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Stability of stationary solutions for the mode with charged particles reflection from the potential barriers in the electron-positron plasma diode

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The stability features of plasma steady-states in the diode with counter flows of electrons and positrons are studied numerically for the case when portion of the particles is reflected from potential barriers and returns to the electrode. The solutions are constructed for the non-monoenergetic particle velocity distribution function. It has been established that solutions with reflection of particles from potential barriers located near the emitting electrodes are stable when the interelectrode distance is less than some threshold value. All other solutions are unstable.

Keywords: Plasma diode, electron and positron beams, solution stability.

Pulsars — sources of pulsed radio emission whose bursts follow each other with very slowly varying periods — were discovered more than 50 years ago. However, there is still no clear idea about either the mechanism of this emission or the reason for the jump between modes [1,2]. Only in recent years has it been realized that pulsar emission is related to collective processes occurring in the electron-positron plasma of the pulsar diode [3].

In [4], it was hypothesized that the pulsar emission is caused by fluctuations of the electric field in the plasma arising from the instability of the stationary states characteristic of diodes with a collisionless plasma [5]. The stationary states of a diode with monoenergetic counter flows of electrons and positrons are studied in detail in [6]. They can be divided into two types: 1) all charged particles reach the opposite boundary; 2) some particles are reflected from the potential barrier in the plasma. The study of the stability of stationary solutions without reflection is carried out in [4,7]. Note that analytical methods for investigating the stability states of diode with electron beam for an inhomogeneous plasma have been proposed in [8,9].

In the present work, the stability of stationary solutions in a planar diode for the regime with reflection of particles from potential barriers is numerically studied. The initial stage of development of small perturbation of solutions is calculated. To validate the results obtained, the calculations are carried out using two numerical codes: EK code and PIC code.

We consider that the nonrelativistic electron flow comes from the left electrode, and the positron flux — from the right electrode with velocity distribution functions (VDFs) slightly different from the monoenergetic ones, with mean velocities equal in modulus: $\overline{v}_{p,0} = -\overline{v}_{e,0}$. A particle reaching any electrode is absorbed at it. The potential difference between the electrodes U is assumed to be zero.

We convert to dimensionless units by choosing the particle energy W_0 and the Debye–Hückel length. $\lambda_D = \left[(2\tilde{\varepsilon}_0 W_0)/(e^2 n_{e,0})\right]^{1/2}$ (e — the electron charge, $n_{e,0}$ — the concentration of electrons as they fly off the emitter, and $\tilde{\varepsilon}_0$ — the dielectric permittivity of the vacuum) as units of energy and length. For dimensionless coordinate, potential and electric field strength we have: $\xi = z/\lambda_D$, $\eta = e\Phi/(2W_0)$ and $\varepsilon = eE\lambda_D/(2W_0)$. In dimensionless form, we define the particle VDF as a small-width gate with $\Delta \ll 1$: $f_0^{(\pm)}(u) = (2\Delta)^{-1}$ in the velosity range $u \in [\mp 1 - \Delta, \mp 1 + \Delta]$ and $f_0^{(\pm)}(u) = 0$ outside this interval (here the signs „minus“ and „plus“ correspond to electrons and positrons).

We first consider stationary solutions. In the $\Delta \ll 1$ case, they are close to the solutions for monokinetic VDFs and are determined by three dimensionless parameters: the interelectrode distance $\delta = d/\lambda_D$, the potential difference between the electrodes $V = eU/(2W_0)$, and the electric field strength at the left electrode ε_0.

The solutions will be represented by points in the plane (ε_0, δ). These points form separate curves called solution branches [6]. For the case of $\Delta = 0$, the solutions are shown in Fig. 1, *a*.

In the case when $V = 0$, and particles enter the plasma from opposite electrodes with charges, masses, and kinetic energies equal in modulus, the potential distributions (PDs) must have odd symmetry about the center of the gap [4], which allows us to reduce the number of solution branches compared to the general case $V \neq 0$ [6], as well as to correct the calculation during simulation.

Stationary solutions are characterized by wave-like PDs. The branches of the solutions for the mode without particle reflection from the extrema of the potential in Fig. 1 are labelled n_k, where the index k — number of extrema on the PD. In the case where there is electron reflection, we call

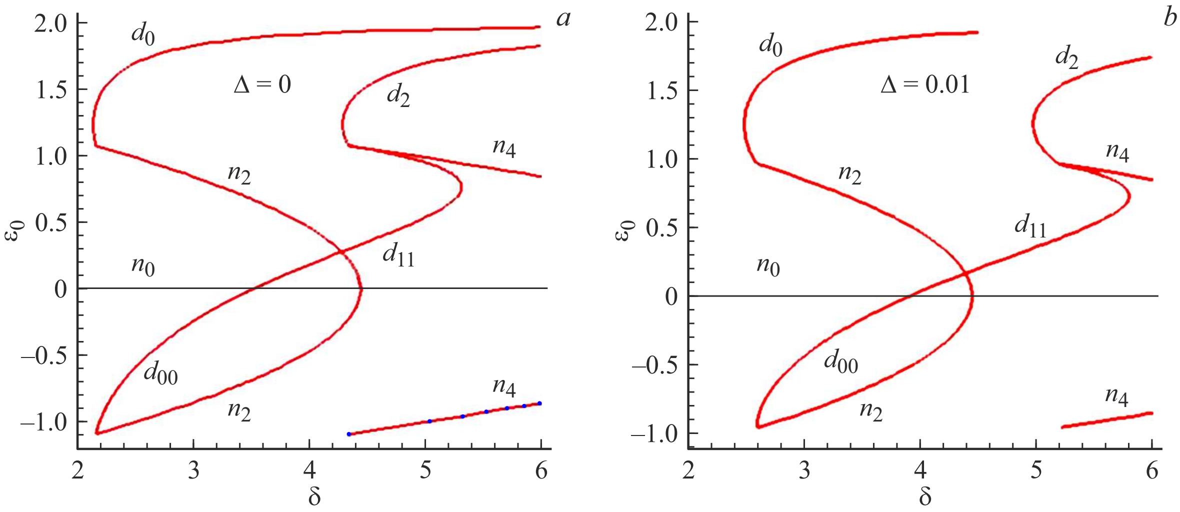

Figure 1. Solution branches for $\Delta = 0$ (a) and 0.01 (b).

the potential minimum at the PD a virtual electron emitter (e-VE) and the potential maximum — a virtual positron emitter (p-VE). There are two types of PDs with particle reflection. When the e-VE is to the left of the p-VE, the corresponding branches in Fig. 1 are labelled d_k, where the index k — the number of extrema on the PD lying between e-VE and p-VE. In the opposite case, when p-VE is located to the left of the e-VE, the corresponding branches in Fig. 1 are labelled d_{ij}, where index i — the number of minima on the PDs lying to the left of p-VE, and index j — the number of maxima on the PDs lying to the right of e-VE. Due to the symmetry in the case of $V = 0$, only solutions belonging to the branches n_k and d_k with $k = 0, 2, 4, \ldots$, as well as d_{ss} with $s = 0, 1, \ldots$ can exist. Examples of PDs corresponding to the branches n_2 and d_0, are shown in Fig. 2, a, and the branches d_{00} and d_{11} — in Fig. 2, b.

The stationary concentrations of electrons and positrons are determined from the law of conservation of energy. To find the PD for each type of solution, the charged particle concentrations are substituted into the Poisson equation, after which it is integrated over the potential between characteristic points (potentials at the left and right boundaries and extrema). The integrals are taken analytically. The equations obtained for the derivative of the potential $\eta'(\xi)$ allow us to relate the values of the potential at the extremes and the electric field strength at the left electrode ε_0, which acts as a parameter. The intervals of variation of ε_0, corresponding to each type of solution, as a functions of Δ are determined by the values of the potential at the extrema at which reflection begins or becomes complete. The potential distributions are found by numerical integration of $\eta'(\xi)$ over the coordinate, taking into account the symmetry with respect to the diode center. The solution branches are constructed by varying ε_0 on the intervals corresponding to these branches.

A typical PD corresponding to the n_2 branch is shown in Fig. 2, a by the red curve. The PD, characteristic of the d_0 branch, is shown in Fig. 2, a by the blue curve.

Figure 1, b shows the solution branches for a diode with VDF having half-width $\Delta = 0.01$, in the region $\delta < 6$. Comparison with similar curves for a diode with $\Delta = 0$ shows that the transition to a narrow rectangular VDF results in a shift of these dependencies to the right and a slight stretch along the axis of δ.

In a numerical study of the stability of stationary solutions in an electron-positron diode, we study the evolution of a small perturbation of the stationary electric field distribution. In all calculations described below, the half-width of the VDF $\Delta = 0.01$. In the case where the solution turns out to be unstable, the evolution of the perturbation causes the solution to move away from stationary. In the opposite case, the solution returns to stationary. We used two different numerical codes to simulate the evolution of the PD in the electron-positron diode: PIC code and EK code.

In the PIC code, the VDF modelling considers individual particles moving in an electric field given in N_ξ equally spaced grid nodes. The model used for the charge density at grid nodes is the „cloud-in cell" (linear particle contribution to the density at neighbouring nodes) [10]. The electric field at the grid nodes is found from Poisson's equation, and a linear approximation [10] is used between the nodes. The position of the particles at the next time instant is calculated using the „leapfrog method" [10] with a constant time step h_τ. At end of each step, particles that hit the electrodes are removed from the calculation and particles that arrived from the electrodes at a random time point distributed uniformly over the interval h_τ, with random velocities distributed uniformly over the intervals $[\mp 1-\Delta, \mp 1 + \Delta]$, are added.

As an initial field distribution at time $\tau = \tau_0$, a stationary field distribution with a perturbation superimposed on it $\tilde{\eta}(\xi, \tau_0) = C \sin(2\pi\xi/\delta)$, where $C \ll 1$, is used. The initial

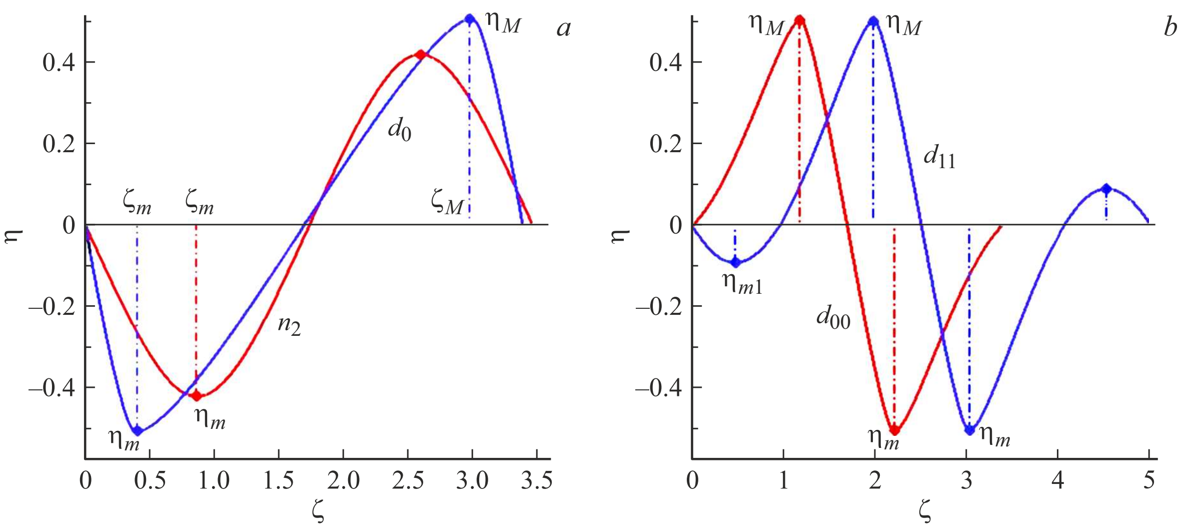

Figure 2. Typical PDs for branches n_2 (red curve) and d_0 (blue curve) (a) and d_{00} (red curve) and d_{11} (blue curve) (b); $V = 0$. A color version of the figure is provided in the online version of the paper.

VDF of particles at each point ξ of the perturbed stationary field has the form of the „gate", the boundaries of which are determined from the law of conservation of energy.

The numerical algorithm implemented in the EK code is described in detail in works [11,12]. The algorithm is based on the sequential calculation of the VDF of charged particles, their concentrations, and the electric field distribution at each time step. To calculate the VDF, the trajectories of the charged particles are traced in the field known at all previous time instants up to the departure from one of the electrodes.

The stability of the solutions belonging to the d_{00} and d_{11} branches was investigated at diode length values δ, equal to 2.7, 2.8, 3, 3.5, 5, 5.6. These values cover almost the entire range of diode lengths for which solutions of this kind exist. At $\delta = 5.6$, there are two stationary solutions of the form d_{11}, differing in the value ε_0 (Fig. 1). In all cases studied, the solutions were unstable. Note that different scenarios for the development of the initial perturbation are possible.

We also investigated the stability of solutions belonging to the branch d_0. Calculations were performed for diode lengths δ, equal to 2.7, 2.8, 2.85, 2.9, 2.95, 3, 3.1, 3.2. As in the calculations for the branches d_{00} and d_{11}, the time dependence of the maximum value of the deviation of the solution from the stationary $\tilde{\eta}_M$ was analyzed. For diode lengths of 2.7, 2.8, and 2.85, the perturbation decreases with time, and the stationary solutions were found to be stable. In the remaining cases, the solutions proved to be unstable. At that, for all diode lengths, except $\delta = 2.7$, at the initial stage of the perturbation evolution after a short transient process, we can distinguish a time interval, where the dependence $\tilde{\eta}_M(\tau)$ has the character of exponentially damped or increasing oscillations of a regular sinusoidal form with a constant frequency: $\eta_M(\tau) = \eta_{M0} + A\exp(\Gamma\tau)\sin(\omega\tau + \phi)$. Note that the time dependences $\varepsilon_0(\tau)$ have the same character.

Table 1. Increment Γ and frequency ω at different diode lengths δ ($\Delta = 0.01$ calculation by EK code)

δ	Γ	ω
2.80	−0.18	2.25
2.85	−0.13	2.17
2.90	0.04	2.18
2.95	0.05	2.18
3.00	0.07	2.17
3.05	0.12	2.15
3.10	0.14	2.15
3.20	0.16	2.14

From the results of the calculations, we determined the increments Γ and oscillation frequencies ω at the selected sites. These are presented in Table 1. The increment sign of Γ changes between points $\delta = 2.85$ and 2.9. In this interval, there is a stability threshold δ^d_{th} for the solutions corresponding to the branch d_0.

In diodes with lengths above the stability threshold, the exponential growth of the oscillation amplitude continues up to values of the order of hundredth. After that, the amplitude growth starts to slow down, and eventually the oscillations turn into periodic oscillations. At diode length δ near the stability threshold the dependences of $\eta(\tau)$ and $\xi(\tau)$ are close to sinusoidal, with increasing δ the oscillation shape becomes distorted.

In Table 2, the amplitude values of the nonlinear periodic oscillations of the PD maximum in the vicinity of the d_0 branch for different diode lengths $\eta^{\max}_M(\delta) - \eta^{\min}_M(\delta)$ are given. Near δ^d_{th}, the dependence of the oscillation amplitude on δ is well approximated by the function

Table 2. Amplitude of steady-state nonlinear oscillations in the diode ($\Delta = 0.01$)

δ	$\eta_M^{\max}-\eta_M^{\min}$	$0.35\sqrt{\delta-2.896}$
2.9	0.022	0.022
2.95	0.081	0.081
3.0	0.112	0.113
3.5	0.235	0.272
4.0	0.289	0.368
4.5	0.302	0.443

$f_H(\delta) = A\sqrt{\delta-\delta_{th}^d}$ at $A = 0.35$, $\delta_{th}^d = 2.896$. This indicates that δ_{th}^d — the position of the stability threshold — is a Hopf bifurcation point.

In the present work, we have actually completed the study of the stability of the stationary states of a diode with counter-flows of electrons and positrons [6] started in [4,7]. In previous works this problem was studied for the regime without reflection of charged particles from the extrema of the potential, i.e., for branches n_k with $k = 0, 2, 4, \ldots$. It was found analytically and numerically that such inhomogeneous stationary solutions are unstable with respect to small perturbations. In addition, it was found that for homogeneous solutions there is a threshold for the dimensionless interelectrode gap (or, that is the same, for the current density), above which instability develops in the diode. In this case, the unsteady process terminates in a new state characterized by nonlinear oscillations that occur near the stationary state corresponding to the mode with particle reflection. The threshold δ_{th}^n turns out to be equal to $\sqrt{2}\pi\lambda_D$.

In this paper, we study the stability of stationary states in the regime with particle reflection from potential barriers, i. e. solutions corresponding to the branches d_0, d_{00} and d_{11}. It is found that the solutions for the d_{00} and d_{11} branches are unstable, and the solutions belonging to the d_0 branch are stable only when the value of the interelectrode gap is smaller than some threshold value $\delta_{th}^d \approx 2.896$. The calculation of the nonlinear stage of the evolution of the perturbed solution for the d_0 branch ends with the nonlinear oscillations in the vicinity of this branch, coinciding with the nonlinear oscillations in which the development of the perturbation of the solutions corresponding to the branches without particle reflection is completed.

The study of the calculated nonlinear oscillations showed that their amplitude near the threshold is proportional to $\left(\delta-\delta_{th}^d\right)^{1/2}$, i.e., a Hopf bifurcation occurs here.

At present, there is no theory of diode stability for the regime with reflection of charged particles from the extremes of the potential. Therefore, we used two numerical codes in our research: EK-code and PIC-code. When the increments and frequencies of PD oscillations can be found from these calculations, their values are close when both codes are used. By doing so, we have demonstrated the correctness of the results obtained.

Conflict of interest

The authors declare that they have no conflict of interest.

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