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On the question of measuring the characteristic voltage of Josephson contacts

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A method is proposed for determining the characteristic voltage of Josephson contacts from the period of oscillations of current steps on the current-voltage characteristics of contacts in the field of microwave radiation, applicable to contacts of all types.

Keywords: Josephson effect, operating speed, resistive model, characteristic voltage of Josephson contact.

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1. Introduction

The Josephson contact (JC) is the main element of superconducting analog and digital electronics [1]. The JC's most important characteristic is its speed of switching. Proper switching time of the ideal tunnel JC $\tau_i \propto \hbar//2\Delta \approx \hbar/eV_c$ (Δ — the energy gap of the superconducting electrode of the JC, $V_C = I_C R_N$ — JC characteristic voltage, I_C — the critical contact current, R_N — the resistance in „the normal" state, $\hbar$ — the Planck's constant, e — the electron charge). This estimate $\tau_i \propto 1/V_C$ is used to determine the performance of any Josephson weak coupling [2]. Therefore, the critical voltage V_C is one of the main characteristics that determine the use of JC in analog and digital circuits. In some JC's, this value can be found from the current-voltage curve (CVC) of the contact, if the latter is approximated fairly accurately by a resistive-shunted model that takes into account contact capacitance and noise (RSJCN) [2,3]. In this model, the JC is represented as an ideal contact connected in parallel, through which only the current of pairs flows, normal resistance R_N and capacitance C, and a noise current is added to the bias current. The RSJCN model allowed describing the characteristics of many Josephson structures and is widely used in interpreting of the properties of JC [1,4]. Currently, the possibility of approximating the current-voltage curve (CVC) of the RSJCN JC model is considered as the basis for estimating V_C [2,3].

Numerous JC with types of weak links different from the insulator have been created and studied in the years since the discovery of the Josephson effect in the tunnel SIS (superconductor-insulator-superconductor) structure. New multi-band superconductors with different order parameters and types of conductivity are used as JC electrodes [5]. All this results in noticeable deviations of the CVC and other characteristics of the JC from those of the standard RSJCN model. The measurement of V_C becomes very relevant because of the predicted application of new structures and new superconductors for creation of JC [2].

The methods of measurement of V_C in JC with superconducting iron-based electrodes (Ferrum Base Superconductors — FBS) are compared in this paper by the current-voltage curve and by the oscillation period of current steps (Shapiro steps) in the field of high-frequency electromagnetic radiation based on our measurements.

2. Experimental procedure

High-quality single crystal plates of $FeSe_xTe_{1-x}$, $K_{1-x}Ba_xFe_2As_2$, KFe_2As_2 were used as JC FBS electrodes in our experiments, [6–9]. We studied the characteristics of two types of point contacts JCs: 1) contacts „ordinary superconductor" — FBS in the standard configuration „needle-anvil" and 2) point contacts on the microcrack (break-junction) in the FBS crystal. A description of the methods for creating JC and electronic devices used in the experimental setup is provided in [6–11].

The CVC series were recorded after the adjustment of the JC at a minimum temperature with an increase of the microwave radiation power P at the contact until signs of heating appeared, resulting in an increase of resistance.

The values I_C and R_N and $V_C^{\mathrm{CVC}} = I_C R_N$ [7–9] were found from the initial section of the CVC ($V \leq 100\,\mu$V) at $P = 0$ without using any model (Table 1). The dependences of the amplitudes of the current steps on the microwave irradiation power $i_n^{\exp}(\sqrt{P}) = I_n(\sqrt{P})/I_C(P=0)$ ($n = 0, 1, 2$ were recovered from CVC recorded at different P, step with $n = 0$ represents the critical contact current I_C).

The measured dependences $i_n^{\exp}(\sqrt{P})$ were compared with those calculated from the RSJ model $i_n^{\mathrm{calk}}(i_{\mathrm{ac}})$ (i_{ac} — alternating current in JC induced by microwave radiation, normalized to I_C) [7–9]. This made it possible to determine the normalized frequency of the microwave signal $\Omega = 2\pi f/\big((2e/\hbar)V_C^{\mathrm{MW}}\big)$ f — the frequency of external microwave radiation) and the characteristic voltage of the JC from the oscillations of the current steps $V_C^{\mathrm{MW}} = V_{n=1}/\Omega$

Table 1. JC characteristics [6-9]: $V_C^{\mathrm{CVC}} = I_C R_N$ — the characteristic voltage found from the CVC, Ω^{MW} — the normalized microwave frequency found from the oscillation period of the current steps, V_C^{MW} — the characteristic voltage calculated from the oscillation period steps of current in the microwave field

№	I_C, mA	R_N, mOhm	V_C^{CVC}, μV	Ω^{MW}	V_C^{MW}, μV	$V_C^{\mathrm{CVC}}/V_C^{\mathrm{MW}}$
1	2.82	16	45.1	0.15	105	43%
2	1.83	5.7	10.4	0.25	62.9	17%
3	1.39	5.5	7.65	0.33	47.6	16%
4	1.27	14	17.8	0.31	50.7	35%
5	1.05	46	48.3	0.22	71.5	68%
6	0.66	100	66	0.25	62.9	105%
7	0.51	330	167	0.16	98.3	171%
8	0.44	80	35.2	0.60	26.2	134%
9	1.45	53	76.9	0.095	166	46%
10	0.66	136	89.8	0.12	131	69%
11	0.95	53	50.4	0.34	46.2	109%
12	0.60	144	86.4	0.091	173	50%

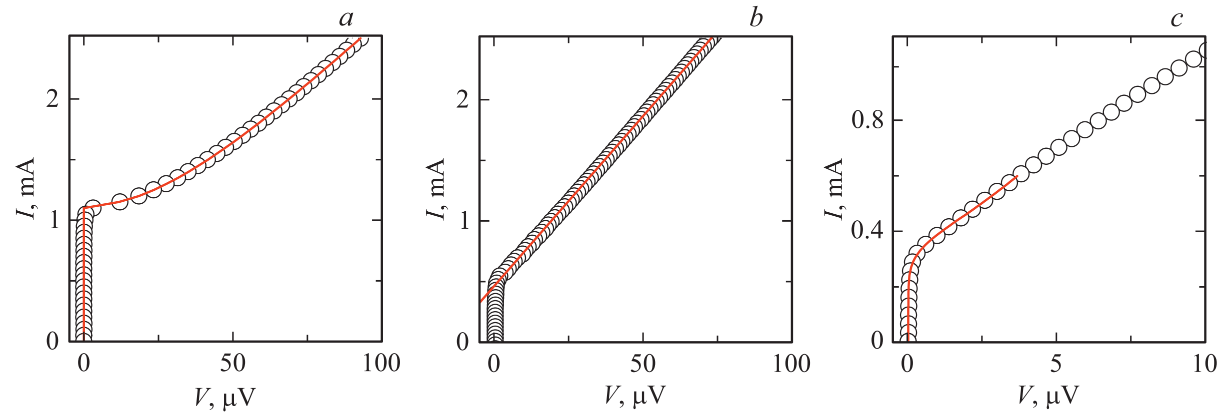

Figure 1. The main types of CVC that were observed when studying the characteristics of JC with ferropnictides. Symbols indicate experimental values, lines indicate approximation. a — the hyperbolic CVC, the initial section is well approximated by the RSJ model, b — the linear CVC, c — the CVC with I_C blurred noise, approximated by theoretical dependence [4], $I_C = 0.43$ mA, $R_N = 8.6$ mOhm, $V_C^{\mathrm{CVC}} = 3.7\,\mu$V, $\gamma = 15.1$ — the parameter characterizing thermal fluctuations.

($V_{n=1}$ — the voltage at which the JC CVC the first step of the current appeared) [7–9].

3. Measurement results

Figure 1, $a-c$ shows the main types of CVC that were observed in JC with FBS. Figure 1, $a-c$ also shows examples of approximation of the initial section of the CVC: a — hyperbola $V = R_N(I^2 - I_C^2)^{0.5}$ corresponding to the RSJ model [4]; b — a straight line; c — a theory that takes into account thermal noise [4].

The critical current JC I_C decreased when the microwave generator was turned on and current steps I_n appeared on CVC at voltages $V_n = 2\pi\hbar f n/2e$ $(n = 1, 2, \ldots)$ corresponding to the harmonics of the microwave signal frequency.

Figure 2 shows the behavior of the current steps on the CVC in the microwave field. It is clearly visible how the height (amplitude) of the steps changed when the power of the microwave signal changed. The height of the steps depended on the value of the normalized frequency of microwave radiation Ω and the degree of „blurring“ of the JC critical current (noise level).

The measured dependences of the amplitudes of the first normalized current steps on the CVC of one of the JC $i_n^{\exp}(k\sqrt{P}) = I_n(\sqrt{P})/I_C(P = 0)$ $(n = 0, 1, 2)$ are shown in Figure 3 symbols. The lines in this figure show the dependencies $i_n^{\mathrm{calk}}(i_{\mathrm{ac}})$ calculated from the RSJ equation (1) [7–9]. The coefficient $k = 3.57$ is selected based on the coincidence of the first minima $i_1^{\exp}(k\sqrt{P})$ and $i_1^{\mathrm{calk}}(i_{\mathrm{ac}})$.

4. Discussion

If the JC CVC can be approximated by the RSJCN model to the voltages $V \geq (3-5)\Delta/e$, then the measurement

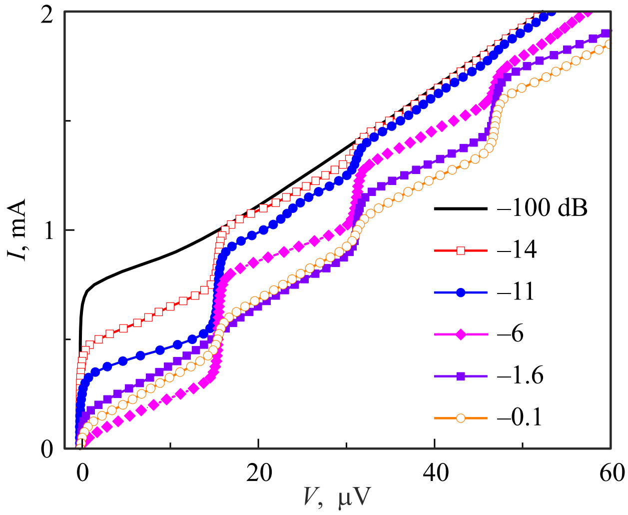

Figure 2. Several current-voltage characteristics of the Josephson contact recorded at different power levels of microwave radiation. The first step of the current $V_{n=1} = 15.7\,\mu$V.

$V_C = I_C R_N$ does not raise questions. I_C and R_N for such JC can be easily found from the calculated CVC approximating the measured one, or I_C can be determined by extrapolating the initial section of the CVC to the current axis, and the resistance in the normal state of R_N can be measured using the linear section of the CVC in the voltage range of $(3-5)\Delta/e$ or with the suppression of the superconductivity in JC electrodes by a magnetic field.

A method is proposed in [12] for determining the components V_C of values from the initial ($V \ll \Delta/e$) section of the CVC applicable to contacts whose CVC is described by a resistive model. At the same time, the RSJCN model with a number of additional complications is used for evaluation of I_C and R_N.

The characteristics of JC from new superconductors, as a rule, noticeably deviate from the standard RSJCN model: the shape of the CVC differs from the hyperbola, I_C is blurred, it is impossible to record the CVC at $V = (3-5)\Delta/e$ for measuring R_N using the standard method because of heating, the value V_C differ from the theory calculated by Ambegaokar, Baratoff. The authors of all the papers that studied JC with electrodes from new superconductors [5,6–9,13–15] evaluated I_C and R_N according to the initial section of the CVC in the region of voltages $V \leq 0.1\Delta/e$. This method did not allow finding the „correct" value V_C and, accordingly, Ω for an accurate approximation of the oscillation period of the measured dependencies $i_n^{\text{exp}}(k \times \sqrt{P})$ calculated [7–9]. Taking into account the capacitance C (as a parameter $\beta_C = (2e/\hbar)I_C R_N^2 C$), to some extent, made it possible to solve this problem [16], and also describe the dependence of the critical JC current on the direction of the bias current (hysteresis) [4]. The inclusion of the noise current i_N allowed describing the blurring of the CVC near I_C and the difference in the amplitudes of the measured current steps from the calculated ones.

According to the authors of [17], it is the possibility of approximation of the oscillations of the current steps on the JC CVC in the microwave field using the RSJ model, and not the shape of the CVC, that proves the applicability of this model to JC. The calculation of CVC in the microwave radiation field is reduced in the simplest RSJ model (without taking into account capacitance and noise) to solving the differential equation [4]:

$$d\varphi/d\tau = i + i_{\text{ac}} \sin \Omega\tau - \sin \varphi, \tag{1}$$

$$\Omega = 2\pi f \Big/ \left(\frac{2e}{\hbar}\right) I_C R_N = \frac{V_{n=1}}{V_C}, \;\; \tau = \left(\frac{2e}{\hbar} I_C R_N\right) t,$$

where φ — the phase difference of the order parameters in the JC electrodes, i and i_{ac} — direct current and current induced by microwave radiation, normalized to I_C, τ — normalized time. From the solution of this equation a constant voltage v, normalized to V_C, is determined. It is equal to

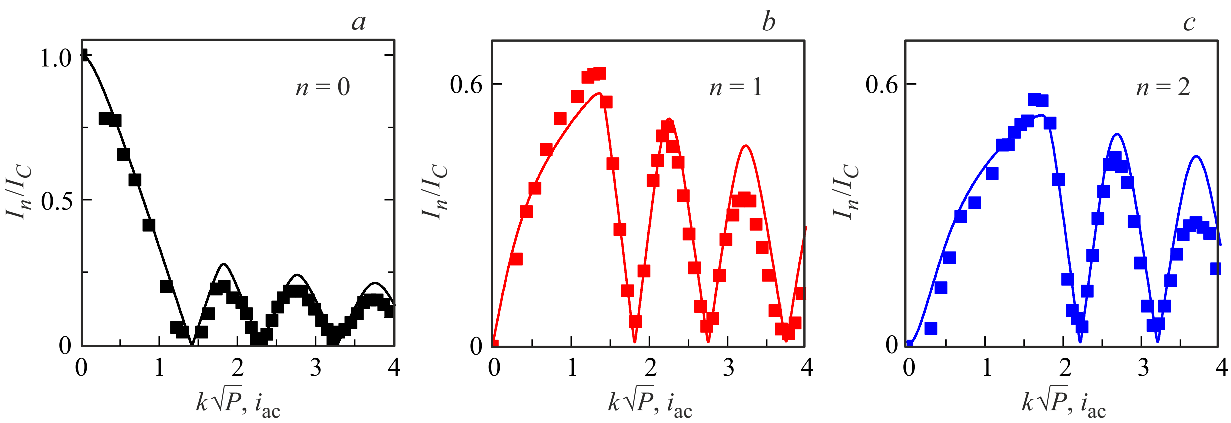

Figure 3. Oscillations of the first normalized steps of the current $i_n^{\text{exp}}(k \cdot \sqrt{P})$ $(n = 0, 1, 2)$ on the JC CVC in the field of microwave radiation with a frequency of $f = 7.6$ GHz. Symbols — measured dependencies $i_n^{\text{exp}}(k \cdot \sqrt{P})$, lines — calculated $i_n^{\text{calk}}(i_{\text{ac}})$. $k = 3.57$, $\eta_1 = 0.52$, $\Omega = 0.34$.

Table 2. Dependence coefficients (2) for current steps with $n = 0, 1, 2$

Coefficient	$n = 0$	$n = 1$	$n = 2$
b_0	-1.81558	-1.91211	-1.73247
b_1	6.80989	8.84761	7.4911
b_2	-19.0752	-32.89914	-51.13829
b_3	33.55267	80.86038	360.77082
b_4	-33.10355	-114.26291	-1533.93039
b_5	16.78838	83.25194	3597.00234
b_6	-3.3055	-22.3944	-4324.91152
b_7	–	–	2093.67881

the time-averaged phase oscillations $\nu(i) = \langle d\varphi/d\tau\rangle$, that is, the normalized CVC. Current steps appear on the CVC at $i_{\mathrm{ac}} > 0$ the amplitude of which oscillates from 0 to a maximum depending on i_{ac}.

We propose to evaluate V_C JC without reference to the type of weak coupling and the type of CVC, using the RSJ contact model in the microwave field (1). This method also allows verifying the applicability of the RSJ model (1) to JC. It is based on the determination of the normalized frequency of electromagnetic radiation Ω^{MW}, which should be found from the normalized oscillation period of one of the steps of the current at the JC CVC in the microwave field of $\eta_n = (i_n^{(2)} - i_n^{(1)})/i_n^{(1)}$ $(n = 0, 1, 2)$. $i_n^{(1)}$ and $i_n^{(2)}$ in this formula are the first and second minima on the dependence of n-th current step $i_n^{\exp}(\sqrt{P})$. We can easily find the characteristic voltage of contact $V_C^{\mathrm{MW}} = V_{n=1}/\Omega^{\mathrm{MW}}$ by defining Ω^{MW}.

The relationship η_n with Ω^{MW} for the first current steps of CVC, following from the RSJ model with dependence $I_S = I_C\sin(\varphi)$, was calculated in [17]. We also obtained the dependencies Ω^{MW} on η_n $(n = 0, 1, 2)$ shown in Figure 4 by numerically solving the equation (1) [7–9]. The symbols in Figure 4 indicate the results of our calculations, solid lines indicate the data approximation with an accuracy better than 1% in the range of values $\Omega^{\mathrm{MW}} = (0.07-1)$. The formula was used for the approximation

$$\log\Omega = \sum_{i=0}^{7} b_i(\eta_n)^i \tag{2}$$

with coefficients b_i, listed in Table 2.

It is necessary to find the normalized frequencies Ω_n^{MW} of the first three current steps $n = 0, 1, 2$ to verify the applicability of the RSJ model. The equality $\Omega_0^{\mathrm{MW}} = \Omega_1^{\mathrm{MW}} = \Omega_2^{\mathrm{MW}} = \Omega^{\mathrm{MW}}$ will confirm the applicability of this model to the studied JC. Or after finding Ω_1^{MW} it is necessary to solve the equation (1) using the data Ω_1^{MW}, find the dependencies $i_n^{\mathrm{calk}}(i_{\mathrm{ac}})$ for the current steps and compare with the measured $i_n^{\exp}(k\sqrt{P})$.

An example of approximation of the measured dependencies $i_n^{\exp}(k\sqrt{P})$ by calculated $i_n^{\mathrm{calk}}(i_{\mathrm{ac}})$ from the equation (1) is shown on Figure 3. The normalized oscillation period of

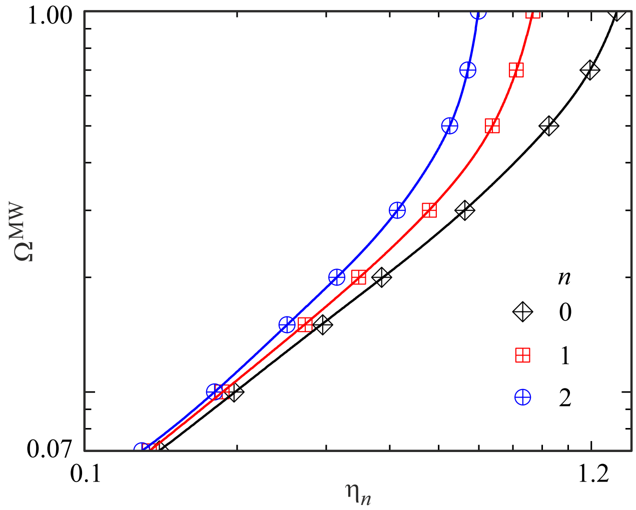

Figure 4. Dependences of the JC normalized frequency Ω^{MW} on the oscillation periods of the current steps η_n $(n = 0, 1, 2)$ within the RSJ model at $I_S = I_C\sin(\varphi)$. The values indicated by symbols are obtained from the solution of equation (1), lines show the approximation by the formula (2).

the first current step $\eta_1 = 0.52$. From dependence $\Omega_1^{\mathrm{MW}}(\eta_1)$ (Figure 4) $\Omega_1^{\mathrm{MW}} = 0.34$. Then the dependencies $i_n^{\mathrm{calk}}(i_{\mathrm{ac}})$ were constructed from the solution (1) and the coefficient $k = 3.57$ was selected, so that the first minima of the calculated $i_1^{\mathrm{calk}}(i_{\mathrm{ac}})$ and measured current steps $i_1^{\exp}(k\sqrt{P})$ coincided. The found k was also used as the scale factor of the abscissa axis for dependencies $i_0^{\exp}$ and $i_2^{\exp}$. The oscillation periods of the measured and calculated dependencies coincide for all three current steps, therefore, this JC is described by the RSJ model, $V_C^{\mathrm{MW}} = 15.7\,\mu\mathrm{V}/0.34 = 46\,\mu\mathrm{V}$. Similar results were obtained for all the JC [7–9] that we studied.

Only one basic characteristic of any JC is used in this method to determine V_C — Josephson superconducting current oscillations at contact voltage $V \neq 0$. The conformance of the shape of the CVC resistive model, measurements I_C, R_N, capacitance and noise current are not required. As practice has shown [7–9], this method works for JC with any type of weak links.

The advantages of this method are related to the fact that the method does not require the input of parameters that take into account the deviation of the CVC from the hyperbolic shape in the RSJ model: noise, capacitance, etc. The disadvantages of the method include: 1) complication of the measurement setup: equipment is needed for supplying and monitoring the power of the microwave signal in addition to the equipment for recording CVC; 2) limitations associated with heating of the JC by microwave radiation.

The critical contact current should be chosen, on the one hand, so that it is not too small for the CVC blur from noise to be small, on the other hand, so that it can be suppressed by microwave irradiation without noticeable heating of the contact. In our experiments, the critical JC current was in the range 0.5−1.5 mA. The frequency of the microwave radiation f should be such that the noise has a slight effect on determination of the position of the current steps, but also not too high, so that $\Omega < 1$. η_n ceases to depend on Ω at $\Omega > 1$, as follows from Figure 4. For Josephson contacts with V_C of the order of tens of microvolts, the microwave frequency should be several GHz.

Let's compare the results of determination of the characteristic voltage of the JC obtained by the proposed method and the standard method for the initial section of the CVC. Table 1 shows some characteristics of the studied JC [7–9] with ferropnictide electrodes. The values V_C^{CVC} found from the initial section of the CVC and from the oscillations of the current steps in the microwave field V_C^{MW} coincide when the CVC of the contact is accurately described by the RSJCN model, in all other cases $V_C^{\mathrm{CVC}} \neq V_C^{\mathrm{MW}}$. The ratio $V_C^{\mathrm{CVC}}/V_C^{\mathrm{MW}}$ can be either greater or less than 1 and is determined by the unknown dimensions and structure of the contact, the properties of the materials from the contact consists. We judge the properties of the contact by its CVC, response to microwave irradiation and the dependence of the critical current on temperature. Unfortunately, these data are not sufficient to make a conclusion about the ratio $V_C^{\mathrm{CVC}}/V_C^{\mathrm{MW}}$.

5. Conclusion

A method is proposed for determination of the characteristic voltage of the Josephson contacts V_C by the period of oscillations of the first stages of the current on the JC CVC in the microwave field, unrelated to the shape of the CVC, the type of weak coupling of the contact, taking into account all the characteristics affecting this value.

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Conflict of interest

The authors declare that they have no conflict of interest.

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