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Extended method of cross-correlation analysis of non-stationary processes

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The paper proposes an extension of the method of cross-correlation analysis of the dynamics of systems with time-varying characteristics, which implies accounting for differences in characteristics of individual segments. The extended method was tested on the example of two-channel recordings of signals of electrical activity of the brain in different functional states.

Keywords: cross-correlation analysis, random process, scaling.

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Existence of cross-correlations is typical of the dynamics of complex systems containing components with different types of links [1,2]. Their quantitative description based on recorded signals allows characterizing specific features of such systems' dynamics and reaviling variations associated with enhancement or reduction of interactions. For this purpose, calculation of the cross-correlation function is typically used, but its estimation needs assuming the dynamics to be stationary. If this assumption is incorrect, it is necessary to transform the processes under analysis to stationary ones through various techniques at the stage of data preprocessing, to use special methods implying elimination of low-frequency variations (trend) in the process of calculation, or to apply these two options simultaneously; the latter is, as a rule, preferable.

At present various methods of cross-correlation analysis of the dynamics of systems with time-varying characteristics are used [3–5]. Among them we can distinguish the DCCA (detrended cross-correlation analysis) method [6,7] which is the fluctuation analysis, namely the DFA (detrended fluctuation analysis) method [8,9], generalized for the case of two simultaneously recorded signals. Works [10,11] proposed to supplement the DFA method with statistical analysis of the signal profile standard deviations from the trend for different segments in order to take into account time variations in characteristics of non-stationary behavior. The proposed modification differs from the conventional approach in that it involves calculation of an extra scaling exponent describing specific features of nonstationarity [12]. Let us discuss the possibility of applying a similar idea to the DCCA method.

In accordance with the calculation algorithm [6], cross-correlation analysis of two time series x_i and $\tilde{x}_i$, $i = 1, \ldots, N$ implies construction of their profiles

$$y_k = \sum_{i=1}^{k} x_i, \quad \tilde{y}_k = \sum_{i=1}^{k} \tilde{x}_i, \quad k = 1, \ldots, N. \qquad (1)$$

Each profile is divided into $M = [(N - n)/\Delta] + 1$ overlapping (in the general case) segments n in length (with overlapping by Δ counts); within individual segments, linear approximation of local trends z_k and $\tilde{z}_k$ is performed. Cross-correlations of detrended signal profiles are first assessed for each segment $j = 1, \ldots, M$:

$$f^2_{\mathrm{DCCA}}(n, j) = \frac{1}{n} \sum_{k=1+(j-1)\Delta}^{(j-1)\Delta+n} (y_k - z_k)(\tilde{y}_k - \tilde{z}_k), \qquad (2)$$

and then are averaged over all the segments:

$$F^2_{\mathrm{DCCA}}(n) = \frac{1}{M} \sum_{j=1}^{M} f^2_{\mathrm{DCCA}}(n, j). \qquad (3)$$

When long-range correlations of signals x_i and $\tilde{x}_i$ take place, the existence of a power-law dependence in the following form is expected:

$$F_{\mathrm{DCCA}}(n) \sim n^{\lambda}. \qquad (4)$$

The general idea of the extended DCCA method is to take into account the differences between cross-correlations of individual segments (2), which may be significant in non-stationary processes. For instance, paper [11] describes the cases when some segments make a predominant contribution, while the effect of other ones appears to be small. The DCCA method does not account for the signal structure heterogeneity causing significant distribution of f_{DCCA} values versus j. Let us consider, along with the mean value of F_{DCCA}, standard deviation σ of the f_{DCCA} values as a function of the segment length n. In the case of the power-law dependence

$$\sigma[f_{\mathrm{DCCA}}](n) \sim n^{\mu} \qquad (5)$$

scalin exponent μ reflects information on specific features of nonstationarity in the dynamics of the system under study.

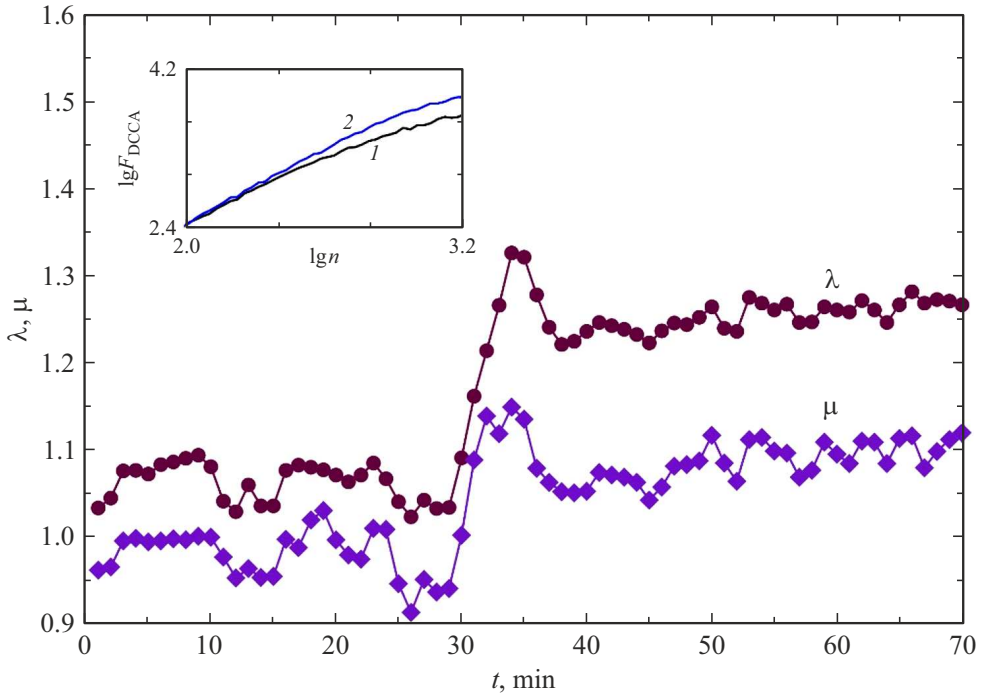

Figure 1. Anesthesia-induced variations in scalin exponents in the conventional (λ) and extended (μ) DCCA methods calculated from two-channel electrocorticography signals for a typical recording. The inset demonstrates characteristic double-logarithmic-scale dependences (4) before (*1*) and after (*2*) anesthesia.

To test the extended DCCA method on non-stationary processes of a complex structure, take as an example two-channel electrical activity recordings (electrocorticograms) of mice brains in two different states: before and after administration of anesthesia in the dose recommended for surgery. Signals were recorded from the left and right hemispheres by using implanted electrodes. Anesthesia-induced variations in electrical activity cause changes in the scaling exponents in both the conventional and extended DCCA methods (Fig. 1). An increase in λ and μ occurs immediately after the anesthesia administration; upon completion of the transient process, the scaling exponents get stabilized at values higher than in the awake state. Double-logarithmic-scale dependences (4) for the signals 10 min prior to and 10 min after anesthesia administration are shown in the inset to Fig. 1. Dependences (5) look similar. Scaling exponents were calculated in the $2.0 \leqslant \lg n \leqslant 3.2$ range where differences in the slopes of double-logarithmic-scale dependences (4) and (5) are most pronounced. At the same time, a nonlinear character of these dependences should be noticed, which provides differences in local exponents. Thereat, the conventional DCCA method reveals the most pronounced differences in the vicinity of $\lg n = 2.5$, while the extended method manifests them in the vicinity of $\lg n = 2.0$ and at $\lg n > 3.0$. This emphasizes that the approaches being applied focus on various distinctive features of the analyzed signals' structure and can complement each other in diagnostic being applied and, hence, reveal differences in experimental data over a wide range of scales.

Let us now turn to the results of statistical analysis for a group of laboratory animals, including eight mice. Figure 2 presents in the double logarithmic scale group-averaged dependences (4) and (5) for electrocorticograms recorded 10 min before (curve *1*) and 10 min after (curve *2*) anesthesia administration. Such dependences in the DCCA method are similar to those presented in the inset to Fig. 1. Visual consideration allows revealing differences in the slopes in the range of high $\lg n$ (Fig. 2, *a*). In the extended method, the differences in slopes are visually less noticeable (Fig. 2, *b*) and can be revealed only by calculations. At the same time, one can notice that anesthesia causes a shift of the dependences themselves, which can serve as an additional diagnostic criterion. Estimates of local slopes of averaged dependences obtained in the window on the $\lg n$ axis 0.6 long allowed revealing maximal differences in the vicinity of $\lg n = 2.7$ in the DCCA method and in the vicinity of $\lg n = 1.9$ in the extended method. This confirms the conclusions made for a typical record (Fig. 1) about complementarity of the two approaches and the fact that the extended DCCA method better reveals variations in the signal structure. In this work, electrocorticogram signals were selected as a challenging example for testing the extended method applicable to processes of various natures in the dynamics of systems with time-varying characteristics.

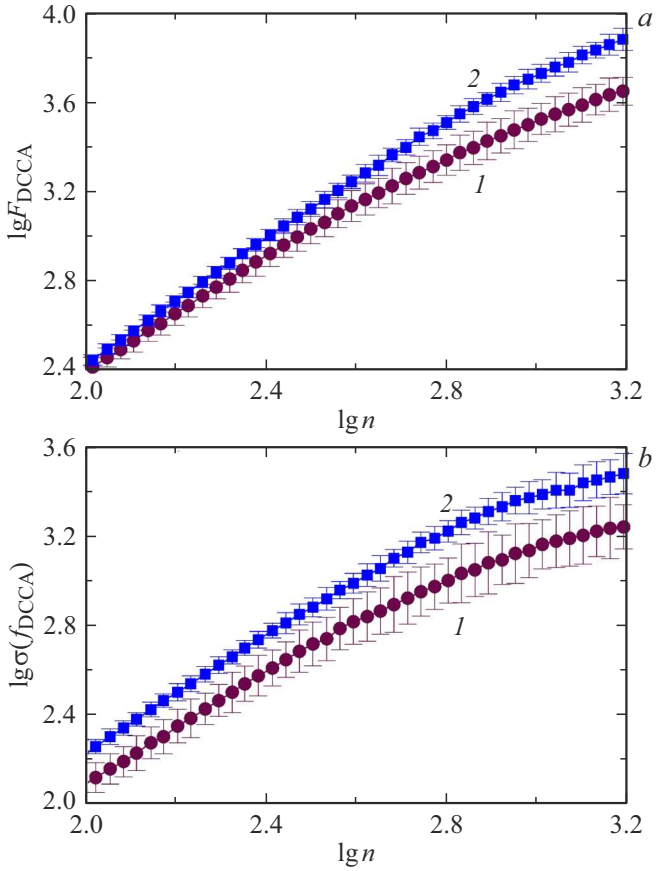

Figure 2. Double-logarithmic-scale dependences $F_{\mathrm{DCCA}}(n)$ (*a*) and $\sigma[f_{\mathrm{DCCA}}](n)$ (*b*) averaged over a group of mice. Mean values and standard deviations are indicated.

The paper does not contain information on any studies involving laboratory animals.

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Conflict of interests

The author declares that he has no conflict of interests.

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