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Creep of vortices and magnetic flux percolation in high-temperature superconducting composites

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The effect of magnetic flux percolation on the creep resistance in superconducting composites containing normal-phase fractal clusters has been studied. An exact solution has been obtained for the voltage induced by the magnetic flux creep taking into account both forward and backward vortex hopping. It has been found that the Anderson−Kim creep resistance in a percolative superconductor exceeds the collective creep resistance at an equivalent height of the pinning barrier.

Keywords: creep, percolation, vortex, pinning, critical current.

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High critical currents can be achieved in superconductors by creating artificial pinning centers [1] whose role is played by various defects, including normal-phase clusters [2]. An example of superconducting composites containing normal-phase inclusions are 1G-wires based on high-temperature superconductors (HTSC) [3] fabricated using the OPIT (oxide powder in tube) technique, as well as 2G-wires fabricated using technique IBAD (ion beam assisted deposition) [4] and RABiTS (rolling-assisted biaxially textured substrates) [5].

HTSCs are type II superconductors having a high Ginzburg−Landau parameter because the depth of magnetic field penetration into them significantly exceeds the coherence length. Magnetic field initiates in such superconductors topological perturbations of the order parameter, namely vortices transporting the magnetic flux. In this paper, we consider a creep of vortices induced in the self-field mode. The magnetic flux motion caused by the creep of vortices results in the critical current dissipation and decrease. The problem of creep suppression is especially important for HTSCs because of high level of thermal fluctuations [6].

Superconducting composites are heterogeneous in structure; hence, they may exhibit percolation effects: supercurrent percolation through a superconducting cluster and magnetic flux percolation through a normal-phase cluster (Fig. 1). In the 2D case, the supercurrent and magnetic flux percolations exclude each other (Fig. 1, a); therefore, to prevent the magnetic flux transfer, it is sufficient to ensure an exceedance of the percolation threshold for the superconducting phase fraction: $\theta > \theta_c$. In the case of the 3D percolation, there exists a range of values of the superconducting phase fraction $\theta_c < \theta < 1 - \theta_c$ where percolation clusters of the superconducting and normal phases coexist (Fig. 1, b). In this case, the magnetic flux motion can be blocked if the superconducting percolation cluster is dense enough to satisfy condition $\theta > 1 - \theta_c$.

Here we consider a superconductor having inside it a superconducting percolation cluster that supports the transport current. Its cells contain normal-phase clusters capturing the magnetic flux. Vortices can move along weak links between the normal-phase clusters. Small coherence length promotes formation of weak links in HTSC [6]. Depending on their configuration, each normal-phase cluster has its own critical depinning current.

In work [2] it was revealed for the first time that normal-phase clusters with fractal boundaries affect the critical current and transport of vortices. The fractal cluster perimeter L and area A obey similarity relation

$$L^{1/D} \propto A^{1/2},$$

where D is the fractal dimensionality of the cluster boundaries [7]. Depinning of vortices via weak links randomly arranged along the fractal boundary gets reduced to the random walks problem for a boundary with discrete absorption points [8] and is described by depinning probability $F(i) = \exp(-Ci^{-2/D})$ equal to probability $F(i) = \Pr\{\forall i_j < i\}$ of that critical current i_j of the j-cluster does not exceed threshold value i, where $i \equiv I/I_c$ is the electric current normalized to the resistive junction current $I_c \equiv \underline{\alpha}(C\overline{A})^{-D/2}$, α is the normal-phase cluster form-factor, $\overline{A}$ is the average cluster area, constant $C \equiv \left((2+D)/2\right)^{2/D+1}$ is determined by fractal dimensionality D of the cluster boundaries.

The number of vortices transporting the magnetic flux due to creeping is characterized by pinning probability $W = 1 - F$ whose dependence on the transport current

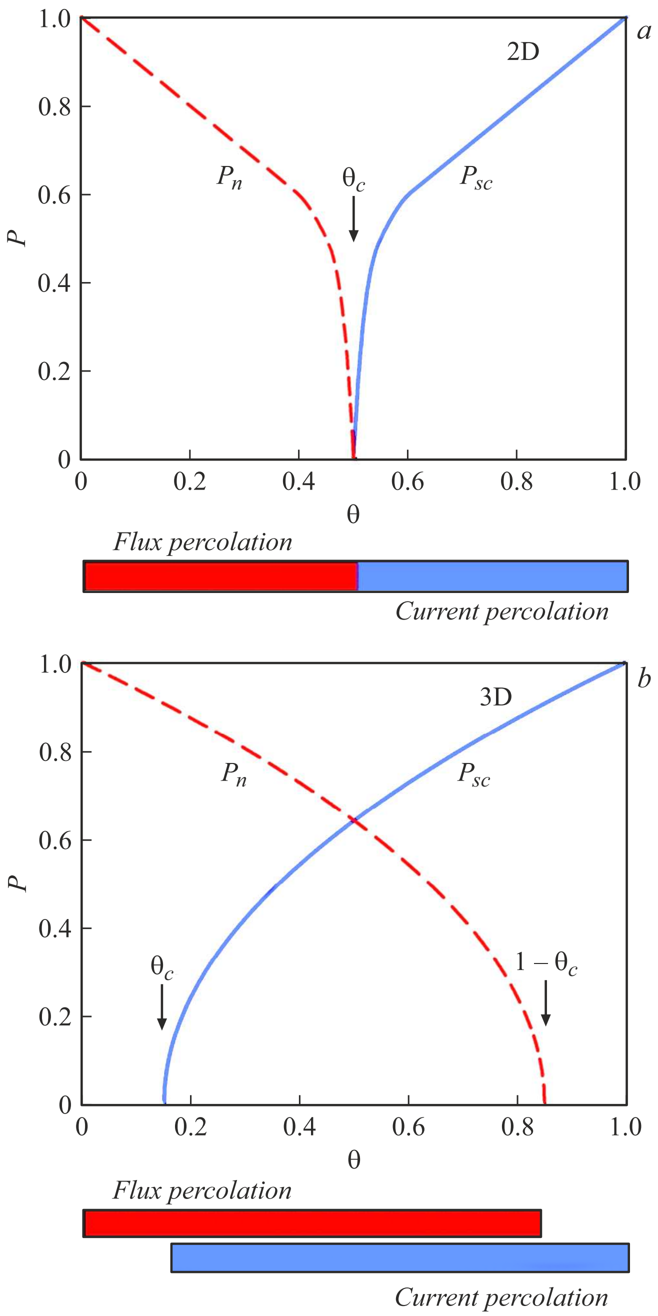

Figure 1. Isotropic percolation of the magnetic flux and superconducting current in the 2D (a) and 3D (b) composite superconductors. P_{sc} and P_n are the densities of percolation clusters of the superconducting and normal phases, respectively, θ is the superconducting phase fraction, and θ_c is the percolation threshold.

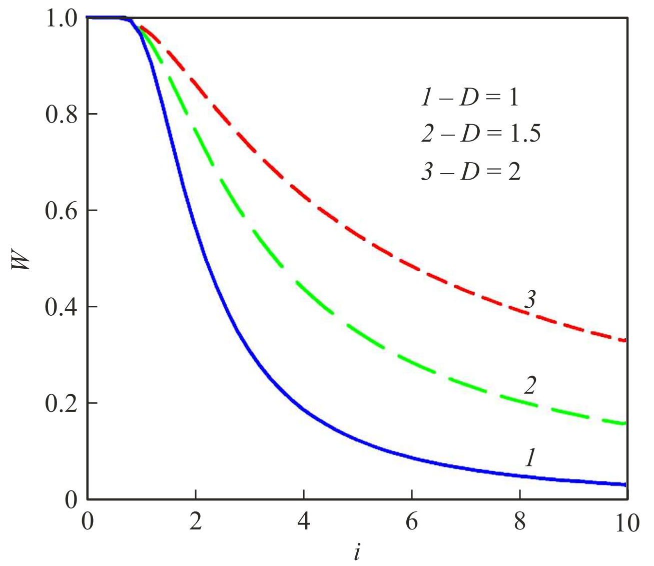

Figure 2. Probability of pinning on normal-phase clusters of different fractal dimensionalities versus the transport current. Value $D = 1$ corresponds to the limiting case of Euclidean clusters, $D = 2$ corresponds to clusters of the maximum fractal dimensionality, $D = 1.5$ is the intermediate value of the cluster boundary fractal dimensionality typical of the experimentally observed situation (analysis of electron micrographs of YBCO films gave $D = 1.44 \pm 0.02$ [2]).

is presented in Fig. 2. One can see that the pinning probability increases with increasing fractal dimensionality. If all pinning centers possess equal critical current i_c, then, when transport current i is passing, the creep induces on the superconductor voltage $v_{fc}(i, i_c)$. When the pinning barrier is linearly biased by transport current $U(i) \propto (1 - i/i_c)$ typical of the Anderson–Kim creep (AKC), this voltage has the following form provided both the direct and reverse (relative to the Lorentz driving force) vortex hops are taken into account:

$$v_{fc}(i, i_c) = R_{fc}\frac{i_c}{2u}\left(1 - \exp\left(-2u\frac{i}{i_c}\right)\right)\exp\left(u\left(\frac{i}{i_c} - 1\right)\right), \tag{1}$$

Here $u \equiv U_0/kT$ is the amplitude of the pinning barrier not biased by the transport current, R_{fc} is the creep resistance.

In the general case, the creep-induced voltage may be defined as an integral response to the transport current in the form of convolution of the depinning currents distribution $f(i_c) = dF(i_c)/di_c$ with kernel (1):

$$V_{fc} = \int_i^\infty v_{fc}(i, i_c) f(i_c) di_c = R_{fc}\frac{C}{2u}e^{-u}$$

$$\times \int_0^{i^{-2/D}} x^{-D/2}\left(1 - \exp(-2uix^{D/2})\right)\exp\left(uix^{D/2} - Cx\right)dx. \tag{2}$$

The magnetic flux transport by creeping begins at low (relative to the resistive junction current) transport currents. When $i \ll 1$, in the case of Euclidean clusters ($D = 1$), expression (2) for the voltage at the sample takes the

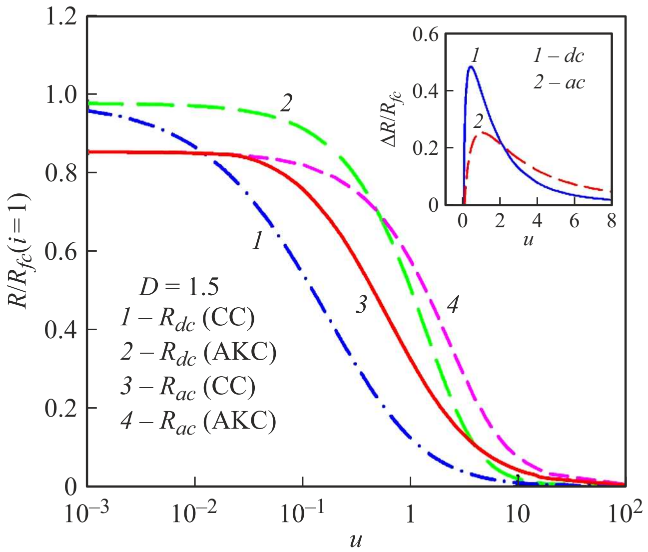

Figure 3. Resistances of the Anderson–Kim creep and collective creep in the resistive junction region for fractal dimensionality $D = 1.5$. The main panel presents the dependences of static resistance R_{dc}/R_{fc} (*1*, *2*) and differential resistance R_{ac}/R_{fc} (*3*, *4*) on the height of unbiased pinning barrier $u \equiv U_0/kT$ for CC (*1*, *3*) and AKC (*2*, *4*). The inset demonstrates the exceedance of the static (*1*) and differential (*2*) resistances of the Anderson–Kim creep over those of the collective creep: $\Delta R/R_{fc} = \big(R(\text{AKC}) - R(\text{CC})\big)/R_{fc}$.

following form:

$$V_{fc} = R_{fc} e^{-u} i \left(1 + \frac{ui}{\beta} \exp\left(\left(\frac{ui}{\beta}\right)^2\right) \times \left(\sqrt{\pi} + 2\frac{ui}{\beta}\, {}_1F_1\left(\frac{1}{2}; \frac{3}{2}; -\left(\frac{ui}{\beta}\right)^2\right)\right)\right),$$

where $\beta = 3(3/2)^{1/2}$, ${}_1F_1(a; b; z)$ is the Kummer degenerate hypergeometric function; accordingly, voltage for clusters of maximum fractal dimensionality ($D = 2$) is

$$V_{fc} = 4R_{fc} \exp(-2/i - u/2)\, {}_0F_1\big(; 3/2; (1/i - u/2)^2\big),$$

where ${}_0F_1(; b; z)$ is the generalized hypergeometric function.

For the collective creep (CC) with a transport-current-induced hyperbolic bias of the pinning barrier $U(i) \propto (i_c/i)^{\mu}$, where $\mu = 2/D$ is the glassiness index, the creep voltage for critical current i_c is

$$v_{fc}(i, i_c) = R_{fc} \frac{i_c}{2u} \left(1 - \exp\left(-2u \frac{i}{i_c}\right)\right) \times \exp\left(-u\left(\left(\frac{i_c}{i}\right)^{\mu} - 1\right)\right),$$

which allows defining the total voltage at the sample as

$$V_{fc} = R_{fc} \frac{C}{2u} e^{u} \int_0^{i^{-\mu}} x^{-1/\mu} \big(1 - \exp(-2uix^{1/\mu})\big) \times \exp\left(-\frac{u}{i^{\mu} x} - Cx\right) dx. \quad (3)$$

In the limit of low currents, expression (3) takes the form of

$$V_{fc} = 2R_{fc} \sqrt{Cu} e^{u} i^{1-\frac{\mu}{2}} K_1\left(2\sqrt{\frac{Cu}{i^{\mu}}}\right),$$

where $K_1(z)$ is the Macdonald function.

Figure 3 presents the dependences of static ($R_{dc} = V_{fc}/i$, curves *1*, *2*) and differential ($R_{ac} = dV_{fc}/di$, curves *3*, *4* /) creep resistances on the pinning barrier height in the region of resistive junction ($i = 1$). Apparently, the AKC resistance at the equivalent pinning barrier height exceeds the CC resistance up to almost complete creep suppression at $u > 20$. The Fig. 3 inset demonstrates the exceedance of AKC resistance over that in the CC mode: $\Delta R/R_{fc} = \big(R(\text{AKC}) - R(\text{CC})\big)/R_{fc}$. This result seems unexpected, since individual pinning causing AKC is typically stronger than the collective one [1], which would lead to a decrease in the vortex mobility and in resistance. The source of this effect may be as follows. In percolation superconductors there is no vortex lattice in the strict sense of translational symmetry; instead of it there are only scattered fragments of a distorted, defective and partially amorphized lattice. Fractal normal-phase clusters are centers of frozen disorder and contribute to amorphization of the vortex lattice. This promotes formation of the vortex structure correlation regions whose capturing in the case of collective pinning reduces the vortex system energy without increasing its elastic energy. Therefore, mobility of collective-creep vortices decreases, thus making the creep resistance lower than that of the Anderson–Kim creep.

Fig. 3 also shows that differential resistance of the Anderson–Kim creep exceeds the static one IFX57XE at the pinning barrier height IFX58XE which coincides in the order of magnitude with the latent melting heat of the vortex lattice per vortex (IFX59XE for YBAIFX60XECUIFX61XEO Runx62xe (YBCO) [9]). The exceedance of static differential resistance over the static one stems from the pinning potential asymmetry caused by a constant subcritical bias current. When $U_0 < 0.5kT$, thermal excitation energy dominates over the pinning energy and, moreover, over the asymmetric component of the pinning potential, and the creep resistance ratio gets reversed: $R_{ac} < R_{dc}$. In the case of collective creep, this limiting value of the pinning barrier height shifts towards lower energies: $R_{ac} > R_{dc}$ at $U_0 > 0.01kT$. This stems from the fact that in collective pinning not a single vortex is captured but vortex bundles within the vortex-structure correlation volume. Therefore, instead of being distributed over a single vortex, energy is distributed over the entire correlation region.

The use of normal-phase fractal clusters as pinning centers provides an extra opportunity to increase the critical current, since inhomogeneities of such fractal-dimensional objects cover a wide range of geometric sizes (up to the vortex core diameter), which ensures efficient pinning. The current-carrying capacity may be enhanced by creating a pinning landscape that provides simultaneous suppression of creeps in the AKC and CC modes. AKC may be suppressed by using correlated defects like clusters of columnar defects; to suppress CC, there may be used randomly arranged particle tracks emerging under ion bombardment which enables formation of different-dimensional defects with controllable density and morphology [10]. For creating such a combined pinning landscape, promising is technique PLD (pulsed laser deposition) [11], as well as MOCVD (metal-organic chemical vapor deposition) [12] in combination with ion bombardment [13].

Conflict of interests

The author declares that he has no conflict of interests.

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