of the runaway area adequately describes the tendency of the RAE current to decrease with increasing magnetic field observed in the experiment [20], since the total current depends on the area of the boundary of the ionized region.

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Conflict of interest

The authors declare that they have no conflict of interest.

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