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Propagation of unipolar impulsive disturbances in crystalline solids with Granato–Lucke dislocation hysteresis

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A theoretical study of the nonlinear propagation of unipolar pulse perturbations in crystalline solids with dislocation hysteresis of Granato–Lucke is carried out. An exact analytical solution has been obtained describing the propagation and evolution of the initial disturbance — the half-period of a sinusoidal oscillation in such medium. The dependences of the parameters of the disturbance in the medium, namely, the amplitude and duration on its initial amplitude and the distance traveled, are determined. Numerical and graphical analysis of the obtained solution is carried out.

Keywords: amplitude-dependent internal friction, elastic waves.

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1. Introduction

The theory of wave processes in ideal (without dissipation) homogeneous media with power (quadratic or cubic) elastic nonlinearity is developed to a fairly complete extent [1–4]. When unipolar pulse perturbations propagate in such media, they are subjected to a nonlinear distortion. At first, the perturbation front is twisted (front or rear, — depending on the sign of the nonlinearity parameter of the medium), and then an ambiguity or „overlap" is formed in its profile. Due to the physical unrealizability of the „overlap", a gap — shock front is artificially introduced into the perturbation profile. As a result, the shape of the perturbation in the medium becomes sawtoothed, while its duration increases, and the amplitude and energy decrease (due to nonlinear discontinuity losses), but the amount of movement of the perturbation remains.

The studies of amplitude-dependent internal friction (ADIF) in crystalline solids (metals, alloys and rocks) containing dislocations indicate that such materials are characterized by hysteresis nonlinearity that significantly exceeds the weak elastic nonlinearity of homogeneous media (without defects). The patterns of wave processes in hysteresis media differ from similar patterns for media with power-law elastic nonlinearity, in particular, twisting of the fronts and „overlap" in the wave profile may not occur, but the wave is nonlinearly distorted and attenuated (because of hysteresis losses) [5]. The theoretical study of nonlinear wave processes in hysteresis media is an urgent task of solid state physics because of the widespread availability of such materials, which constitutes the basis for studying the dynamics of dislocations under the action of alternating elastic stresses and determination of the mechanisms of hysteresis nonlinearity of crystalline solids [6–8].

To date, the dislocation theory of Granato–Lucke absorption is the only microscopic theory defining the hysteresis equation of state of crystalline solids, i. e. the dependence $\sigma = \sigma(\varepsilon, \dot{\varepsilon})$, where σ, ε and $\dot{\varepsilon}$ — the stress, strain and velocity strain [9–11]. The hysteresis of the crystal equation of state in this theory is associated with the separation of dislocation segments from impurity atoms and their different behavior at the loading and unloading stages. The area of the hysteresis loop determines the nonlinear losses of the wave, and the derivatives $\sigma_\varepsilon(\varepsilon, \dot{\varepsilon})$ determine the propagation velocities of the leading and trailing wave fronts. The expression for hysteresis losses in the Granato–Lucke theory is obtained in the non-wave approximation, when the sample length is much less than the wavelength, and the amplitude of the stress in the medium is equal to the amplitude of the harmonic voltage at the boundary of the medium. Theoretical studies of the nonlinear propagation of elastic waves in hysteresis solids were not conducted within the framework of the Granato–Lucke dislocation theory. Meanwhile, the patterns of the ADIF wave effects in media with hysteresis nonlinearity will differ from the patterns of the same effects in their non-wave description, since manifestations of the hysteresis properties of the medium accumulate in nonlinear distortions of the wave during its propagation. The identification of patterns of wave effects can be aimed at determining the mechanisms of hysteresis nonlinearity of crystalline solids and studying the dynamics of dislocations in various crystals, as well as to create methods for their nonlinear acoustic diagnostics and non-destructive testing.

It should be noted that the amplitude dependences of the effects of ADIF for many crystalline solids [5,6,8,12] do not correspond to the Granato–Lucke hysteresis. Nevertheless, the solution of the problem of nonlinear propagation of elastic waves and pulsed perturbations in solids with Granato–Lucke dislocation hysteresis is of particular interest, since it gives a correct qualitative understanding of nonlinear wave processes in such media. Such a solution is also useful as a reference for analyzing and comparing the patterns of nonlinear wave processes in solids with other types of hysteresis nonlinearity.

The propagation of unipolar pulsed perturbations in crystalline solids with Granato–Lucke dislocation hysteresis nonlinearity is theoretically studied in this paper. The specificity of such a problem lies in the fact that it is not possible to represent the Granato–Lucke hysteresis as a power series — Taylor series, and thus obtain a solution to the wave problem from solutions for media with power hysteresis [5]. Here all the wave effects of the ADIF will also be associated with the manifestation of hysteresis within the framework of the Granato–Lucke dislocation theory, but their patterns will be different, different from the patterns for media with power-law hysteresis nonlinearity.

2. Main equations

It follows from the dislocation theory of Granato–Lucke [9,10] that the hysteresis equation of state of a crystalline solid (for shear stresses $\sigma = \sigma_{xy}$ and deformations $\varepsilon = \partial U_y/\partial x$) has the following form

$$\sigma(\varepsilon, \dot{\varepsilon}) = G_0[\varepsilon - f(\varepsilon, \dot{\varepsilon})], \quad (1)$$

$$f(\varepsilon, \dot{\varepsilon}) = D \begin{cases} [1+(\varepsilon/\beta)]\exp(-\beta/\varepsilon), & \varepsilon \geq 0, \dot{\varepsilon} > 0, \\ [1+(\varepsilon_m/\beta)](\varepsilon/\varepsilon_m)\exp(-\beta/\varepsilon_m), & \varepsilon \geq 0, \dot{\varepsilon} < 0, \\ -[1-(\varepsilon/\beta)]\exp(\beta/\varepsilon), & \varepsilon \leq 0, \dot{\varepsilon} < 0, \\ [1+(\varepsilon_m/\beta)](\varepsilon/\varepsilon_m)\exp(-\beta/\varepsilon_m), & \varepsilon \leq 0, \dot{\varepsilon} > 0, \end{cases} \quad (2)$$

where $U_y = U_y(x, t)$ — y-displacement component,

$$G_0 = G/(1+QG), \quad D = \gamma^3 \Gamma Q/6, \quad \gamma = L_N/L_c \gg 1,$$

$$\Gamma = \pi f_m/4aL_c, \quad f_m \approx U_0/a,$$

$$Q = 48a^2\Lambda L_c^2/\pi^4 C = 24(1-\nu)L_c^2\Lambda/\pi^3 G,$$

$$C = 2Ga^2/\pi(1-\nu), \quad \beta = \Gamma/G_0,$$

G — the shear modulus of a dislocation-free crystal, a — the modulus of the Burgers vector, U_0 — the binding energy of a dislocation with an impurity atom, Λ — the dislocation density, L_c — the distance between impurity atoms along the dislocation axis, L_N — the dislocation length, ν — the Poisson's ratio, $f(\varepsilon, \dot{\varepsilon})$ — the hysteresis function, $|f(\varepsilon, \dot{\varepsilon})| \ll |\varepsilon| \ll 1$, $f(\varepsilon = 0, \dot{\varepsilon}) = 0$, $\varepsilon_m/\beta < 1$, ε_m — the strain amplitude.

We give characteristic estimates for the parameters of G_0, β and D of the hysteresis equation of state (1), (2). Assuming that $G = 4 \cdot 10^{10}\,\mathrm{kg/m \cdot s^2}$, $\nu = 0.25$, $a = 4 \cdot 10^{-10}\,\mathrm{m}$, $L_c = 5 \cdot 10^{-8}\,\mathrm{m}$, $\Lambda = 10^{12}\,\mathrm{m^{-2}}$, $\gamma = 50$, $U_0 = 2 \cdot 10^{-20}\,\mathrm{J}$, we get:

$$QG = 24(1-\nu)L_c^2\Lambda/\pi^3 \approx 1.5 \cdot 10^{-3} \ll 1,$$

$$G_0 \approx G, \quad f_m \approx 5 \cdot 10^{-11}\,\mathrm{kg \cdot m/s^2}, \quad D \approx 10^{-3},$$

$$\beta = \Gamma/G_0 \approx 2.5 \cdot 10^{-5}.$$

It should be noted that the amplitude ε_m in the expression (2) is not the initial amplitude of the strain ε_0 set at the polycrystal boundary (as in [9,10]), i.e. $\varepsilon_m \neq \varepsilon_0$. The amplitude ε_m is determined by the maximum strain of the wave in the medium; the amplitude ε_m decreases as the wave propagates (along the axis x) and its nonlinear attenuation, therefore $\varepsilon_m = \varepsilon_m(\varepsilon_0, x) \neq \varepsilon_0$.

Generally speaking, it is necessary to take into account the linear dissipative term $\eta\dot{\varepsilon}$ in the equation of state (1), where η — the coefficient of linear dissipation of the medium, however, it can be neglected if we consider sufficiently strong and slow perturbations for which the following inequality holds: $G_0|f(\varepsilon, \dot{\varepsilon})| \gg \eta|\dot{\varepsilon}|$ [1]. In this case, it is possible to obtain accurate solutions for simple waves [1,3]. (Taking into account the linear dissipative term prevents the formation of „overlap" and smoothes „sharp corners" in the profile of a nonlinear wave, but it is not possible to obtain an analytical expression for the wave itself.)

It should be noted that a similar hysteresis occurs for longitudinal stresses and strains. So-called orientation multipliers are used in the Granato–Lucke theory [9,10] for the transition from shear stresses and strains to longitudinal ones, taking into account the direction of propagation of the longitudinal wave with respect to the planes and directions of sliding in the crystal and the distribution of dislocations over all sliding systems, and the shear modulus G should be replaced in the equation (1) by $K + 4G/3$ (K — the all-round compression modulus) — for a limitless environment or by a Young's modulus — for a rod. Thus, the hysteresis equation of state for longitudinal stresses and strains will be the same as for shear stresses in the Granato–Lucke dislocation theory, with an accuracy of up to constant coefficients.

We obtain a single-wave equation for simple shear strain waves $\varepsilon(x, \tau) = \partial U_y(x, \tau)/\partial x$ spreading along the axis x by substituting the equation of state (1) into the equation of motion $\rho U_{tt} = \sigma_x(\varepsilon, \dot{\varepsilon})$ [13], and proceeding to the accompanying coordinate system $\tau = t - x/C_0$, $x' = x \geq 0$ [1]:

$$\frac{\partial \varepsilon}{\partial x} = -\frac{1}{2C_0}\frac{\partial f(\varepsilon, \varepsilon_\tau)}{\partial \tau}, \quad (3)$$

where $U_y = U_y(x, \tau)$ — y-the displacement component, ρ — density, $C_0 = (G_0/\rho)^{1/2}$ — the linear wave velocity.

The boundary condition is given in the form of a unipolar perturbation — a half-period of a sinusoidal oscillation with a frequency of ω:

$$\varepsilon(x = 0, t) = \varepsilon_0 \sin\omega t, \quad 0 \leq \omega t \leq \pi, \quad (4)$$

where ε_0 and $T = \pi/\omega$ — the initial amplitude and duration of the perturbation. We will assume for the sake of certainty that $\varepsilon_0 > 0$.

The wave equation (3) and the boundary condition (4) for the normalized strain $e(z,\theta) = \varepsilon(z,\theta)/\varepsilon_0 \geq 0$ have the following form in dimensionless variables:

$$\frac{\partial e}{\partial z} = -\begin{cases} \left(1 + \frac{\beta}{\varepsilon_0 e} + \frac{\beta^2}{\varepsilon_0^2 e^2}\right)\exp\left(-\frac{\beta}{\varepsilon_0 e}\right)\frac{\partial e}{\partial \theta}, & e_\theta(z,\theta) > 0,\\ \left(1 + \frac{\beta}{\varepsilon_0 e_m(z)}\right)\exp\left(-\frac{\beta}{\varepsilon_0 e_m(z)}\right)\frac{\partial e}{\partial \theta}, & e_\theta(z,\theta) < 0, \end{cases} \quad (5)$$

$$e(z=0,\theta) = \sin\theta, \quad 0 \leq \theta \leq \pi, \quad (6)$$

where

$$\theta = \omega(t - x/C_0) = \omega\tau, \ \ z = \frac{D\omega x}{2\beta C_0} = \frac{\gamma^3 Q G_0 k x}{12},$$

$$e_\theta(z,\theta) = \frac{\partial e(z,\theta)}{\partial \theta}, \ \ e_m(z) = \frac{\varepsilon_m(z)}{\varepsilon_0} \leq 1,$$

$k = \omega/C_0$, $\varepsilon_0/\beta < 1$. It is noteworthy that the equation (5) and its solution in dimensionless variables depend only on the ratio of the initial amplitude of the perturbation ε_0 to the parameter β.

3. Evolution of unipolar deformation pulses

For solving the equation (5) we will use the method of „stitching" of simple waves corresponding to each branch of the hysteresis (2) [5]. Such „stitching" occurs in case of deformation of $e(z,\theta)$ equal to the amplitude $e_m(z)$ of perturbation at $\theta = \theta_m(z)$. The exact solution of the equation (5) with boundary condition (6) is written implicitly and has the following form:

$$e(z,\theta) = \begin{cases} \sin\left(\theta - \left(1 + \frac{\beta}{\varepsilon_0 e} + \frac{\beta^2}{\varepsilon_0^2 e^2}\right)\exp\left(-\frac{\beta}{\varepsilon_0 e}\right) z\right), \\ \qquad\qquad e_\theta(z,\theta) > 0, \\ \sin\left(\theta - \int\limits_0^z \left(1 + \frac{\beta}{\varepsilon_0 e_m(z_1)}\right)\exp\left(-\frac{\beta}{\varepsilon_0 e_m(z_1)}\right) dz_1\right), \\ \qquad\qquad e_\theta(z,\theta) < 0. \end{cases} \quad (7)$$

Figure 1 shows the evolution of the shape of the pulse perturbation (6) at $\varepsilon_0/\beta = 1/5$ and various values of z. It can be seen from Figure 1 that the shape of the perturbation (7), its amplitude $e_m(z)$ and duration $\theta^*(z)$ strongly change with the growth of z: at the beginning, the peak of the perturbation sharpens, then its shape tends to trapezoidal, at the same time, the amplitude $e_m(z)$ decreases, and the duration $\theta^*(z)$ increases. (The perturbation duration $\theta^*(z)$ is determined from the equation $e(z,\theta^*(z)) = 0$ at $e_\theta(z,\theta) < 0$). All this is associated with the fact that the distortion of the leading ($e_\theta(z,\theta) > 0$) and trailing ($e_\theta(z,\theta) < 0$) edge of perturbation are determined by different branches of hysteresis (2), while the velocity of the leading edge of the perturbation is greater than the velocity of its trailing edge.

The amplitude $e_m(z)$ is determined from the equation (7) at the point $\theta = \theta_m(z)$ of the intersection of the leading ($e_\theta(z,\theta) > 0$, $0 \leq \theta \leq \theta_m(z)$) and trailing ($e_\theta(z,\theta) < 0$, $\theta_m(z) \leq \theta \leq \theta^*(z)$) edges of perturbation, i.e. at its vertex, when $e(z,\theta_m(z)) = e_m(z)$:

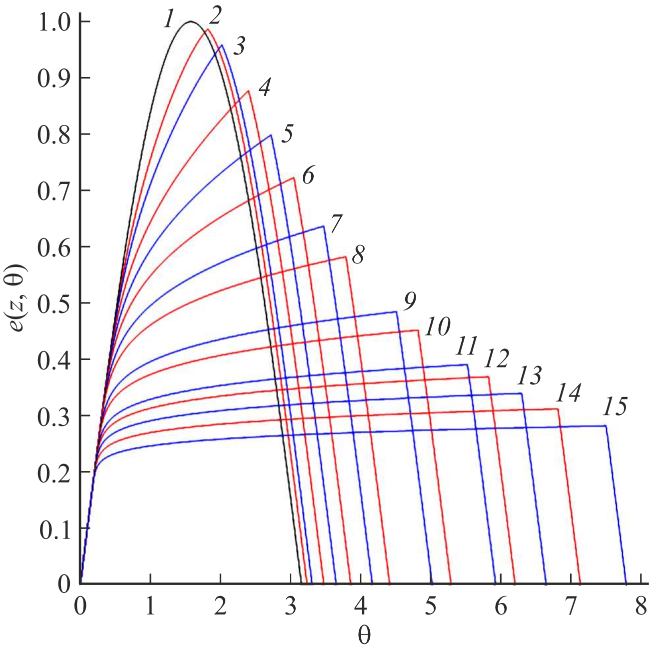

Figure 1. Evolution of the shape of perturbation $e(z,\theta)$ at $\varepsilon_0/\beta = 1/5$ and various values of z: line *1* — $z = 0$, *2* — $z = 2$, *3* — $z = 4$, *4* — $z = 10$, *5* — $z = 2 \cdot 10$, *6* — $z = 4 \cdot 10$, *7* — $z = 10^2$, *8* — $z = 2 \cdot 10^2$, *9* — $z = 10^3$, *10* — $z = 2 \cdot 10^3$, *11* — $z = 10^4$, *12* — $z = 2 \cdot 10^4$, *13* — $z = 6 \cdot 10^4$, *14* — $z = 2 \cdot 10^5$, *15* — $z = 10^6$.

$$e_m(z) = \\ = \begin{cases} \sin\left(\theta_m(z) - \left(1 + \frac{\beta}{\varepsilon_0 e_m(z)} + \frac{\beta^2}{\varepsilon_0^2 e_m^2(z)}\right)\exp\left(-\frac{\beta}{\varepsilon_0 e_m(z)}\right) z\right), \\ \sin\left(\theta_m(z) - \int\limits_0^z \left(1 + \frac{\beta}{\varepsilon_0 e_m(z_1)}\right)\exp\left(-\frac{\beta}{\varepsilon_0 e_m(z_1)}\right) dz_1\right). \end{cases} \quad (8)$$

We obtain the equation for $z = z(e_m)$ from this expression:

$$\frac{dz}{de_m} - \left(1 - \frac{\beta}{\varepsilon_0 e_m}\right)\frac{z}{e_m} + \frac{2\varepsilon_0^2 e_m^2}{\beta^2\sqrt{1 - e_m^2}}\exp\left(\frac{\beta}{\varepsilon_0 e_m}\right) = 0. \quad (9)$$

We find a transcendental solution for $z = z(e_m)$ from the equation (9) and expressions for $\theta_m(z)$ and $\theta^*(z)$ are found from the equation (8):

$$z = \frac{2\varepsilon_0^2 e_m(z)\sqrt{1 - e_m^2(z)}}{\beta^2}\exp\left(\frac{\beta}{\varepsilon_0 e_m(z)}\right). \quad (10)$$

$$\theta_m(z) = \arcsin e_m(z) + \frac{2\sqrt{1 - e_m^2(z)}}{e_m(z)} \times \left(1 + \frac{\varepsilon_0 e_m(z)}{\beta} + \frac{\varepsilon_0^2 e_m^2(z)}{\beta^2}\right), \quad (11)$$

$$\theta^*(z) = 2\arcsin e_m(z) + \frac{2\sqrt{1-e_m^2(z)}}{e_m(z)} \times \left(1 + \frac{\varepsilon_0 e_m(z)}{\beta} + \frac{\varepsilon_0^2 e_m^2(z)}{\beta^2}\right). \quad (12)$$

The asymptotic solutions of equation (10) have the following form: $e_m(z) \approx 1 - (z/z_0)^2$ — at $(z/z_0) \ll 1$, where $z_0 = 2^{3/2}\varepsilon_0^2 \exp(\beta/\varepsilon_0)/\beta^2$, and $e_m(z) \approx (\beta/\varepsilon_0)/\ln(\beta^2 z/2\varepsilon_0^2) \ll 1$ — at $z \gg 2\varepsilon_0^2\beta^2$, or $\varepsilon_m(z) \approx \varepsilon_0[1-(z/z_0)^2] \propto \varepsilon_0$ — at $(z/z_0) \ll 1$, and $\varepsilon_m(z) \approx \beta/\ln(\beta^2 z/2\varepsilon_0^2) \ll \varepsilon_0$ — at $z \gg 2\varepsilon_0^2/\beta^2$.

Figure 2 shows the dependences of the perturbation amplitude $e_m(z)$ and duration $\theta^*(z)$ on z at different values of $\varepsilon_0\beta$. It follows from the expressions (10)–(12) that the area $S(z)$ under the curve $e = e(z,\theta)$ is preserved:

$$S(z) = \int_0^{\theta^*} e(z,\theta)d\theta = 2 = \text{const}.$$

More informative manifestations of the hysteresis nonlinearity of the medium are the dependences of the shape of the perturbation $e = e(z,\theta)$, its amplitude $e_m(z)$ and duration $\theta^*(z)$ on the initial amplitude ε_0 (at $z = \text{const}$), since it is difficult for the body to change the position of the receiver in a solid (coordinate z, i.e. x), but the amplitude of ε_0 can be easily changed. Figure 3 shows the evolution of the shape of the perturbation $e = e(z,\theta)$ depending on ε_0/β at $z = 3\cdot10^4$. Here the qualitative behavior of $e = e(z,\theta)$ is the same as in Figure 1.

Figure 4 shows the dependences of the perturbation amplitude $\varepsilon_m(z)$ — (I) and the duration $\theta^*(z)$ — (II) on ε_0 at $z = 10^3$ and various values of parameter β. The amplitude of the disturbance $\varepsilon_m(z)$ initially grows linearly ($\varepsilon_m(z) \propto \varepsilon_0$) with an increase of ε_0, then — logarithmically slowly ($\varepsilon_m(z) \approx \beta/\ln(\beta^2 z/2\varepsilon_0^2) \ll \varepsilon_0$), i.e. there is a tendency to saturation of the amplitude $\varepsilon_m(z)$, and the

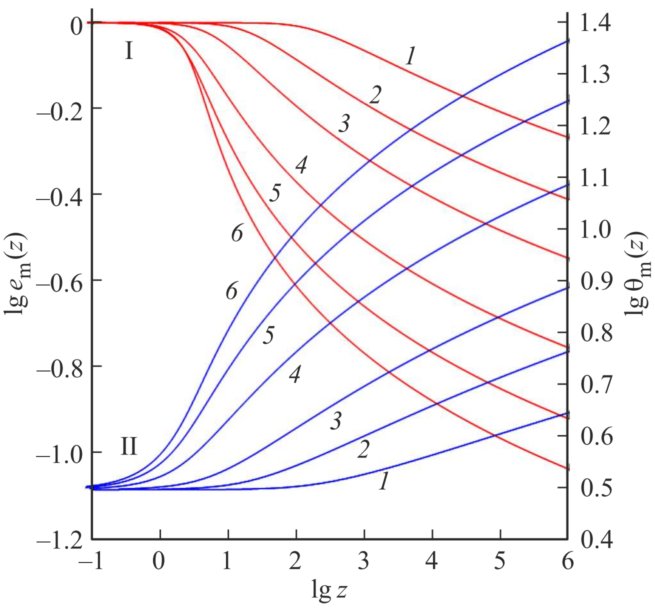

Figure 2. The dependences of amplitude $e_m(z)$ — (I) and duration of perturbation $\theta^*(z)$ — (II) on z at different values ε_0/β: line *1* — $\varepsilon_0/\beta = 1/10$, *2* — 1/7, *3* — 1/5, *4* — 1/3, *5* — 1/2, *6* — 2/3.

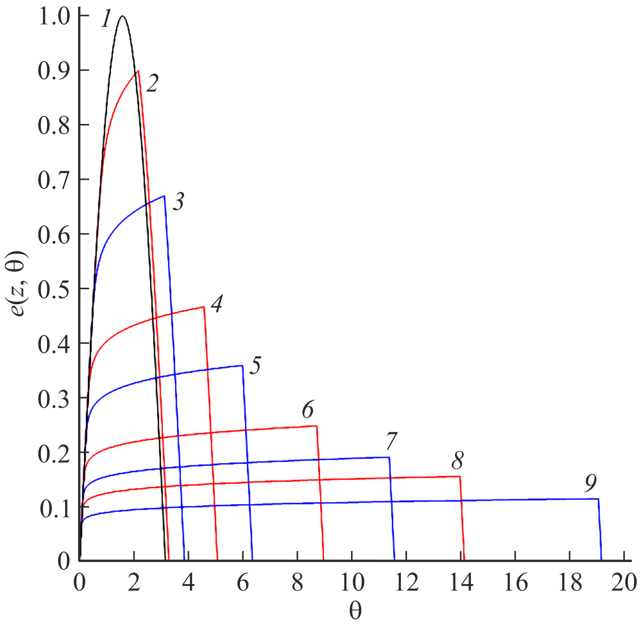

Figure 3. Evolution of the shape of perturbation $e = e(z,\theta)$ depending on ε_0/β at $z = 3\cdot10^4$, line *1* — $\varepsilon_0/\beta \le 3\cdot10^{-2}$, *2* — $\varepsilon_0/\beta = 7\cdot10^{-2}$, *3* — $\varepsilon_0/\beta = 10^{-1}$, *4* — $\varepsilon_0/\beta = 1.5\cdot10^{-1}$, *5* — $\varepsilon_0/\beta = 2\cdot10^{-1}$, *6* — $\varepsilon_0/\beta = 3\cdot10^{-1}$, *7* — $\varepsilon_0/\beta = 4\cdot10^{-1}$, *8* — $\varepsilon_0/\beta = 5\cdot10^{-1}$, *9* — $\varepsilon_0/\beta = 7\cdot10^{-1}$.

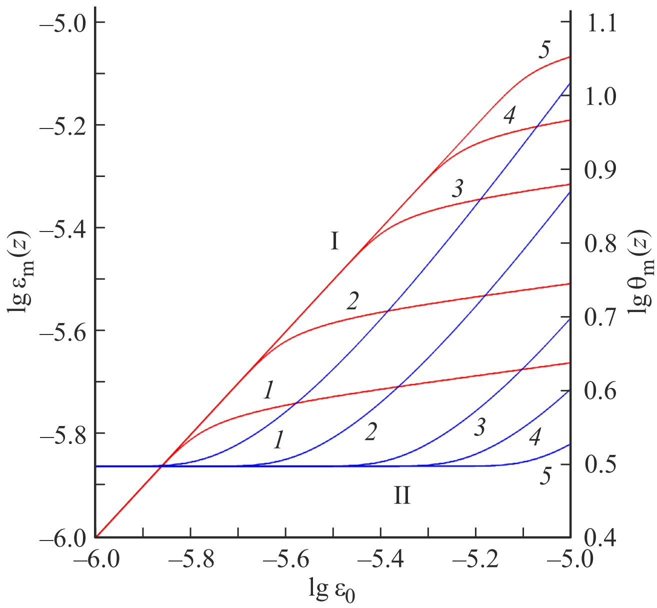

Figure 4. Dependences of amplitude $\varepsilon_m(z)$ — (I) and duration of disturbance θ^* — (II) on ε_0 at $z = 10^3$ and various values β: line *1* — $\beta = 2\cdot10^{-5}$, *2* — $\beta = 3\cdot10^{-5}$, *3* — $\beta = 5\cdot10^{-5}$, *4* — $\beta = 7\cdot10^{-5}$, *5* — $\beta = 10^{-4}$.

duration $\theta^*(z)$ at the beginning $\theta^*(z) = \text{const}$, and then — $\theta^*(z) \propto \varepsilon_0$.

4. Conclusion

It should be noted in conclusion that experiments on the propagation of unipolar disturbances were conducted in of [14,15], where the evolution of longitudinal compression pulses in rods made of unannealed and annealed polycrystalline aluminum was studied. The above-mentioned patterns were observed in these studies with an increase of the initial amplitude of the perturbation, namely, a change of the shape of the perturbation (from bell-shaped — close to (4) to trapezoidal) without twisting of the leading and trailing edges, as well as saturation of the amplitude of the perturbation and an increase in its duration. It was also found that the acoustic nonlinearity of polycrystalline aluminum increases with an increase of the annealing temperature, accompanied by an increase of grain size (and, accordingly, a decrease of the dislocation density [16]). Qualitative explanations of the observed effects in [14] were provided within the framework of elastic quadratic nonlinearity characteristic of homogeneous solids [1], and within the framework of phenomenological hysteresis in [15], for which $\varepsilon \neq 0$ and $f(\varepsilon, \dot{\varepsilon}) \neq 0$ with $\sigma = 0$. The latter corresponds to the fact that irreversible plastic strains occur after the passage of the first (and each subsequent) perturbation in a solid. As a result, after exposure to each subsequent disturbance, the hysteresis equation of state must change, which, apparently, can occur at high stresses exceeding the elastic limit of a solid, but not for acoustic disturbances of moderate amplitude. The developed theory of nonlinear propagation of unipolar pulsed disturbances within the framework of the Granato–Lucke dislocation hysteresis explains all the observed patterns without the occurrence of plastic strains in a crystalline solid, since $\varepsilon = 0$ and $f(\varepsilon, \dot{\varepsilon}) = 0$ at $\sigma = 0$.

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Conflict of interest

The authors declare that they have no conflict of interest.

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